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Inverse Mellin Transform of Holonomic Sequences

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We describe a method to compute the inverse Mellin transform of holonomic sequences, that is based on a method to compute the Mellin transform of holonomic functions. Both methods are implemented in the computer algebra package HarmonicSums.

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1. Introduction

In this paper we will present a method to compute the inverse Mellin transform of holonomic sequences and related to it we will revisit a method from [5] to compute the Mellin transform of holonomic functions. We emphasize that these methods are implemented in the computer algebra package `HarmonicSums`. Now let $\mathbb{K}$ be a field of characteristic 0. A function $f = f(x)$ is called *holonomic* (or *D-finite*) if there exist polynomials $p_d(x), p_{d-1}(x), \ldots, p_0(x) \in \mathbb{K}[x]$ (not all p_i being 0) such that the following holonomic differential equation holds:

$$p_d(x)f^{(d)}(x) + \cdots + p_1(x)f'(x) + p_0(x)f(x) = 0. \tag{1.1}$$

We emphasize that the class of holonomic functions is rather large due to its closure properties. Namely, if we are given two such differential equations that contain holonomic functions $f(x)$ and $g(x)$ as solutions, one can compute holonomic differential equations that contain $f(x) + g(x)$, $f(x)g(x)$ or $\int_0^x f(y)dy$ as solutions. In other words any composition of these operations over known holonomic functions $f(x)$ and $g(x)$ is again a holonomic function $h(x)$. In particular, if for the inner building blocks $f(x)$ and $g(x)$ the holonomic differential equations are given, also the holonomic differential equation of $h(x)$ can be computed.
Of special importance is the connection to recurrence relations. A sequence $(f_n)_{n\geq 0}$ with $f_n \in \mathbb{K}$ is called holonomic (or *P-finite*) if there exist polynomials $p_d(n), p_{d-1}(n), \ldots, p_0(n) \in \mathbb{K}[n]$ (not all p_i being 0) such that a holonomic recurrence

$$p_d(n)f_{n+d} + \cdots + p_1(n)f_{n+1} + p_0(n)f_n = 0 \tag{1.2}$$

holds for all $n \in \mathbb{N}$ (from a certain point on). In the following we utilize the fact that holonomic functions are precisely the generating functions of holonomic sequences: if $f(x)$ is holonomic, then the coefficients f_n of the formal power series expansion

$$f(x) = \sum_{n=0}^{\infty} f_n x^n$$

form a holonomic sequence. Conversely, for a given holonomic sequence $(f_n)_{n\geq 0}$, the function defined by the above sum (i.e., its generating function) is holonomic (this is true in the sense of formal power series, even if the sum has a zero radius of convergence). Note that given a holonomic differential equation for a holonomic function $f(x)$ it is straightforward to construct a holonomic recurrence for the coefficients of its power series expansion. For a recent overview of this holonomic machinery and further literature we refer to [13].

The paper is organized as follows. In Section 2 we revisit a method from [5] to compute the Mellin transform of holonomic functions, while in Section 3 we present a method to compute the inverse Mellin transform of holonomic functions.

2. The Mellin Transform of Holonomic Functions

In the following, we deal with the problem:
Given a holonomic function $f(x)$.

Find, whenever possible, an expression $F(n)$ given as a linear combination of indefinite nested sums such that for all $n \in \mathbb{N}$ (from a certain point on) we have

$$\mathbf{M}[f(x)](n) = F(n). \tag{2.1}$$

In [5] three different but similar methods to solve the problem above were presented. All three methods are implemented in the Mathematica package `HarmonicSums` [8, 6, 7, 4]. All of these methods rely on the holonomic machinery sketched above. In addition the symbolic summation package `Sigma` [15, 16] is used which is based on an algorithmic difference field theory. Here one of the key ideas is to derive a recurrence relation that contains the Mellin transform as solution and to execute `Sigma`'s recurrence solver that finds all solutions that can be expressed in terms of indefinite nested sums and products [14, 9, 12, 10]; these solutions are called d'Alembertian solutions. In the following we revisit one of the methods form [5].

We state the following proposition.

Proposition 1. *If the Mellin transform of a holonomic function is defined i.e., the integral*

$$\int_0^1 x^n f(x)dx$$

exists, then it is a holonomic sequence.

Proof. Let $f(x)$ be a holonomic function such that the integral $\int_0^1 x^n f(x)dx$ exists. Using the properties of the Mellin transform we can easily check that

$$\mathbf{M}[x^m f^{(p)}(x)](n) = \frac{(-1)^p(n+m)!}{(n+m-p)!}\mathbf{M}[f(x)](n+m-p) + \sum_{i=0}^{p-1}\frac{(-1)^i(n+m)!}{(n+m-i)!}f^{(p-1-i)}(1). \tag{2.2}$$

Finally, we apply the Mellin transform to the holonomic differential equation of $f(x)$ using the relation above, and we get a holonomic recurrence for $\mathbf{M}[f(x)](n)$. □

Now, a method to compute the Mellin transform is obvious:

Let $f(x)$ be a holonomic function. In order to compute the Mellin transform $\mathbf{M}[f(x)](n)$, we can proceed as follows:

1. Compute a holonomic differential equation for $f(x)$.
2. Use the proposition above to compute a holonomic recurrence for $\mathbf{M}[f(x)](n)$.
3. Compute initial values for the recurrence.
4. Solve the recurrence (with `Sigma`) to get a closed form representation for $\mathbf{M}[f(x)](n)$.

Note that `Sigma` finds all solutions that can be expressed in terms of indefinite nested sums and products. Hence as long as such solutions suffice to solve the recurrence in item 4, we succeed to compute the Mellin transform $\mathbf{M}[f(x)](n)$.

Example 2. We want to compute the Mellin transform of

$$f(x) := \int_0^x \frac{\sqrt{1-\tau}}{1+\tau}\, d\tau.$$

We find that

$$(-3+x)f'(x)+2(-1+x)(1+x)f''(x)=0$$

holds, which leads to the recurrence

$$6\int_0^1 \frac{\sqrt{1-\tau}}{1+\tau}\, d\tau \;=\; -2(n-1)n\,\mathbf{M}[f(x)](n-2)+3n\,\mathbf{M}[f(x)](n-1)+(n+1)(2n+3)\,\mathbf{M}[f(x)](n).$$

Initial values can be computed easily and solving the recurrence leads to

$$\mathbf{M}[f(x)](n) \;=\; \frac{(1+(-1)^n)\int_0^1 \frac{\sqrt{1-\tau}}{1+\tau}\, d\tau + (-1)^n\left(6+8\sum\limits_{i=1}^{n}\frac{(-4)^i}{\binom{2i}{i}}\right)}{1+n} - \frac{4(5+4n)\,(2^n)^2}{(1+2n)(3+2n)\binom{2n}{n}}.$$

Note that this method can be extended to compute regularized Mellin transforms: given a holonomic function $f(x)$ such that

$$\int_0^1 (x^n-1)f(x)dx$$

exists, then we can compute

$$\mathbf{M}[[f(x)]_+](n) := \int_0^1 (x^n-1)f(x)dx$$

using a slight extension of the method above. For example we can compute

$$\mathbf{M}[[\frac{\log(x)}{1-x}]_+](n) = \int_0^1 (x^n-1)\frac{\log(x)}{1-x}dx = \sum_{i=1}^{n}\frac{1}{i^2}.$$

3. The Inverse Mellin Transform of Holonomic Sequences

In the following, we deal with the problem:
Given a holonomic sequence $F(n)$.
Find, whenever possible, an expression $f(x)$ given as a linear combination of indefinite iterated integrals such that for all $n \in \mathbb{N}$ (from a certain point on) we have

$$\mathbf{M}[f(x)](n) = F(n).$$

As a first step we want to compute a differential equation for $f(x)$ given a holonomic recurrence for $\mathbf{M}[f(x)](n)$.

Analyzing (2.2) we see that

$$\begin{aligned}\mathbf{M}[(-1)^p x^{m+p} f^{(p)}(x)](n) \;=\; & \frac{(n+m+p)!}{(n+m)!}\mathbf{M}[f(x)](n+m) \\ & +\sum_{i=0}^{p-1}\frac{(-1)^{i+p}(n+m+p)!}{(n+m+p-i)!} f^{(p-1-i)}(1).\end{aligned} \tag{3.1}$$

Hence we get

$$\begin{aligned} n^p\,\mathbf{M}[f(x)](n+m) &= \mathbf{M}[(-1)^p x^{m+p} f^{(p)}(x)](n) - a(n)\,\mathbf{M}[f(x)](n+m) \\ &\quad -\sum_{i=0}^{p-1} \frac{(-1)^{i+p}(n+m+p)!}{(n+m+p-i)!} f^{(p-1-i)}(1), \end{aligned} \tag{3.2}$$

where $a(n) \in \mathbb{K}[n]$ with $\deg(a(n)) < p$. We can use this observation to compute the differential equation recursively: Let

$$p_d(n) f_{n+d} + \cdots + p_1(n) f_{n+1} + p_0(n) f_n = 0 \tag{3.3}$$

be the holonomic recurrence for $\mathbf{M}[f(x)](n)$. Let $k := \max_{0\le i\le d}(\deg(p_i(x)))$ and let c be the coefficient of n^k in the recurrence i.e.,

$$c = \sum_{i=0}^{d} c_i f_{n+i}$$

for some $c_i \in \mathbb{K}$. For $0 \le i \le d$ we replace $c_i n^k f_{n+i}$ by

$$c_i n^k f_{n+i} + c_i(-1)^k x^{k+i} f^{(k)}(x) - c_i\,\mathbf{M}[(-1)^k x^{k+i} f^{(k)}(x)](n)$$

and apply (2.2). Considering (3.2) we conclude that we reduced the degree of n. We apply this strategy until we have removed all appearences of f_{n+i}. At this point we have an equation of the form

$$q_l(x) f^{(l)}(x) + \cdots + q_1(x) f'(x) + q_0(x) f(x) + \sum_{j=0}^{k-1} r_j(n) f^{(j)}(1) = 0.$$

where $r_i(n) \in \mathbb{K}[n]$. If all $r_i(n) = 0$, we are done. If not, we differentiate the differential equation. In both cases we end up with a holonomic differential equation for $f(x)$.
Let us illustrate this strategy using an example.

Example 3. Consider the recurrence

$$(2+n) f_{n+2} - f_{n+1} - (n+1) f_n = 0. \tag{3.4}$$

The maximal degree of the coefficients f_{n+i} with $0 \le i \le 2$ is 1 and the coefficient of n of the left hand side of (3.4) is $f_{n+2} - f_n$. We substitute

$$\begin{aligned} n f_{n+2} &\to n f_{n+2} - x^3 f'(x) + \mathbf{M}[x^3 f'(x)](n) \\ -n f_n &\to -n f_n + x f'(x) - \mathbf{M}[x f'(x)](n) \end{aligned}$$

in (3.4) and apply (2.2). This yields

$$(-x^3 + x) f' - f_{n+2} - f_{n+1} = 0. \tag{3.5}$$

since $\mathbf{M}[x^3 f'(x)](n) = -(n+3) f_{n+2} + f(1)$ and $\mathbf{M}[x f'(x)](n) = -(n+1) f_n + f(1)$. Next we substitute

$$-f_{n+2} \to -f_{n+2} - x^2 f(x) + \mathbf{M}[x^2 f(x)](n)$$

$$-f_{n+1} \to -f_{n+1} - xf(x) + \mathbf{M}[xf(x)](n)$$

in (3.5) and apply (2.2). Since $\mathbf{M}[x^2 f(x)](n) = f_{n+2}$ and $\mathbf{M}[xf(x)](n) = f_{n+1}$, this yields the differential equation

$$(-x^3 + x)f'(x) - (x^2 + x)f(x) = 0. \tag{3.6}$$

Our strategy to compute the inverse Mellin transform of holonomic sequences can be summarized as follows:

1. Compute a holonomic recurrence for $\mathbf{M}[f(x)](n)$.
2. Use the method above to compute a holonomic differential equation for $f(x)$.
3. Compute a linear independent set of solutions of the holonomic differential equation (using `HarmonicSums`).
4. Compute initial values for $\mathbf{M}[f(x)](n)$.
5. Combine the initial values and the solutions to get a closed form representation for $f(x)$.

Note that `HarmonicSums` finds all solutions that can be expressed in terms of iterated integrals over hyperexponential alphabets [14, 9, 12, 11, 5]; these solutions are called d'Alembertian solutions. Hence as long as such solutions suffice to solve the differential equation in item 3 we succeed to compute $f(x)$.

Example 4. We want to compute the inverse Mellin transform of

$$f_n := (-1)^n \left(\sum_{i=1}^{n} \frac{(-1)^i \sum_{j=1}^{i} \frac{1}{j^2}}{i} - \sum_{i=1}^{\infty} \frac{(-1)^i \sum_{j=1}^{i} \frac{1}{j^2}}{i} \right)$$

We find that

$$\begin{aligned} 0 = {} & (n+1)(n+2)^2 f_n - (n+2)\left(n^2+7n+11\right) f_{n+1} \\ & + \left(-n^3 - 5n^2 - 6n + 1\right) f_{n+2} + (n+3)^3 f_{n+3} \end{aligned}$$

which leads to the differential equation

$$\begin{aligned} 0 = {} & -(x-1)^2(x+1)x^3 f^{(3)}(x) - (x-1)(2x-1)(3x+1)x^2 f''(x) \\ & -(x-1)(7x-1)x^2 f'(x) - (x-1)x^2 f(x) \end{aligned}$$

that has the general solution

$$s(x) = \frac{c_1}{x+1} + \frac{c_2}{x+1} \int_0^x \frac{1}{y-1} dy + \frac{c_3}{x+1} \int_0^x \frac{\log(y)}{y-1} dy,$$

for some constants c_1, c_2, c_3. In order to determine these constants we compute

$$\int_0^1 x^0 s(x) dx = c_1 \log(2) + c_2 \frac{\log(2)^2 - \zeta_2}{2} + c_3 \frac{2\zeta_3 - \log(2)\zeta_2}{2},$$

$$\int_0^1 x^1 s(x)dx = c_1(1-\log(2))+c_2\frac{-\log(2)^2+\zeta_2-2}{2}+c_3\frac{\log(2)\zeta_2-2\zeta_3+2}{2},$$
$$\int_0^1 x^2 s(x)dx = c_1\frac{2\log(2)-1}{2}+c_2\frac{2\log(2)^2-2\zeta_2+1}{4}+c_3\frac{-4\log(2)\zeta_2+8\zeta_3-3}{8}.$$

Since

$$f_0=-\sum_{i=1}^{\infty}\frac{(-1)^i\sum_{j=1}^{i}\frac{1}{j^2}}{i};\ f_1=1+\sum_{i=1}^{\infty}\frac{(-1)^i\sum_{j=1}^{i}\frac{1}{j^2}}{i};\ f_2=-\frac{3}{8}-\sum_{i=1}^{\infty}\frac{(-1)^i\sum_{j=1}^{i}\frac{1}{j^2}}{i}$$

we can deduce that $c_0=0, c_1=0$ and $c_2=1$ and hence

$$f_n=\mathbf{M}\left[\frac{1}{x+1}\int_0^x\frac{\log(y)}{y-1}dy\right](n).$$

Note that the method above only works if the result is of the form

$$c^n\,\mathbf{M}[f(x)](n)+d$$

for some $c,d\in\mathbb{R}$. However, in general we will find results of the form

$$\sum_{i=0}^{k}c_i^n\,\mathbf{M}[f_i(x)](n)+d$$

for some $c_i,d\in\mathbb{R}$. Hence in order to deal with more general functions we refine our approach and compute $f_i(x)$ and c_i for $i=1$ to $i=k$ one after another. We illustrate this using the following example.

Example 5. We want to compute the inverse Mellin transform of

$$f_n:=\sum_{i=1}^{n}\frac{(-1)^i}{i}\sum_{j=1}^{i}\frac{1}{2^j j}.$$

We find that

$$0 = 4(1+n)(2+n)f_n-2(2+n)(7+2n)f_{n+1}+\left(2-2n-n^2\right)f_{n+2}+(3+n)^2f_{n+3}$$

which leads to the differential equation

$$\begin{aligned}0 =\ & \left(-2x+3x^2\right)f(x)+\left(4x-16x^2+13x^3\right)f'(x)+\left(8x-10x^2-9x^3+8x^4\right)f''(x)\\ &+\left(4x^2-4x^3-x^4+x^5\right)f^{(3)}(x)\end{aligned}$$

with the following three linear independent solutions

$$\begin{aligned}&\frac{1}{2+x},\frac{-\log(1-\frac{x}{2})}{2+x},\frac{1}{4(2+x)}\Bigg(\pi^2+2\log^2(2)+4\log(1-x)\log(2-x)-2\log^2(2-x)\\ &+4\pi\log\left(\frac{x}{2}-1\right)+\log\left(\frac{(-2+x)^2}{4x^2}\right)-4\mathrm{Li}_2\left(\frac{2}{2-x}\right)+4\mathrm{Li}_2(x-1)\Bigg).\end{aligned}$$

We know that f_n has to appear in at least one of the Mellin transforms of these solutions. Indeed we get

$$\mathbf{M}[\frac{-\log(1-\frac{x}{2})}{2+x}](n) = (-2)^n\Bigg(\sum_{i=1}^{n}\frac{(-1)^i}{i}\sum_{j=1}^{i}\frac{1}{2^j j}+\log(2)\Bigg(\log(3)-\sum_{i=1}^{n}\frac{(-1)^i}{i} + \sum_{i=1}^{n}\frac{1}{(-2)^i i}\Bigg)+\frac{\pi^2}{12}-\frac{5\log(2)^2}{2}-\mathrm{Li}_2\left(\frac{1}{4}\right)\Bigg).$$

Now we can write

$$\begin{aligned} f_n &= f_n+\frac{1}{(-2)^n}\mathbf{M}[\frac{-\log(1-\frac{x}{2})}{2+x}](n)-\Bigg(\sum_{i=1}^{n}\frac{(-1)^i}{i}\sum_{j=1}^{i}\frac{1}{2^j j}+\log(2)\Bigg(\log(3)-\sum_{i=1}^{n}\frac{(-1)^i}{i}\\ &\quad+\sum_{i=1}^{n}\frac{1}{(-2)^i i}\Bigg)+\frac{\pi^2}{12}-\frac{5\log(2)^2}{2}-\mathrm{Li}_2\left(\frac{1}{4}\right)\Bigg)\\ &= \frac{1}{(-2)^n}\mathbf{M}[\frac{-\log(1-\frac{x}{2})}{2+x}](n)-\log(2)\left(\log(3)-\sum_{i=1}^{n}\frac{(-1)^i}{i}+\sum_{i=1}^{n}\frac{1}{(-2)^i i}\right)\\ &\quad-\frac{\pi^2}{12}+\frac{5\log(2)^2}{2}+\mathrm{Li}_2\left(\frac{1}{4}\right) \end{aligned}$$

Next we compute the inverse Mellin transform of $g_n=\sum\limits_{i=1}^{n}\frac{1}{(-2)^i i}$. We find that

$$0 = -2(2+n)g_{n+1}+(4+n)g_{n+2}+(3+n)g_{n+3}$$

which leads to the differential equation

$$0 = x^2(1-x)g(x)+x^2(2-x-x^2)g'(x)$$

with the solution $\frac{1}{2+x}$. Note that

$$\mathbf{M}[\frac{1}{2+x}](n)=(-2)^n\left(\log(3)-\log(2)+\sum_{i=1}^{n}\frac{1}{(-2)^i i}\right),$$

hence we can write

$$f_n = \frac{-1}{(-2)^n}\mathbf{M}[\frac{\log(2-x)}{2+x}](n)+\frac{18\log(2)^2-\pi^2}{12}+\mathrm{Li}_2\left(\frac{1}{4}\right)+\log(2)\sum_{i=1}^{n}\frac{(-1)^i}{i}.$$

It remains to compute the inverse Mellin transform of $h_n=\sum_{i=1}^{n}\frac{(-1)^i}{i}$. We derive the recurrence

$$0 = -(1+n)h_n+h_{n+1}+(2+n)h_{n+2}$$

which gives rise to the differential equation

$$0 = x(1-x)h(x)+x(1-x^2)h'(x)$$

with the solution $\frac{1}{1+x}$. Since

$$\mathbf{M}[\frac{1}{1+x}](n)=(-1)^n\left(\log(2)+\sum_{i=1}^{n}\frac{(-1)^i}{i}\right)$$

we finally get

$$f_n=\frac{-1}{(-2)^n}\mathbf{M}[\frac{\log(2-x)}{2+x}](n)+(-1)^n\log(2)\mathbf{M}[\frac{1}{1+x}](n)+\frac{6\log(2)^2-\pi^2}{12}+\mathrm{Li}_2\left(\frac{1}{4}\right).$$

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References

[1] J. Ablinger, *The package HarmonicSums: Computer Algebra and Analytic aspects of Nested Sums*, PoS LL **2014** (2014) 019 [arXiv:1407.6180 [cs.SC]].

[2] J. Ablinger and J. Blümlein, *Harmonic Sums, Polylogarithms, Special Numbers, and their Generalizations*, in *Computer Algebra in Quantum Field Theory: Integration, Summation and Special Functions, Texts & Monographs in Symbolic Computation*, Springer, Wien, 2013. [arXiv:1304.7071 [math-ph]].

[3] J. Ablinger, J. Blümlein and C. Schneider, *Generalized Harmonic, Cyclotomic, and Binomial Sums, their Polylogarithms and Special Numbers*, J. Phys. Conf. Ser. **523** (2014) 012060 doi:10.1088/1742-6596/523/1/012060 [arXiv:1310.5645 [math-ph]].

[4] J. Ablinger, *Computer Algebra Algorithms for Special Functions in Particle Physics*, *PhD Thesis, J. Kepler University Linz*, 2012. arXiv:1305.0687 [math-ph].

[5] J. Ablinger, J. Blümlein, C. G. Raab and C. Schneider, *Iterated Binomial Sums and their Associated Iterated Integrals*, J. Math. Phys. **55** (2014) 112301 doi:10.1063/1.4900836 [arXiv:1407.1822 [hep-th]].

[6] J. Ablinger, J. Blümlein and C. Schneider, *Harmonic Sums and Polylogarithms Generated by Cyclotomic Polynomials*, J. Math. Phys. **52** (2011) 102301 doi:10.1063/1.3629472 [arXiv:1105.6063 [math-ph]].

[7] J. Ablinger, J. Blümlein and C. Schneider, *Analytic and Algorithmic Aspects of Generalized Harmonic Sums and Polylogarithms*, J. Math. Phys. **54** (2013) 082301 doi:10.1063/1.4811117 [arXiv:1302.0378 [math-ph]].

[8] J. Ablinger, *A Computer Algebra Toolbox for Harmonic Sums Related to Particle Physics*, arXiv:1011.1176 [math-ph].

[9] S.A. Abramov and M. Petkovšek, *D'Alembertian solutions of linear differential and difference equations*, in proceedings of *ISSAC'94*, ACM Press, 1994.

[10] J. Blümlein, M. Kauers, S. Klein and C. Schneider, *Determining the closed forms of the* $O(a_s^3)$ *anomalous dimensions and Wilson coefficients from Mellin moments by means of computer algebra*, Comput. Phys. Commun. **180** (2009) 2143 doi:10.1016/j.cpc.2009.06.020 [arXiv:0902.4091 [hep-ph]].

[11] M. Bronstein, *Linear Ordinary Differential Equations: breaking through the order 2 barrier*, in proceedings of *ISSAC'92*, ACM Press, 1992.

[12] P.A. Hendriks and M.F. Singer, *Solving difference equations in finite terms*, *J. Symbolic Comput.* **27**, 1999.

[13] M. Kauers and P. Paule, *The Concrete Tetrahedron*, Text and Monographs in Symbolic Computation, Springer, Wien, 2011.

[14] M. Petkovšek, *Hypergeometric solutions of linear recurrences with polynomial coefficients*, *J. Symbolic Comput.* **14**, 1992.

[15] C. Schneider, *Symbolic summation assists combinatorics*, *Sém. Lothar. Combin.* **56**, 2007.

[16] C. Schneider, *Simplifying Multiple Sums in Difference Fields*, in *Computer Algebra in Quantum Field Theory: Integration, Summation and Special Functions, Texts & Monographs in Symbolic Computation*, Springer, Wien, 2013. [arXiv:11304.4134 [cs.SC]].