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From macro to micro: universal properties of neutron stars

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A recent discovery has pointed out that the moment of inertia, the tidal Love number and the spin-induced quadrupole moment of a neutron star, are linked together by semi-analytic relations which are almost insensitive to the star equations of state. These I-Love-Q relations open the opportunity to combine gravitational and electromagnetic observations and to devise new strategies, which will provide a better insight on the neutron star structure and the nature of gravity in the strong field regime. In this paper we review the main features of these universal relations, studying four scenarios in which they have been analysed to extend their domain of validity. We focus first on compact binaries close to the coalescence. Then, we investigate how strong magnetic fields, fast rotation, and hot equations of state affect the I-Love-Q universality.

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1. Introduction

Neutron stars (NSs) are among the most exotic objects of our Universe. They provide an unique intersection between astrophysics, gravity and particle physics, and represent natural laboratories for experimental tests under extreme conditions, which cannot be reproduced on Earth. NSs are well established sources of electromagnetic signals, which have already provided precious information on their internal and external structural properties [1]. Moreover, they are powerful emitters of gravitational radiation as isolated and binary systems. Second generation detectors like Advanced LIGO and Virgo are going to make gravitational wave astronomy a routine, unveiling the physical phenomena occurring in highly dynamical and high-curvature regimes in the NS surroundings [2, 3, 4]. Combined measurements in both these observational windows offer the chance to test gravity in the strong field regime, and to put constraints on the holy grail of NS astrophysics, i.e. its equation of state (EoS).

The largest obstacle in fully exploiting such objects as probe of new physics, lies in our ignorance about their internal structure. Some macroscopical features, like the star radius or its moment of inertia, strongly depend on the underlying EoS, which is nowadays highly uncertain [1]. This lack of knowledge affects our understanding of the stellar matter at supranuclear densities, and has prevented model-independent tests of gravity.

However, in this scenario a breakthrough is represented by the *I-Love-Q* relations. Recently, it has been shown that in slowly rotating and weakly magnetised neutron stars, universal relations do exist between the quadrupole moment (Q), the moment of inertia (I) and the tidal Love number (λ), which are almost insensitive to the star equation of state [5, 6]. These relations have several applications, and may be able to provide redundancy tests of General Relativity (GR), and shed new light on the NS inner regions [7]. Moreover, they can be used to break degeneracies between astrophysical parameters from electromagnetic and gravitational signals, as those between the quadrupole and the spins from GW observations of binary NSs [5], or in X-ray pulse profiles detected by future space satellites [8, 9].

Confirmed in [10], these relations have been deeply analysed to assess their domain of validity, considering highly dynamical binaries close the coalescence [11], strong magnetic fields [12], fast rotating objects, [13, 14, 15, 16, 17, 18], and newly-born proto-neutron stars [19]. Moreover, similar universal relations have been discovered for higher order multipole moments [13, 17, 16, 18, 20], tidal coefficients of NS binaries [21], other physical quantities [11, 22, 23], and in different classes of theories, alternative to GR [24, 25, 26, 27, 28, 29].

In the following sections we will provide a brief overview of the I-Love-Q relations (we refer the reader to the seminal papers [5, 6] for all the mathematical details and the possible applications), and then we will present four examples in which they have been investigated to test and extend their domain of validity. Moreover, although the I-Love-Q relations have also be proven to exist for strange stars, hereafter we will mainly focus on normal EoS. Throughout this paper we use units in which $G = c = 1$.

2. I-Love-Q

The I-Love-Q relations connect the moment of inertia I, the spin-induced quadrupole moment

Q, and the tidal Love number λ of an isolated neutron star, through semi-analytic fourth order fits which are independent of the star internal composition:

$$\ln y = a + b\ln x + c(\ln x)^2 + d(\ln x)^3 + e(\ln x)^4 \,, \tag{2.1}$$

where $(a\ldots e)$ are numerical coefficients given in Table 1. The $\bar{I},\bar{Q},\bar{\lambda}$ trio is normalised such that:

$$\bar{I} = \frac{I}{M^3} \quad , \quad \bar{Q} = -\frac{Q}{M^3\chi^2} \quad , \quad \bar{\lambda} = \frac{\lambda}{M^5} \,, \tag{2.2}$$

where M is the NS gravitational mass, and $\chi = J/M^2$ the spin parameter, J being its angular momentum. Top panels of Fig. 1 show the $\bar{I}$-$\bar{\lambda}$ and the $\bar{Q}$-$\bar{\lambda}$ relations tested against the numerical

y	x	a	b	c	d	e
$\bar{I}$	$\bar{\lambda}$	1.47	0.0817	0.0149	0.000287	$-3.64\cdot10^{-5}$
$\bar{I}$	$\bar{Q}$	1.35	0.697	-0.143	0.0994	$-1.24\cdot10^{-2}$
$\bar{Q}$	$\bar{\lambda}$	0.194	0.0936	0.0474	-0.00421	$1.23\cdot10^{-4}$

Table 1: Best-fit coefficients of Eq. (2.1) for the I-Love-Q relations [5].

results for a representative set of EoS and different NS configurations, obtained varying the central pressure. The bottom plots show the relative errors between the data and the analytic fits (2.1), and demonstrate that these relations are EoS independent with an accuracy better that 1%.

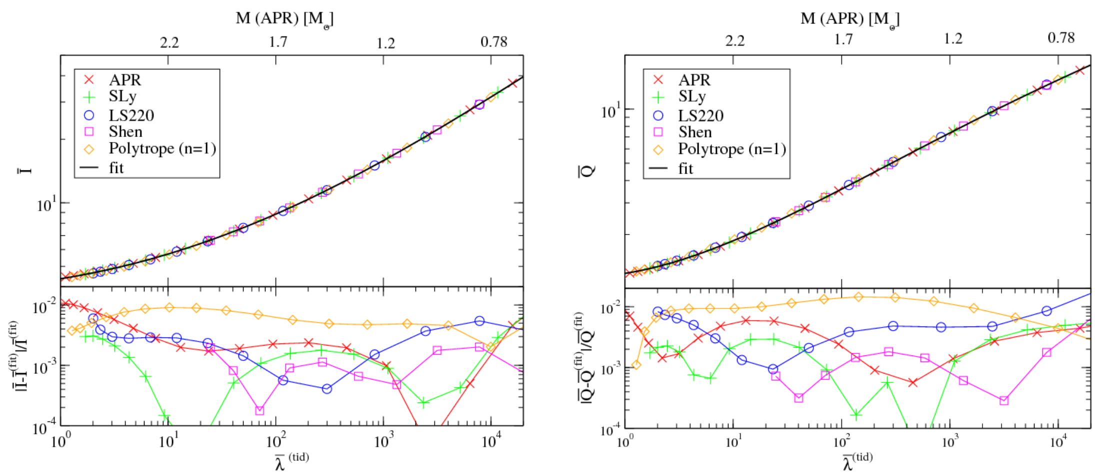


Figure 1: (Top) Analytic fits (solid black curves) and numerical results for the $\bar{I}$-$\bar{\lambda}$ and $\bar{Q}$-$\bar{\lambda}$ universal relations. Different points refer to distinct equations of state. The top x-axes show the corresponding NS masses for the APR EoS [30]. (Bottom) Relative errors between the data and the analytic expression (2.1) (taken from [6]).

Although the reason of such universality is not completely understood, Yunes and Yagi have already outlined some convincing arguments supporting their discovery [5]. The origin of the I-Love-Q relations has a fascinating explanation in terms of the *no-hair* theorem, for which the gravitational field of a rotating, non-charged black hole, solely depends on its mass and spin angular momentum, with *all* the other multipole moments related to the first two [31]. Although this conjecture does not apply to ordinary stars, approximate universality of Eq. (2.1) might suggest that I-Love-Q relations approach the black hole limit, as the stellar compactness increases. This argument seems also to be supported by the discovery of no-hair universal relations which connect *some* of the NS multipole moments ($l \leq 10$) to the first three [13, 16, 17]. However, there is no continuous limit which brings a NS sequence to a BH.

The reason of universality has been further investigated in [32, 33, 34], in which the authors made a more detailed analysis to connect the NS structural properties with the origin of the I-Love-Q relations. The main result point out that the EoS independence relies on the assumption that the star is modeled by isodensity contours which are self-similar ellipsoids, with large variations of the eccentricity being able to destroy the universality. This idea has also been corroborated by a different study carried out on hot proto-neutron stars, showing that the I-Love-Q lose their validity when entropy gradients are active inside the star, which reflect in considerable changes of the ellipticity of the isodensity contours [19].

Finally, comparing the numerical data obtained for the I-Love-Q trio, computed for incompressible stars and normal EoS, a different interpretation of the universality has been given in [35, 36]. The result of this analysis shows that Eq. (2.1) is stationary at first order under perturbations of the EoS around the incompressible limit, suggesting that EoS independence could be related to the proximity of NSs to incompressible objects.

3. I-Love forever

A first study devoted to extend the domain of validity of the I-Love Q relations was made in [11] to investigate the role of Eq. (2.1) during the coalescence of compact binary systems.

Neutron star tidal deformations are characterised by a set of coefficients, the *Love numbers*, which are computed assuming that tidal effects are produced by an external, time-independent gravitational field. However, this assumption makes the star effectively at isolation. The dominant contribution, described by the $l = 2$ component λ, is defined by the relation

$$Q_{ij} = \lambda \mathscr{E}_{ij} \ , \tag{3.1}$$

where $\mathscr{E}_{ij}$ and Q_{ij} are the tidal tensor and the star quadrupole tensor, respectively. Equation (3.1) is grounded on the *adiabatic approximation,* which states that the timescale of the orbital evolution is much longer than the timescale associated with the stellar deformations [37, 38, 39, 40]. Such hypothesis becomes less accurate as the binary approaches the merging phase, and *dynamic* tides start to provide a significant contribution [41]. Moreover, current GWs interferometers are expected to detect signals coming from the last stage of the inspiral, when binaries orbit at small distances, in an highly dynamical regime. It is therefore crucial to determine whether, in this scenario, the I-Love-Q relations would retain their universal character.

To this aim, the authors of [11] used a semi-analytical approach called the *post-newtonian affine model*, which has been developed to describe tidal deformations in compact binaries, and does not rely on the adiabatic approximation [42, 43]. In this framework the spherical star is deformed by the tidal field into an ellipsoid, preserving this shape during the orbital motion. This approach allows to compute Q_{ij} and $\mathscr{E}_{ij}$ in terms of the dynamical variables, and to evaluate their ratio as function of the orbital frequency f. A new *Love function* $\lambda(f)$ is then introduced:

$$\lambda(f) = \frac{\mathscr{E}_{ij}(f)}{Q_{ij}(f)} , \tag{3.2}$$

whose limit of zero frequency (or infinite orbital separation) corresponds to the constant λ obtained in the stationary approach. The post newtonian affine model also leads to define the moment of inertia:

$$I_i = I \times \frac{a_i^2}{R^2} , \tag{3.3}$$

where $a_{i=1,2,3}$ are the axes of the deformed ellipsoidal star, being R and I its radius and moment of inertia at spherical equilibrium. Three realistic EoS have been taken into account, `APR4` [30], `H4` [44] and `MS1` [45], which are expected to cover a wide range of NS deformability [46]. The authors computed the normalised Love function $\bar{\lambda} = \lambda(f)/M^5$, and the normalised moment of inertia corresponding to the axis pointing towards the companion $\bar{I} = I_1/M^3$, as function of the orbital frequency, testing the data against Eq. (2.1). Several NS-NS binary configurations have been considered, with mass ratio equal to one, and star masses in the range $M \in [1.2-2]M_\odot$.

In the left panel of Fig. (2) the $\bar{I}$-$\bar{\lambda}$ relation is plotted for three values of the GW frequency $f_{GW} = 2f = (170, 500, 850)$Hz, with the dashed curves corresponding to new semi-analytic fits of the data obtained at fixed orbital distances:

$$\ln \bar{I} = b_0 + b_1 \ln \bar{\lambda} + b_2 (\ln \bar{\lambda})^2 + b_3 (\ln \bar{\lambda})^3 + b_4 (\ln \bar{\lambda})^4 . \tag{3.4}$$

The coefficients b_i depend on f_{GW} and are listed in Table 1 of [11]. The top-right panel of Fig. (2) shows the relative errors $(\bar{I} - \bar{I}_{fit})/\bar{I}_{fit}$, between the data points and Eq. (3.4): for the three samples considered, such discrepancies are always $\lesssim 2\%$. In the lower panel the ratio $\bar{I}/\bar{I}_0$ is plotted against $\bar{\lambda}$, being $\bar{I}_0$ the value of $\bar{I}$ at isolation. We note that, as the binary approaches the final phase of the inspiral, the moment of inertia changes with respect to its asymptotic value, and increases up to 30%, depending on the EoS. The relative errors found are weakly dependent on the star internal composition, and suggest the existence of a more general, f-independent fit between $\bar{I}$ and $\bar{\lambda}$, which captures the whole range of frequencies considered. Such relation has been found with the same functional form of Eq. (3.4) and (2.1):

$$\ln \bar{I}_{uni} = 1.95 - 0.373 \ln \bar{\lambda} + 0.155 (\ln \bar{\lambda})^2 - 0.0175 (\ln \bar{\lambda})^3 + 0.000775 (\ln \bar{\lambda})^4 . \tag{3.5}$$

This fitting curve is drawn against all the numerical data in the upper panel of Fig. 3, with the bottom section displaying the relative errors obtained by means of Eq. (3.5) (black squares), and using the analytical result of Yunes and Yagi (red circles). The new fit reproduces the $\bar{I}$-$\bar{\lambda}$ relation with 5% accuracy throughout the inspiral, with the original formulation which becomes less accurate as the binary approaches the coalescence, with relative errors of the order of 10%. The universal relation (3.5) has also been tested for unequal NS-NS systems, and mixed BH-NS binaries, finding similar results with the same degree of accuracy.

We finally note that in this study, a new relation has been found between the NS compactness $C = M/R$ and the constant tidal Love number. For the same sets of binary configurations previously analysed, the authors found that the fit

$$C = 3.71 \times 10^{-1} - 3.91 \times 10^{-2} \ln \bar{\lambda} + 1.056 \times 10^{-3} (\ln \bar{\lambda})^2 , \tag{3.6}$$

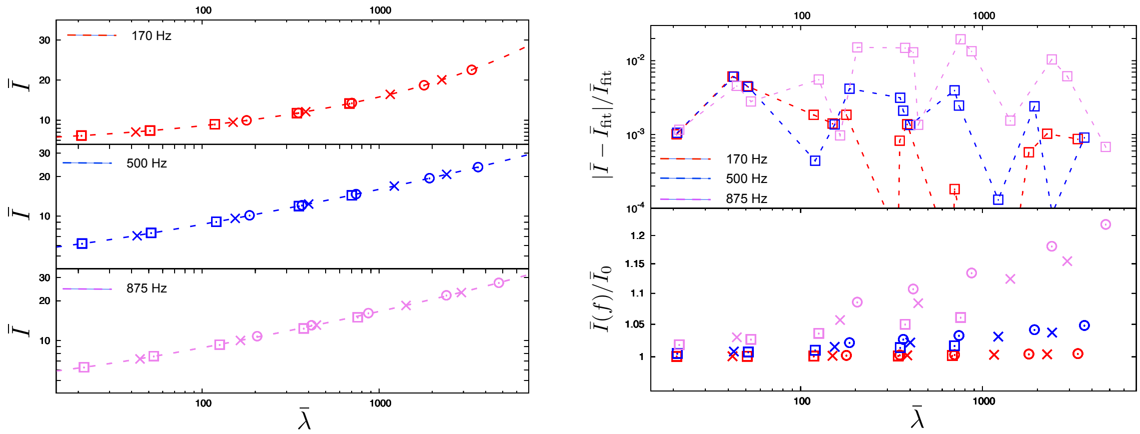


Figure 2: (Left) $\bar{I}-\bar{\lambda}$ relation for three values of the gravitational wave frequency $f_{\rm GW} \equiv 2f = (170, 500, 875)$ Hz and for the EoS `APR4` ($\times$), `MS1` ($\bigcirc$), `H4` ($\Box$). The dashed curves refer to the fits (3.4). (Right Top) Relative fractional errors between fits and numerical results. (Right Bottom) $\bar{I}$-$\bar{\lambda}$ relation with moment of inertia normalised by its value at infinity (taken from [11]).

reproduces the compactness with relative errors smaller than 2%. The C-$\bar{\lambda}$ relation could be extremely useful to extract information on the star EoS from future GW observations of binary coalescences, leading to estimates of the NS radius with uncertainties of $\lesssim 10\%$ [11].

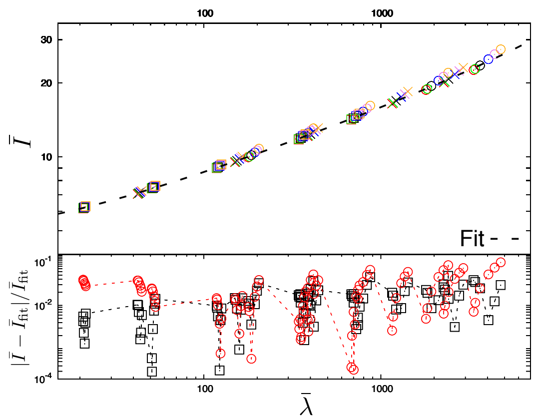


Figure 3: (Top) Fitting curve (3.5) (dashed line) and numerical results of the $\bar{I}$-$\bar{\lambda}$ relation, for data-set including points up to $f_{\rm GW} = 875$ Hz. (Bottom) Relative fractional errors between fits and numerical results. Black squares and red circles refer to the fit of Eq. (3.5), and to the analytical relation found by Yunes and Yagi, respectively (taken from [11]).

4. Universality for fast-rotating stars

A natural extension of the original discovery made by Yunes and Yagi was to consider fast rotating objects. This problem was first addressed in [14].

Fig. 4 shows the data of $\bar{I}$ and $\bar{Q}$ computed through the `rns` code [47, 48], and tested against the analytic relation (2.1) for different rotation frequencies f. Several realistic EoS have been taken into account. The figure suggests that universality is completely broken for frequencies greater than $\sim$ 160Hz, with the spread of the data due to the equation of state being comparable to the spread due to rotation. The bottom panel also displays the relative errors between numerical values and Eq. (2.1) for two of the cases considered, showing that deviations from the fourth order fit can be $\mathcal{O}(10\%)$ for stars close to the fastest millisecond pulsar known ($f \sim 700$Hz). Such differences, as the span related to the EoS, decrease for lower values of $\bar{I}$ and $\bar{Q}$, for which stellar configurations are characterised by larger compactness. As pointed out by the authors, this result is somehow expected, as for these configurations the NSs approach the BH limit. Although a universal $\bar{I}$-$\bar{Q}$

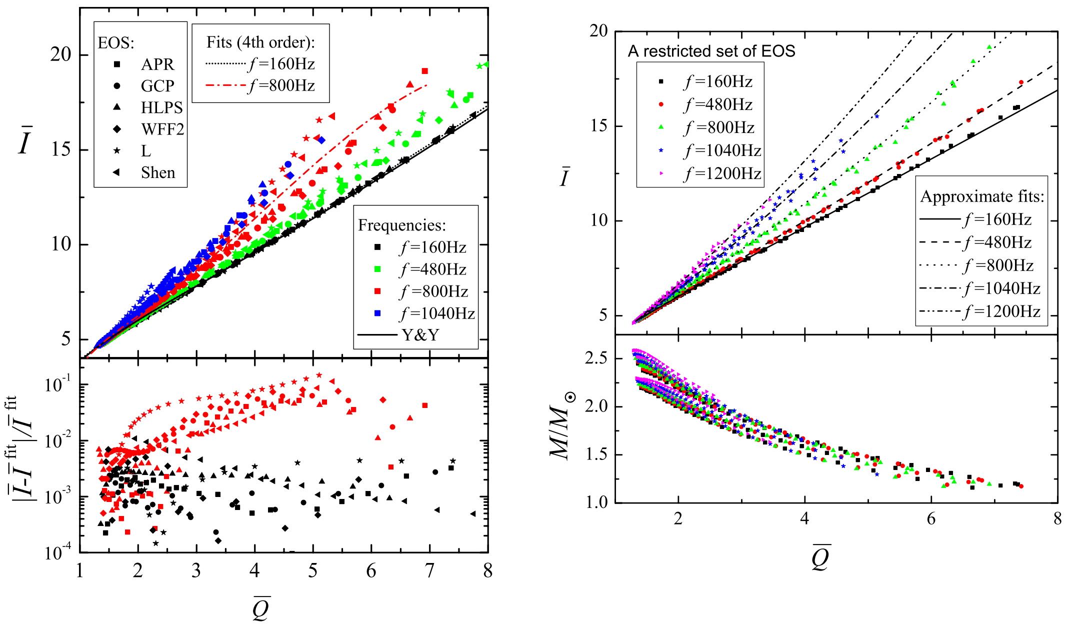


Figure 4: (Left) Numerical data obtained for the normalised moment of inertia and spin-induced quadrupole, for NSs with rotation frequency f. Different point shapes correspond to distinct EoS. The analytic relations (2.1) (black solid curve) and (4.1) (red dashed and black dotted curves) are also drawn against the data. The bottom panel shows the relative errors between the numerical values and the fit derived by Yunes and Yagi (Y&Y in the label). (Right) The top panel shows the new $\bar{I}$-$\bar{Q}$ universal relations (4.1) for fixed sequences of rotational frequency. The bottom figure displays the numerical values for gravitational mass as function of the normalised quadrupole (taken from [14]).

does not exist, new semi-analytic fits can be derived, which capture the universality for sequences of fixed rotational frequency:

$$\ln \bar{I} = a_0 + a_1 \ln \bar{Q} + a_2 (\ln \bar{Q})^2 \ , \tag{4.1}$$

where $a_i = a_i(f)$ are fitting coefficients described by a third order relation:

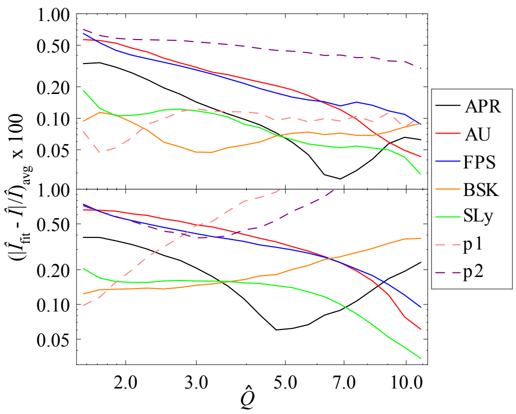
$$a_i = c_0 + c_1 \frac{f}{1 \text{ kHz}} + c_2 \left(\frac{f}{1 \text{ kHz}}\right)^2 + c_3 \left(\frac{f}{1 \text{ kHz}}\right)^3 \ , \tag{4.2}$$

(see Table 1 of [14] for the numerical values of c_i). The right plot of Fig. 4 shows these relations for a restricted sample of EoS. Relative errors with respect to the numerical data decrease to 2% for all the frequencies $\lesssim$ 500Hz, and up to $\sim 5\%$ for higher rotations. Deviations for slowly rotating stars are mainly due to the accuracy of Eq. (4.1), while for faster spins are related to the spread of different EoS.

This problem was also tackled in [15], where new analytic fits have been discovered, which improve the universality of the $\bar{I}$-$\bar{Q}$ relations even for frequencies greater than 500 Hz. The authors focused on finding a dimensionless parameter α, which characterises the star rotation, and such that the function $\bar{I}(\bar{Q},\alpha)$ is independent from the EoS. Universality is maximised assuming the following functional form:

$$\ln \bar{I} \approx \sum_{ij} \mathscr{A}_{ij} a^i \ln^j \bar{Q} \approx \sum_{ij} \mathscr{B}_{ij} \tilde{f}^i \ln^j \bar{Q} \,, \tag{4.3}$$

where $\mathscr{A}_{ij}$ and $\mathscr{B}_{ij}$ are fitting coefficients (listed in Table 1 of [15]), and the two parametrisations have been chosen such that $a = J/M^2$, and $\tilde{f} = 20Rf$. Figure 5 shows the relative percentage errors between Eq. (4.3) and 3×10^4 data in the parameter space $0.1 < a < 0.6$, $0.2 < \tilde{f} < 1.2$, $1.5 < \bar{Q} < 15$, for the EoS specified on the side. Deviations from Eq. (4.3) are always $\lesssim 1\%$, and on average $\sim 0.3\%$. We note that, in both cases, the new universal relations depend on an extra parameter, being a or $\tilde{f}$. Future simultaneous measurements of the $\bar{I}$-$\bar{Q}$-a trio, must be consistent with the curve of the right panel of Fig. 5, if GR holds. Finally, the authors suggested the chance to use the $\tilde{f}$ parametrisation to constrain the NS radius, once the dimensionless $\bar{I}$,$\bar{Q}$ and the dimensionful stellar frequency f have been measured.



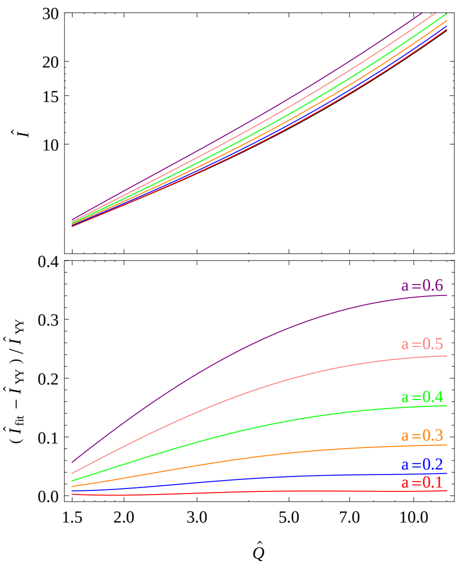


Figure 5: (Right) Percentage relative errors between numerical data computed through the `rns` code, and the universal relations (4.3). Top and bottom panels refer to the errors averaged on a and $\tilde{f}$, respectively. (Left) $\bar{I}$-$\bar{Q}$-a relations (top) and relative differences with respect to the Yunes-Yagi fit (2.1) (bottom), for different values of the parameter a (taken from [15]).

5. I-Love-Q magnetically

NSs are characterised by strong magnetic fields, with surface values of the order of 10^{12} G for radio pulsars, and up to 10^{15} G for magnetars. Their neat effect is to deform the star spherical shape, and then to modify its quadrupole moment. It is therefore expected that magnetic fields would affect the I-Love-Q relations. A systematic study about this topic has been first addressed in [12], focusing on the *I*-*Q* pair [1].

As argued by the authors, a simple Newtonian framework is already able to capture the main features of the magnetised $\bar{I}$-$\bar{Q}$ relation. Given a standard polytropic EoS with polytropic index $\gamma = 2$, for a purely poloidal magnetic field the normalised quadrupole and moment of inertia are related at the lowest order by:

$$\bar{Q} = \bar{Q}_r + \bar{Q}_m \simeq 4.9\bar{I}^{1/2} + 10^{-3}\bar{I}\left(\frac{B_p}{10^{12}\mathrm{G}}\right)^2\left(\frac{P}{1s}\right)^2 , \tag{5.1}$$

where the first term is due to rotation ($\bar{Q}_r$), and the second ($\bar{Q}_m$) to the magnetic field B_p at the pole, being P the star period. In the same spirit, for a purely toroidal configuration:

$$\bar{Q} \simeq 4.9\bar{I}^{1/2} - 3\times 10^{-5}\bar{I}\left(\frac{\langle B\rangle}{10^{12}\mathrm{G}}\right)^2\left(\frac{P}{1s}\right)^2 , \tag{5.2}$$

where $\langle B\rangle$ is the field averaged on the star volume. Equations (5.1)-(5.2) show that the rotational contribution is always dominant, except for very strong fields and large periods. Moreover, the quadrupole is proportional to the product $B \times P$, and therefore is not universal, since it will in general depend on the NS parameters.

This analysis has been extended to a fully relativistic treatment, by considering the two configurations described above separately, and a *twisted-torus* model, in which both fields are present. The left panel of Fig. 6 shows the magnetic quadrupole, for purely poloidal ($\bar{Q}_m > 0, B_p = 10^{12}$G) and purely toroidal ($\bar{Q}_m < 0, \langle B\rangle = 10^{12}$G) fields, for a non-rotating star, with different EoS. It is clear that only once the magnetic field configuration has been specified, the $\bar{I}$-$\bar{Q}_m$ relation acquires an universal character, which weakly relies on the star internal composition. The right panel of the same figure also shows the strong dependence on the product $B \times P$. In this case a pure toroidal set up has been considered as function of P, with $\langle B\rangle = 10^{14}$G and one realistic EoS. Different periods lead to distinct $\bar{I}$-$\bar{Q}$ relations, which are now dominated by the rotational component.

Universality is also lost for twisted-torus configurations, as displayed in Fig. 7. In the right panel the authors considered a NS with internal toroidal-to-total magnetic field energy ratio $\mathscr{F} = E_{rot}/E_m^{int} = 50\%$, a surface poloidal field $B_p = 5\times 10^{12}$G with $P = 10$s, and five EoS (see [12] for more details). In this case the equation of state plays a crucial role, completely spoiling out the validity of the $\bar{I}$-$\bar{Q}$ relation. Moreover, these results seem to strongly depend on the values of $\mathscr{F}$, and on the prescription for the internal currents. Different field strengths, specified by the parameter $\bar{k}$, yield distinct slopes and then destroy the universal behaviour.

[1] Magnetic corrections to the Love number are currently not known, and would be of higher order.

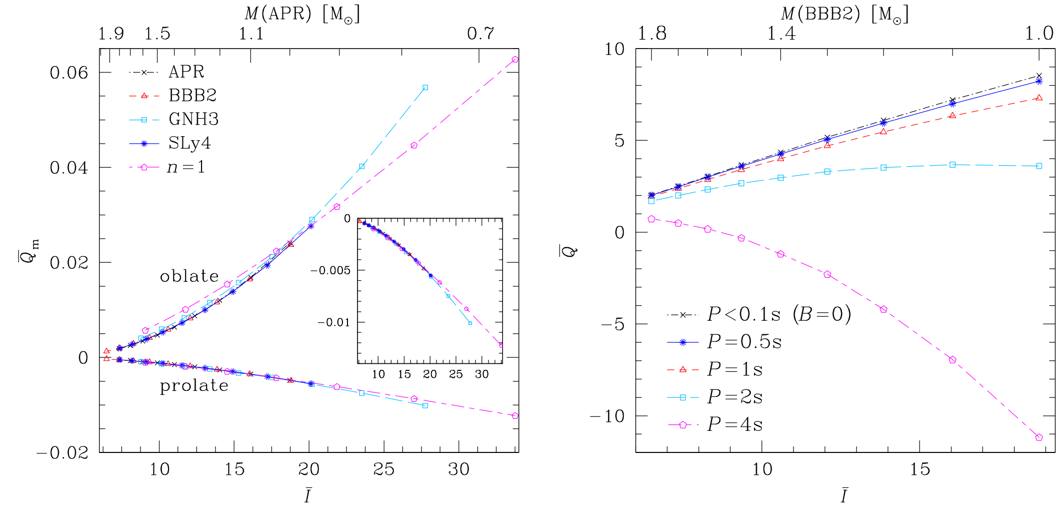


Figure 6: (Left) Normalised magnetic quadrupole as function of the normalised moment of inertia, for purely poloidal ($\bar{Q}_m > 0, B = 10^{12}$G) and toroidal ($\bar{Q}_m < 0, \langle B\rangle = 10^{12}$G) fields. The NS is non-rotating and modeled by the realistic EoS APR [30], with masses identified by the upper x-axis. (Right) $\bar{I}$-$\bar{Q}$ relation for a purely toroidal magnetic field for different rotation rates and $\langle B\rangle = 10^{14}$G (taken from [12]).

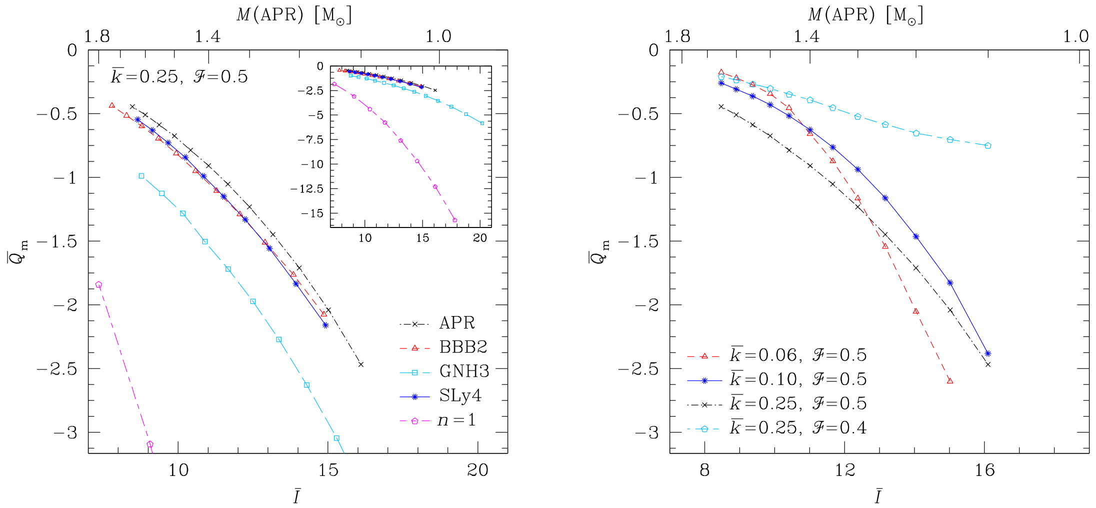


Figure 7: (Left) $\bar{Q}_m$ relation for twisted-torus configurations with $\mathscr{F} = 0.5$, $B_p = 5\times 10^{12}$, $\bar{k} = 0.25$ and $P = 10$s. (Right) Same as left plot, but for different $\bar{k}$ and $\mathscr{F}$, and the EoS APR [30] (taken from [12]).

6. Universal relations for hot stars

I-Love-Q relations have been originally tested for cold equations of state, which are expected to provide a reliable description of old neutron stars. Although some finite-temperature EoS have been considered [5, 6], they have been treated as barotropic, assuming a uniform temperature of $T = 10^9$K. Such configurations yield a good picture of the star only 1 minute after the supernova explosion. This scenario has been extended in [19], in which newly-born neutron stars were studied, taking into account hot EoS where entropy gradients between the core and the envelope are still active.

The authors considered a sequence of mass-energy, pressure, and lepton fraction profiles, which describe a NS with baryonic mass $M_b = 1.6M_\odot$ during the first minute of the stellar life. Then, they computed the I-Love-Q trio at different stages of the star evolution, comparing the numerical data with the theoretical results given by Eq. (2.1). Figure 8 shows the relative errors $\Delta I/\bar{I}_{fit} = |\bar{I} - \bar{I}_{fit}|/\bar{I}_{fit}$ and $\Delta Q/\bar{Q}_{fit} = |\bar{Q} - \bar{Q}_{fit}|/\bar{Q}_{fit}$ for the normalised moment of inertia and spin-induced quadrupole moment, at $t = (0.2, 0.3, 0.5, 1, 2.5, 20)$ seconds after the star birth. The plots clearly show that the analytical fits lose their validity during the first stages after the bounce, with discrepancies up to 30% for the $\bar{I}$-$\bar{Q}$ and $\bar{Q}$-$\bar{\lambda}$ relations. These errors rapidly decrease, reduc-

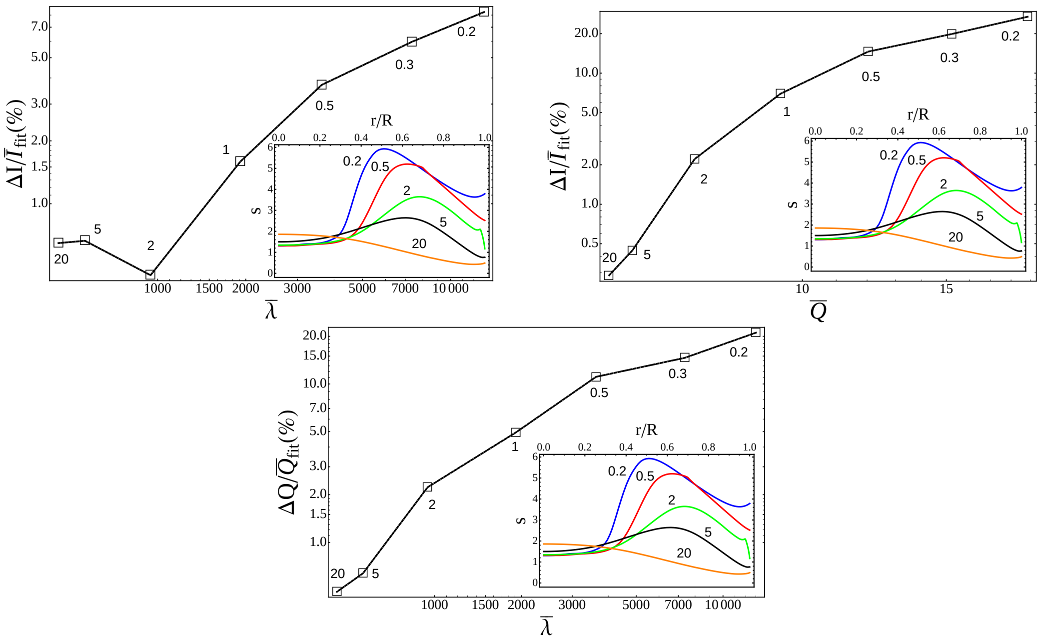


Figure 8: The plots show the relative differences between the universal relations (2.1) and the numerical data obtained from hot EoS (see text), at different times (in seconds) after the proto-neutron star birth. The insets display the entropy gradient per unit baryon, as function of the normalised radius of the star, for some configurations (taken from [19]).

ing to values $\lesssim 1\%$ after 2s, and then restoring the I-Love-Q universal character. The latter seems therefore to depend on how fast the newly-born star reaches a quiet and cold state.

The inset in each plot also shows, for some of the configurations considered, the entropy per baryon as function of the star radius. It is interesting to note that largest deviations from universality occur when the entropy gradient is maximum. This point has been examined more in detail, computing the ellipticity of the isodensity contours e inside the star, normalised to the star rotation rate Ω. From Fig. 9 it is clear that the ratio e/Ω changes with variations greater than 200% during the first second. However, for $t \gtrsim 2$s, when entropy gradients smoothen, the profiles become nearly flat. These results seem to support the hypothesis made in [32], for which I-Love-Q relations lose their accuracy when the ellipticity of the isodensity contours inside the NS presents large changes.

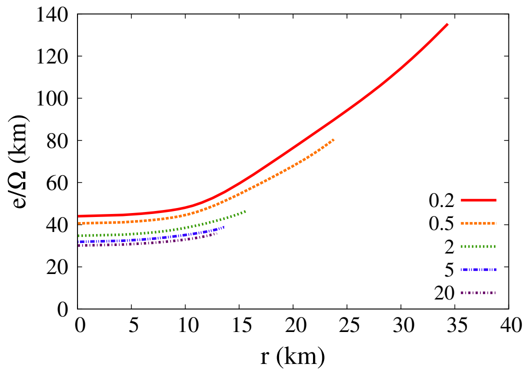


Figure 9: Ellipticity of the isodensity contours normalised to the star rotation rate, as function of the equatorial radius, at different times of the star evolution after the bounce (taken from [19]).

7. Conclusions

Neutron stars are the golden mine of relativistic astrophysics, as they represent the ideal arena to study physics in the most extreme conditions. However, our ignorance of their internal structure, i.e of the equation of state at supranuclear densities, has limited our ability to address some fundamental questions related to their properties. In this scenario, new possibilities are offered by the recently discovered I-Love-Q relations, which link together the moment of inertia, the tidal Love number and spin-induced quadrupole of isolated binary NSs, in an EoS independent way.

In this paper we have given a brief overview of the main features of such universal relations, and presented four scenarios in which they have been deeply investigated adding new physical degrees of freedom. In particular, we have shown that universality survives in highly dynamical regimes, when binary mergers close to the coalescence are considered, also leading to define new semi-analytic fits valid throughout the inspiral. Conversely, it has been pointed out that strong magnetic fields or large entropy gradients inside hot neutron stars are able to destroy I-Love-Q relations. Finally, we have described how different parametrisations may lead to the extension of their domain of validity to fast rotating objects.

Although the origin of such universality is still debated, I-Love-Q relations promise to offer new chances to combine current and future gravitational and electromagnetic observations, and to provide a new understanding of the astrophysical phenomena involving neutron stars.

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