PROCEEDINGS OF SCIENCE

Low-cost GPS/SINS Integrated Navigation System by Using MEKF to Estimate the Attitude for Land-Vehicles

Haifeng Liu[1]
School of Mechatronics Engineering and Automation, Shanghai University
Shanghai, 200072, China
E-mail: LiuHaiFeng_Arvin@163.com

Tongyue Gao
School of Mechatronics Engineering and Automation, Shanghai University
Shanghai, 200072, China
E-mail: gty@shu.edu.cn

Heding Shi
School of Mechatronics Engineering and Automation, Shanghai University
Shanghai, 200072, China
E-mail: shdxs1990@163.com

Based on the complementary feature of GPS and SINS, a low-cost, high precision and robust navigation system for land-vehicles can be designed by integrating them effectively. It is a hot topic in the field of the navigation system for land-vehicles. Many researchs show that there are two key points to design this GPS/SINS integrated navigation system and some problems remain to be resolved, such as the problem that the system can't position accurately when the car turns sharply. The first issue is to estimate the attitude matrix of the body coordinate frame relative to the navigation coordinate frame, and the second one is to integrate the data of GPS with SINS. In order to estimate the attitude matrix and deal with the problem that the system can't position accurately when the car turns sharply, the MEKF algorithm is introduced in our system. EKF is used to fuse the data of GPS and SINS so as to improve precision and reliability of the system. Effectiveness of this method is verified by experiments conducted in this paper.

ISCC2015
18-19, December, 2015
Guangzhou, China

[1]Speaker and Correspongding Author

PoS(ISCC2015)060

1. Introduction

The concept of the position and navigation system for land-vehicles began in the 1960s, but it is until the 1990s that it entered the practical stage with the rapid development of the computer and communications technology. Nowadays, the most widely used methods of navigation for land-vehicles are GPS (Global Position System), SINS (Strapdown Inertial Navigation System), MM (Map Matching) and so on. All of these methods have their advantages and disadvantages. Among them, GPS and SINS have complementary features: first, SINS's errors will accumulate over time and thus decrease the location accuracy of navigation system, while GPS's location accuracy is fairly stable because its error is timeless; second, SINS can output continuous signals even if there is a shelter upon it, while GPS can not; third, it will take a long time for SINS to output accurate navigation massage after it starts, while GPS can provide this promptly; fourth, a high precision SINS device is expensive, however, a GPS receiver device is cheap [1-2]. Based on their complementary features , a low-cost, high precision and robust navigation system for land-vehicles can be designed by integrating GPS with SINS effectively. There are many useful attempts and studies on this integrated navigation system.

Pavel Davidson et al. studied the integrated navigation system for land-vehicles, combined with GPS and low-cost inertial sensors in the urban environment, and found out that the use of inertial sensors can improve accuracy of the system significantly in the case that the GPS signal is not available at short term [3]. L. Zhao's research results showed that excellent navigation performance can be obtained in most situations by using EKF (Extended Kalman Filter) in the low-cost GPS/DR integrated navigation system for cars, in addition to the case that the car turns sharply [4]. Sameh Nassar et al. proposed a two-filter smoothing algorithm to deal with problems that the signal of GPS will be lost frequently in an urban environment and errors of INS will accumulate rapidly when it is used independently. The experiment they conducted indicated that the two-filter smoothing algorithm is workable [5]. Besides, Haihong Zhang and her colleages proposed an adaptive Kalman Filter algorithm based on different measurement characteristics of GPS and INS, and simulation results showed that it can overcome the filter divergence problem underlying the Sage-Husa adaptive filter algorithm, especially when the low-precision INS device is used [6]. These research results showed that there are two key points to design this GPS/SINS integrated navigation system. The first one is to estimate the attitude matrix of the body coordinate frame relative to the navigation coordinate frame. The second one is to integrate data of GPS and SINS. The problem that the system can't position accurately when the car turns sharply needs to be resolved as well.

In order to deal with the first problem, there are two algorithms for choice: AEKF (Additive Extended Kalman Filter) and MEKF (Multiplicative Extended Kalman Filter). The essential difference between AEKF and MEKF is whether the unit norm constraint of the quaternion is met or not when the quaternion error is corrected. Shuster noted that if we apply the constraint to AEKF properly, it will be equivalent to MEKF, but this process needs more calculation [7]. Therefore, the MEKF algorithm is used in this paper to deal with the first problem. As for the data fusion of GPS and SINS, KF (Kalman Filter), EKF (Extended Kalman Filter) and UKF (Unscented Kalman Filter) are often used. Among them, the KF algorithm requires that the system's mathematical model must be linear, while the system we often use in engineering is non-linear. Besides, UKF is a good choice for the non-linear system, but its advantage can be represented only when the system is highly non-linear and it is very difficult to design and implement [8]. Considering the methods mentioned above, EKF is used to integrate data of GPS and SINS in our navigation system.

In section 2, the MEKF algorithm and the structure of the navigation system we designed in this paper will be represented briefly. In section 3, we show how to estimate the attitude matrix of the body coordinate frame relative to the navigation coordinate frame based on MEKF. In section 4, the filter model of the system will be presented. At last, experiments are conducted in order to verify the effectiveness of the method proposed in this paper.

2. MEKF Representation

The basic idea of MEKF is to estimate the three components' attitude error parameter and use the quaternion multiplication as a carrier to provide the global non-singular gesture description [9]. Several parameters can be used to describe the attitude error, such as the infinitesimal rotation vector, the double vector part of the quaternion, the double Gibbs vector, and so on [10]. However, we use the Euler attitude error vector to describe the attitude error parameter in this paper, and establish the Euler attitude error quaternion and multiplicative formula according to it. The use of this algorithm can solve the problem effectively that the system can't position accurately when the car turns sharply.

(1) Establish the Euler attitude error vector based on the measured and predicted value of the vector. Take $\vec{X}_m$ and $\vec{X}_p$ as the measured and the predicted value of the vector $\vec{X}$ respectively, and establish the unit error vector (error-axis) and the error angle (in radians):

$$\vec{E}=\frac{\vec{X}_m}{\|\vec{X}_m\|}\times\frac{\vec{X}_p}{\|\vec{X}_p\|} \tag{2.1}$$

$$\Theta=\arccos\left(\frac{\vec{X}_m}{\|\vec{X}_m\|}\cdot\frac{\vec{X}_p}{\|\vec{X}_p\|}\right) \tag{2.2}$$

Euler attitude error vector is defined as the product of the error angle and unit error vector:

$$\vec{a}=\Theta\,\vec{E} \tag{2.3}$$

(2) Establish normalized Euler attitude error quaternion $\delta\vec{q}(\vec{a})$ by using the Euler attitude error vector, i.e., project the attitude error message to the attitude error quaternion:

$$\delta\vec{q}(\vec{a})=\begin{bmatrix}\cos\left(\frac{\|\vec{a}\|}{2}\right)\\ \left(\frac{\|\vec{a}_x\|}{\|\vec{a}\|}\right)\sin\left(\frac{\|\vec{a}\|}{2}\right)\\ \left(\frac{\|\vec{a}_y\|}{\|\vec{a}\|}\right)\sin\left(\frac{\|\vec{a}\|}{2}\right)\\ \left(\frac{\|\vec{a}_z\|}{\|\vec{a}\|}\right)\sin\left(\frac{\|\vec{a}\|}{2}\right)\end{bmatrix} \tag{2.4}$$

(3) Multiplicative formula:

$$\vec{q}=\delta\vec{q}(\vec{a})\otimes\hat{\vec{q}} \tag{2.5}$$

According to the definition of the Euler attitude error vector, we know that the Euler attitude error quaternion $\delta\vec{q}(\vec{a})$ is the error quaternion between the actual quaternion $\vec{q}$ and the predicted quaternion $\hat{\vec{q}}$. Therefore, the predicted quaternion $\hat{\vec{q}}$ can be corrected by using the attitude error message through this formula. Accurate attitude information can be obtained eventually.

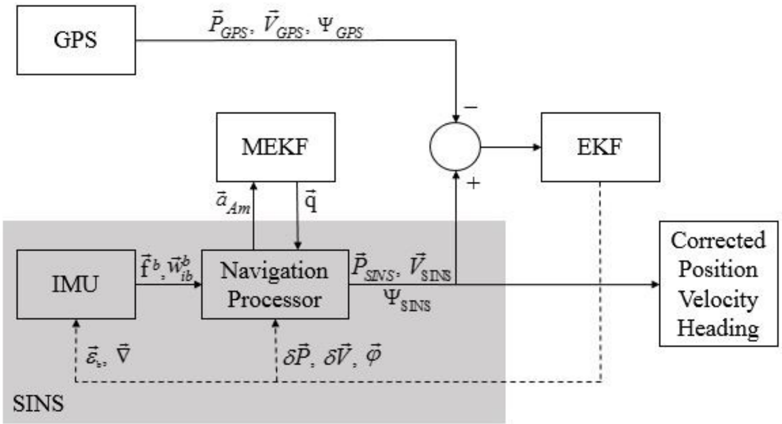

Fig. 1: the Structure of GPS/SINS Integrated Navigation System for Land-vehicles

The structure of GPS/SINS integrated navigation system for land-vehicles designed in this paper is shown in Fig. 1. There are two filters, MEKF and EKF, in the system. At first, we solve the attitude quaternion $\vec{q}$ by using the MEKF algorithm. Second, we take the position, velocity and heading differences of SINS and GPS (two antennas) as the measurement vector and take the error of SINS as the state vector to design the EKF filter. Third, SINS is corrected based on the output value of EKF, i.e., the error estimated value of SINS. So we can get the corrected value of the position, velocity and heading at last. In this way, a low-cost, high precision and robust GPS/SINS integrated navigation system for land-vehicles can be achieved.

3. Implementation of MEKF

In this section, the attitude matrix of the body coordinate relative to the navigation coordinate (the local E-N-U frame), denoted by $\boldsymbol{C}_b^n$, should be calculated. At the first place, the attitude quaternion of the body coordinate frame relative to the inertial coordinate frame should be calculated and used to solve the corresponding rotation matrix, namely $\boldsymbol{C}_b^i$. In the second place, $\boldsymbol{C}_b^n$ can be solved after the transformation of the matrix.

(1) the MEKF state equation

The linearized dynamic model of the attitude error vector defined in the body frame, is presented as follows [11]:

$$\dot{\vec{a}}=-(\vec{w}\times)\vec{a}+\vec{w}(t) \tag{3.1}$$

where $\vec{w}$ is the angular velocity vector of the body coordinate frame relative to the inertial coordinate frame, namely $\vec{w}^b$ measured by the gyroscope. $\vec{w}(t)$ is the noise sequence of the system, an independent and zero mean Gaussian white noise sequence. Take the

Euler attitude error vector $\vec{a}$ as the state vector of the MEKF filter, then the noise matrix of the system, denoted as $\boldsymbol{G}$, is a unit matrix, and we can get the updated matrix of the state:

$$\boldsymbol{F}=\frac{\partial f(\vec{x},\vec{u},t)}{\partial \vec{x}}=-(\vec{w}\times) \tag{3.2}$$

where $(w\times)$ is a skew-symmetric matrix, and its elements are elements of $\vec{w}^b$:

$$(\vec{w}\times)=\begin{bmatrix} 0 & -w_z^b & w_y^b \\ w_z^b & 0 & -w_x^b \\ -w_y^b & w_x^b & 0 \end{bmatrix} \tag{3.3}$$

(2) the MEKF measurement equation

$$\vec{z}=\boldsymbol{H}\vec{a}+\vec{v}(t) \tag{3.4}$$

where the output matrix $\boldsymbol{H}$ of the measurement is a 3x3 unit matrix, and the noise sequence $\vec{v}(t)$ of the measurement is an independent and zero mean Gaussian white noise sequence.

(3) Time update

After the predicted attitude quaternion is corrected by the error message of the attitude in every measurement update, it can be considered that the attitude quaternion corrected is accurate, i.e., its value of error is zero. Therefore, we reset the Euler attitude error vector $\hat{\vec{a}}_{k-1}$ at zero after each measurement update. In this way, the discrete updated Euler integral equation can be written as:

$$\hat{\vec{a}}_{k/k-1}=\hat{\vec{a}}_{k-1}+Tf[\hat{\vec{a}}_{k-1},t_{k-1}]=\vec{0} \tag{3.5}$$

$$\boldsymbol{P}_{k/k-1}=\boldsymbol{P}_{k-1}+T[\boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1}+\boldsymbol{P}_{k-1}\boldsymbol{F}_{k-1}^T+\boldsymbol{Q}_{k-1}-\boldsymbol{P}_{k-1}\boldsymbol{R}_{k-1}^{-1}\boldsymbol{P}_{k-1}] \tag{3.6}$$

where T is the step-length of integration; the subscript k-1 represents the previous time, and k/k-1 represents the time update from the previous time k-1 to the present time k. Concurrent with the propagation of the state and the covariance matrix, we also propagate the attitude quaternion by using the Euler integral equation:

$$\hat{\vec{q}}_{k/k-1}=\hat{\vec{q}}_{k-1}+\frac{T}{2}\left(\begin{bmatrix} 0 \\ \vec{w}_k \end{bmatrix}\otimes\hat{\vec{q}}_{k-1}\right) \tag{3.7}$$

The normalization constraint must be conducted after the time transfer of the attitude quaternion is completed to meet the unit-norm requirement of the multiplicative formula:

$$\hat{\vec{q}}_{k/k-1}=\frac{\hat{\vec{q}}_{k/k-1}}{\|\hat{\vec{q}}_{k/k-1}\|} \tag{3.8}$$

(4) Measurement update

The measurement of the gravity vector $\vec{g}$ is conducted through the accelerometer. The z axis of the inertial frame can be transferred to the body frame through the rotation matrix $\boldsymbol{R}(\vec{q}_{k-1})$. It is solved based on the attitude quaternion $\vec{q}_{k-1}$ obtained in the previous measurement update.

$$\vec{z}_i^b=\boldsymbol{R}(\hat{q}_{k-1})\vec{z}_i \tag{3.9}$$

According to the equation (2.1) and (2.2), the attitude error axis and error angle based on the accelerometer can be obtained:

$$\vec{E}_A = \frac{\vec{g}}{\|\vec{g}\|} \times \vec{z}_i^b \tag{3.10}$$

$$\Theta_A = \arccos\left(\frac{\vec{g}}{\|\vec{g}\|} \cdot \vec{z}_i^b\right) \tag{3.11}$$

The measurement value $\vec{a}_{Am}$ of the Euler attitude error vector can be acquired by putting the equation (3.10) and (3.11) into the equation (2.3). The measurement output matrix $\boldsymbol{H}$ is an unit matrix, and the equation to calculate the gain of Kalman filter is presented as follows:

$$\boldsymbol{K}_{\boldsymbol{k}} = \boldsymbol{P}_{\boldsymbol{k/k-1}} [\boldsymbol{P}_{\boldsymbol{k/k-1}} + \boldsymbol{R}_{\boldsymbol{k}}]^{-1} \tag{3.12}$$

where $\boldsymbol{R}$ is the covariance matrix of the measurement white noise. The updated state will be conducted subsequently. According to the equation (3.5), it is known that:

$$\hat{\vec{a}}_{Ak} = \hat{\vec{a}}_{k/k-1} + \boldsymbol{K}_{\boldsymbol{k}} [\vec{a}_{Am} - \boldsymbol{H} \hat{\vec{a}}_{k/k-1}] = \boldsymbol{K}_{\boldsymbol{k}} \vec{a}_{Am} \tag{3.13}$$

Calculate the newly predicted covariance matrix:

$$\boldsymbol{P}_{\boldsymbol{k}} = \boldsymbol{P}_{\boldsymbol{k-1}} - \boldsymbol{P}_{\boldsymbol{k-1}}^{\boldsymbol{T}} [\boldsymbol{P}_{\boldsymbol{k-1}} + \boldsymbol{R}_{\boldsymbol{k}}]^{-1} \boldsymbol{P}_{\boldsymbol{k-1}} \tag{3.14}$$

The attitude error quaternion can be obtained by putting $\hat{\vec{a}}_{Ak}$ into the equation (2.4). Then, the attitude is corrected by the multiplicative formula (2.5):

$$\hat{\vec{q}}_k = \delta \vec{q}(\hat{\vec{a}}_{Ak}) \otimes \hat{\vec{q}}_{k/k-1} \tag{3.15}$$

The method proposed in this paper is to project the attitude error message from the Euler attitude error vector to the Euler attitude error quaternion at first. Then it wil use the quaternion containing the attitude error message to correct the attitude quaternion obtained in the time update step. At last, the accurate attitude quaternion can be acquired, denoted as $\hat{\vec{q}}_k = [q_0 \;\; q_1 \;\; q_2 \;\; q_3]^T$. The attitude matrix, namely the body coordinate frame relative to the inertial coordinate frame, is presented as follows:

$$\boldsymbol{C}_{\boldsymbol{b}}^{\boldsymbol{i}} = \begin{bmatrix} (q_0^2 + q_1^2 - q_2^2 - q_3^2) & 2(q_1 q_2 - q_0 q_3) & 2(q_1 q_3 + q_0 q_2) \\ 2(q_1 q_2 + q_0 q_3) & (q_0^2 - q_1^2 + q_2^2 - q_3^2) & 2(q_2 q_3 - q_0 q_1) \\ 2(q_1 q_3 - q_0 q_2) & 2(q_2 q_3 + q_0 q_1) & (q_0^2 - q_1^2 - q_2^2 + q_3^2) \end{bmatrix} \tag{3.16}$$

The attitude matrix, namely the body coordinate frame relative to the navigation coordinate frame, is obtained by transfer of the matrix:

$$\boldsymbol{C}_{\boldsymbol{b}}^{\boldsymbol{n}} = \boldsymbol{C}_{\boldsymbol{i}}^{\boldsymbol{n}} \boldsymbol{C}_{\boldsymbol{b}}^{\boldsymbol{i}} \tag{3.17}$$

4. The Filter Model of GPS/SINS Integrated Navigation System

The SINS system is taken as the main system of GPS/SINS integrated navigation system. Based on the error equation of SINS and integrated navigation system, the state equation and measurement equation of the integrated navigation system are established respectively. Then, the minimum variance estimation of the SINS system error is acquired by the EKF filter. At last, the SINS system is corrected by the estimated error value error to get the accurate navigation message [12-13].

(1) State equation

We choose the local E-N-U frame as the navigation frame, and establish the state equation based on the positioning error, velocity error, digital platform attitude error, and the inertial error.

$$\dot{\vec{X}}_I(t)=\boldsymbol{F}_I(t)\vec{X}_I(t)+\boldsymbol{G}_I(t)\vec{w}_I(t) \tag{4.1}$$

where,

$\vec{X}_I=[\delta L \;\; \delta\lambda \;\; \delta h \;\; \delta V_E \;\; \delta V_N \;\; \delta V_U \;\; \varphi_E \;\; \varphi_N \;\; \varphi_U \;\; \varepsilon_{bx} \;\; \varepsilon_{by} \;\; \varepsilon_{bz} \;\; \nabla x \;\; \nabla y \;\; \nabla z]^T$

represent the state vector and its components :

$\delta L \;\; \delta\lambda \;\; \delta h$ is the latitude error, longitude error, and the height error respectively;

$\delta V_E \;\; \delta V_N \;\; \delta V_U$ is the error of velocity;

$\varphi_E \;\; \varphi_N \;\; \varphi_U$ is the attitude angle error of the digital platform;

$\varepsilon_{bx} \;\; \varepsilon_{by} \;\; \varepsilon_{bz}$ is the random constant drift of the gyroscope;

$\nabla x \;\; \nabla y \;\; \nabla z$ is the random constant drift of the accelerometer.

Subscripts E, N, U represent the east, north, and the upper axial of the navigation coordinate frame, and x, y, z represent the right, front, and upper axial of the body coordinate frame.

The state matrix can be acquired based on the component form of the error equation of the SINS system:

$$\boldsymbol{F}_I=\begin{bmatrix} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \boldsymbol{F}_{9\times9} & \mathbf{0}_{3\times3} & \boldsymbol{C}_b^n \\ & \boldsymbol{C}_b^n & \mathbf{0}_{3\times3} \\ \mathbf{0}_{6\times9} & \boldsymbol{F}_M & \mathbf{0}_{3\times3} \\ & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix} \tag{4.2}$$

where $\boldsymbol{C}_b^n$ is the attitude rotation matrix from the body coordinate frame to the navigation coordinate frame, so $\boldsymbol{F}_M=diag\left(-\frac{1}{\tau_{gx}} \;\; -\frac{1}{\tau_{gy}} \;\; -\frac{1}{\tau_{gz}}\right)$. Besides, the drift of the gyroscope is seen as the Markov process here.

The noise of the system is $\vec{w}_I=[w_{gx} \;\; w_{gy} \;\; w_{gz} \;\; w_{ax} \;\; w_{ay} \;\; w_{az}]^T$, where w_{gx}, w_{gy}, w_{gz} are random drifts of the gyroscope, and w_{ax}, w_{ay}, w_{az} are random drifts of the accelerometer. The assignment matrix of the system noise is presented as follows:

$$\boldsymbol{G}_I=\begin{bmatrix} \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \boldsymbol{C}_b^n \\ \boldsymbol{C}_b^n & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \end{bmatrix} \tag{4.3}$$

(2) Measurement equation

The position, velocity, and heading difference of SINS and GPS is taken as the measurement vector. Therefore, the measurement equation is given by:

$$\vec{z}_I(t)=\begin{bmatrix} \vec{P}_{SINS}-\vec{P}_{GPS} \\ \vec{V}_{SINS}-\vec{V}_{GPS} \\ \Psi_{SINS}-\Psi_{GPS} \end{bmatrix}=\boldsymbol{H}_I(t)\vec{X}_I+\vec{V}_I(t) \tag{4.4}$$

where the output matrix of the measurement is:

$$\boldsymbol{H}_I = \begin{bmatrix} \boldsymbol{I}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times6} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{I}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times6} \\ \boldsymbol{0}_{1\times3} & \boldsymbol{0}_{1\times3} & \vec{H}_\Psi & \boldsymbol{0}_{1\times6} \end{bmatrix} \tag{4.5}$$

where $\vec{H}_\Psi = [-\frac{T_{11}T_{31}}{T_{11}^2+T_{21}^2} \quad -\frac{T_{21}T_{31}}{T_{11}^2+T_{21}^2} \quad 1]$. T_{11}, T_{21}, T_{31} is the element of the rotation matrix $\boldsymbol{C}_b^n$ in the first column.

The noise sequence of the measurement is $\vec{V}_I = [N_E \quad N_N \quad N_U \quad M_E \quad M_N \quad M_U \quad N_\Psi]^T$, and $N_E, N_N, N_U, M_E, M_N, M_U, N_\Psi$ is the position, velocity and heading error of the GPS receiver respectively.

5. Experiments

Experiments are conducted in open areas where visibility of the satellite is good. Two antennae of GPS are mounted on of the car roof, and along the centerline respectively. The baseline length between two antennae is 1.2m. Besides, SINS is mounted on the center of chassis of the car. The lever arm compensation between SINS and GPS has been treated properly.

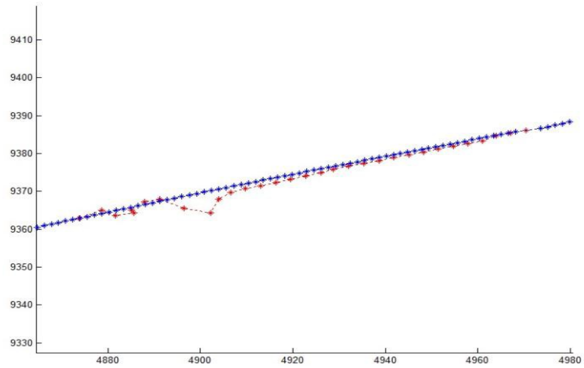

Figure 2 : The locus of straight maneuvers

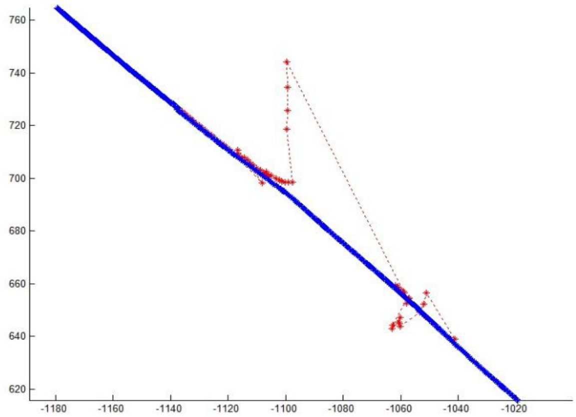

Figure 3: the Locus of Straight Maneuvers

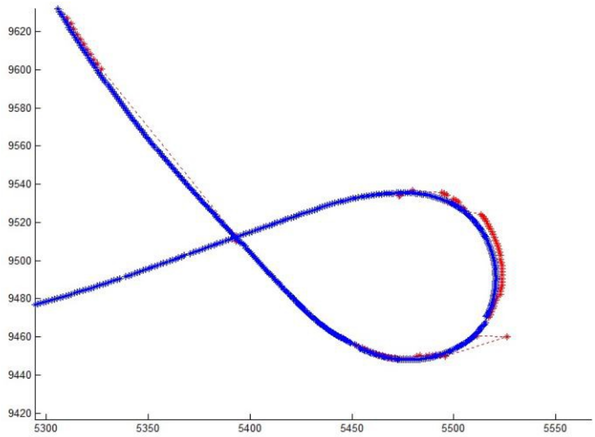

Figure 4: the Locus of Loop Maneuver with a Small Radius

At first, two straight maneuvers are conducted and their locus is shown in Fig. 2 and 3 respectively. The red locus is generated by the raw GPS data, and the blue is obtained from data of the GPS/SINS integrated navigation system by using MEKF to estimate the attitude at first and EKF to fuse data coming from GPS and SINS . The length unit in this figure is meter.

As we can see in Fig. 2 and 3, there are severe jumps in these loca generated by data of GPS when the car is running straight. But, the locus generated by data of the GPS/SINS integrated navigation system is stable and consistent with the actual path. These results show that the integrated navigation system designed in this paper can mask the outliers information of GPS and trace the position of land-vehicles accurately when they are running straight. After these straight maneuvers, a loop maneuver with a small radius is executed, and the locus is shown in Fig. 4. Meaning of the color in Fig. 4 is same as that in Fig. 2 and 3. Just like the straight maneuver locus, there are severe jumps in the locus generated by data of GPS, while the locus generated by data of the GPS/SINS integrated navigation system is stable and consistent with the actual path. This result shows that the integrated navigation system designed in this paper can mask the outliers information of GPS and trace the position of land-vehicles accurately when they are running with a small radius. In other words, it can provide accurate navigation messages when the car makes turns.

6. Conclusion

Nowadays, there are several kinds of navigation methods for land-vehicles. All of these methods have their advantages and disadvantages. Based on complementary features of GPS and SINS, a low-cost, high precision and robust navigation system for land-vehicles can be designed by integrating them effectively. In order to achieve this goal, MEKF is introduced in this paper to estimate the attitude matrix of the body coordinate frame relative to the navigation coordinate frame, and EKF is used to integrate data of GPS and SINS. Experimental results show that the outliers data of GPS can be effectively masked by the integrated navigation system, and the system designed in this paper can trace the position of land-vehicles accurately whether it drives straight or makes turns. we can improve accuracy and robustness of the navigation system for land-vehicles with this method.

References

[1] G. T. Schmidt. *Navigation sensors and systems in GNSS degraded and denied environments*[J]. Chinese Journal of Aeronautics. 28(1): 1-10(2015)

[2] D. H. Titterton, J. L. Weston. *Strapdown Inertial Navigation Technology(2nd Edition)*[M]. The Institution of Electrical Engineers, United Kingdom. 287-293(2004)

[3] P. Davidson, J. Hautamäki, J. Collin, J. Takala. *Improved Vehicle Positioning in Urban Environment through Integration of GPS and Low-Cost Inertial Sensors*[C]. Proceedings of the 17th ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems, Washington, USA. 352-361(2009)

[4] L. Zhao, W. Y. Ochieng, M. A. Quddus, R. B. Noland. *An extended Kalman filter algorithm for integrating GPS and low cost dead reckoning system data for vehicle performance and emissions monitoring*[J]. The Journal of Navigation. 56(02):257-275(2003)

[5] H. Liu, S. Nassar, N. El-Sheimy. *Two-Filter Smoothing for Accurate INS/GPS Land-Vehicle Navigation in Urban Centers*[J]. IEEE Transactions on Vehicular Technology. 59(9):4256-4267(2010)

[6] Y. H. Chang, H. Zhang, Q. F. Zhou. *A Measurement-Based Robust Adaptive Kalman Filtering Algorithm*[C]. International Conference on Materials Science and Information Technology (MSIT 2011), TRANS TECH PUBLICATIONS LTD, Switzerland. 3773-3779(2012)

[7] L. J. Zhang, S. F. Zhang, H. B. Yang, S. Qian. *Multiplicative filtering for spacecraft attitude determination*[J]. Journal of National University of Defense Technology. 35(6):46-52(2013)

[8] Y. Y. Qing, H. Y. Zhang, S. H. Wang. *Kalman filter and the principle of integrated navigation(2nd Edition)*[M]. Northwestern Polytechnic University Press, Xi'an, China.198-222(2012)

[9] M. F. Landis. *Multiplicative vs. Additive Filtering for Spacecraft Attitude Determination*[C]. Proceedings of 6th Cranfield Conference on Dynamics and Control of Systems and Structures in Space, Riomaggiore, Italy. 467-474(2004)

[10] M. F. Landis. *Attitude Error Representations for Kalman Filtering*[J]. Journal of Guidance, Control, and Dynamics. 26(2):311-317(2003)

[11] J. K. Hall, N. B. Knoebel, T. W. McLain. *Quaternion Attitude Estimation for Miniature Air Vehicles Using a Multiplicative Extended Kalman Filter*[C]. Position, Location and Navigation Symposium, 2008 IEEE. 1230-1237(2008)

[12] H. Zhang, Y. H. Chang, H. Che. *Measurement-based adaptive kalman filtering algorithm for GPS/INS integrated navigation system*[J]. Journal of Chinese Inertial Technology. 18(6):696-701 (2010)

[13] M. Ilyas, Y. C. Yang, Q. S. Qian, R. Zhang. *Low-cost IMU/Odometer/GPS Integrated Navigation Aided with Two Antennae Heading Measurement for Land Vehicle Application*[C]. Control and Decision Conference (CCDC), 2013 IEEE. 4521-4526(2013)