De-noising Method based on Median Filter and Wavelet Transform

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As to the hybrid noise characteristic, a mixed de-noising method is hereby proposed in combination with the median filter and the wavelet transform. Firstly, conduct the median filter to the pending image; then apply an improved threshold function to the wavelet transform (WT) to eliminate noise; finally, adopt Signal Noise Ratio (SNR) to evaluate the image de-noising effect. The experiment results show that de-noising effect of the new algorithm is more excellent than the traditional wavelet transform and can eliminate hybrid noise so that the image quality can be improved effectively.

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1. Introduction

The image contains various kinds of noise due to the influence of complicated background, illumination and acquisition system. The existence of image noise leads to the image quantity reduction, which has exerted strong impact on the accuracy of industrial inspection, face identification and feature detection [1].

In this sense, it is a fatal issue of image preprocessing to eliminate noise. The image noise mainly behaves as Gaussian and Impulse noise [2]. Median filter has good inhibiting effect on the impulse noise, well protects the detail information such as margin and acute angle, etc [3]; but the inhibition ability to Gaussian distribution is weaker. The de-noising method based on WT can focus on detail area of the image, magnify small change, well inhibit the noise of Gaussian distribution, but partial impulse noise will be left [4], as a result, combine median filter and wavelet transform together to realize image de-noising.

2. Theoretical Basis

2.1 Median filtering

The median filter is a typical nonlinear spatial filter method for removing noise. It is the main idea that a point in value of the digital image is replaced by the median value of each point in the neighborhood [5]. The mathematical definition of median filter is shown as follows: given an array $A = [x_1, x_2, ..., x_n]$, x_i is the gray value of the pixel, as shown in Formula (2.1).

$$M = Med\left\{x_1, x_2, ... x_n\right\} = \begin{cases} x_{\frac{n+1}{2}} & n \ is \ odd \\ \frac{1}{2}\left[x_{\frac{n}{2}} + x_{\left(\frac{n}{2}+1\right)}\right] & even \ n \end{cases} \tag{2.1}$$

2.2 Filter Window Size Selection

The size of the median filter window affects the filter performance. p_k is the intensity of noise interference. If p_k is small, the image details can be retained well. If p_k is large, it performs well in the filter ability and makes the image details blurred. With this principle, if a square window is selected for filtering, the window length is shown as follows.

$$l_k = \begin{cases} 0, & p_k < 0.2\,\% \\ 3, & 0.2\,\% \leqslant p_k \leqslant 3\,\% \\ 5, & P_k > 3\,\% \end{cases} \tag{2.2}$$

2.3 Wavelet Transform

Basically, the wavelet transform through space and frequency of the local transformation, extracting information from signals [6]. A detailed analysis of the signal can be used for image compression and filtering, etc. Wavelet transform de-noising is used to complete the image wavelet decomposition, threshold and reconstruct wavelet coefficient. The digital image basically can be viewed as a finite energy signal $f(x, y)$. If $c_{j,m,n}(x, y)$ is the scale function，$\varphi_{j,m,n}^l(x, y)$ is the translation base function, and $l \in [1,2,3]$, the wavelet transform of discrete function $f(x, y)$ of M*N is shown as follows [7]:

$$W_c(j_0,m,n)=\frac{1}{\sqrt{MN}}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)c_{j_0,m,n}(x,y) \qquad (2.3)$$

$$W_\varphi^l(j,m,n)=\frac{1}{\sqrt{MN}}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)\varphi_{j,m,n}^l(x,y) \qquad (2.4)$$

Where j_0 is an arbitrary scale. It is usually 0. And $M=N=2^j$, $j=0,1,...J-1$, $m=n=0,1,...2^j-1$. (2.3) and (2.4) are wavelet transform of two-dimensional signal. W_c stands for the approximation in the scale of j. W_φ^1,W_φ^2, W_φ^3 denotes details of vertical, horizontal, diagonal. $W_c(j_0,m,n)$ and W_φ^l refer to the wavelet coefficient.

3. Median Filter and Wavelet Transform Hybrid De-noising Algorithm

3.1 Basic Idea of the Algorithm

Firstly, remove the salt & pepper noise with median filter to obtain images only containing the Gaussian noise; then eliminate Gaussian noise by using wavelet transform. The wavelet transform de-noising is adopted to remove the non-useful information. The non-useful often exists in high-frequency part. The main information is mostly distributed in low-frequency sub-band upon the wavelet decomposition. To avoid destructing main information, the high-frequency wavelet coefficients are disposed by threshold and 5*5 median filter. Finally, the wavelet coefficients of each sub-band are reconstructed by wavelet, so we obtain the de-noising image.

3.2 Determination of Decomposition Level

The wavelet decomposition of images is that images are decomposed by selecting appropriate wavelet basis and layers of the wavelet decomposition [8]. It is obviously different to the analysis of different signals with different wavelet basis functions, so it is critical to choose the wavelet basis function. According to principles of wavelet basis selection and the experience of image preprocessing, Db3 wavelet basis is selected.

As to the two-dimensional discrete wavelet transform, we get four sub-images: $W_c,W_\varphi^1,W_\varphi^2$, W_φ^3 in the wavelet decomposition of each layer. W_c is only decomposed in the decomposition of the next layer. We obtain another four sub-images until it predetermines the number of decomposition [9]. Four sub-images are produced in the wavelet decomposition of every layer. The cascaded filters increase with the decomposition layers, which leads to the signal shifts to get enlarged with less and less information of each high frequency sub-graph [10]; therefore, the three-layer wavelet decomposition is selected.

3.3 Improvement of Threshold Function

After the wavelet decomposition, most energy concentrates on the low-frequency wavelet coefficient and few energy distracts in high-frequency wavelet coefficient. The wavelet coefficient value of useful signal is larger than that of noise. To choose the thresholds directly affecting the image quality, the selection of threshold function is the different processing methods of wavelet coefficients that are larger or smaller threshold.

Hard threshold function does well in reserving the local feature, but its discontinuity can make the reconstructed image to produce the gibbes, etc. The treatment of soft threshold is relatively smooth; but the error in estimating the wavelet coefficients makes the image edge to be fuzzy. To avoid the gibbes and overcome the disadvantage of soft threshold in de-noising, we form a new threshold function. The expressions are given as follows.

$$\hat{W}_{j,m,n}=\begin{cases} sign(W_{j,m,n})\mu(|W_{j,m,n}|-\alpha\lambda)(0\le\alpha\le1) \\ 0 \end{cases} \qquad (3.1)$$

$$\mu = \begin{cases} 1 & |W_{j,m,n}| \geq \lambda \\ \dfrac{\lambda}{\lambda - \lambda_0}, & \lambda_0 \leq |W_{j,m,n}| < \lambda \\ 0 & |W_{j,m,n}| < \lambda_0 \end{cases} \tag{3.2}$$

Where sign (.) is sign function, λ is threshold. Due to limitations of soft and hard functions, the weight function μ is used to adjust the soft and hard functions. It not only preserves the edge details but also does not lose useful information. We select $\lambda_0 = 0.6\lambda$ through experiments.

3.4 New Threshold Determination

According to (3.1) and (3.2), it is important to select the threshold. If the threshold is too large, the most wavelet coefficients are set to zero, which leads to loss of too many detail signals. If the threshold is too small, it can not reach the expected effect. The fixed threshold method is used to estimating threshold.

$$\lambda = \sigma \sqrt{2 \ln N} \tag{3.3}$$

Where N is the number of wavelet decomposition coefficients all scales, σ is the noise variance. The valid signal wavelet transform is still clear on large scale. The threshold varies with different scales. The threshold varies inversely with the scale. Eq. (3.4) is an improved threshold function. According to (3.4), the threshold is a fixed threshold if $j = 1$, while the threshold is changing with the scale of j if $j > 1$.

$$\lambda_j = \sigma \sqrt{2 \log N} \Big/ \ln(e + j - 1) \tag{3.4}$$

4 Experimental Results and Analysis

4.1 Color Image De-noising

We conduct an experiment on standard image in California Institute of Technology Library. The image added noise of different intensities is processed with median filter, wavelet transform and hybrid de-noising algorithm. The SNR shown in Table1 is used to evaluate results. Fig. 1 shows the results of different de-noising algorithms. Fig. (1-c) and Fig. (1-d) are not clearer than Fig. (1-e) and Fig. (1-f), even lose some important details. There is a small part of vagueness or distortion in Fig. (1-e). Fig. (1-f) performs better than Fig. (1-e) in reserving the original image information.
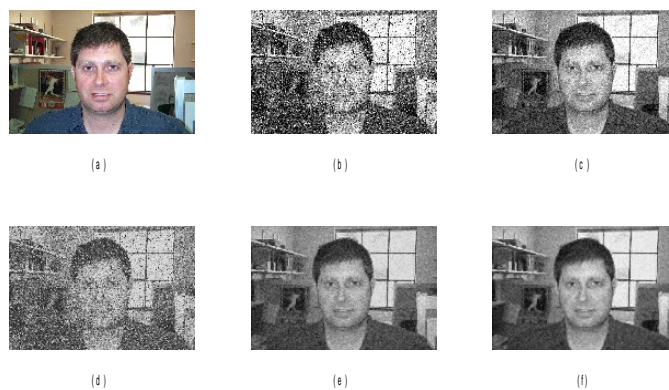

(a)   (b)   (c)

(d)   (e)   (f)

Figure 1 : Results of Different De-noising Algorithms: (a) Original Image; (b) Noisy Image; (c) Median Filter; (d) Wavelet Transform De-noising; (e) Wavelet Transform and 3*3 Median Filter; (f) Wavelet Transform and 5*5 Median Filter($\sigma = 0.05$, $\rho = 0.05$)

Noise intensity σ / density ρ	No de-noising	Median filter		Wavelet transform		Combining wavelet transform and median filtering			
						Db3		Db6	
		3*3	5*5	Db3	Db6	3*3	5*5	3*3	5*5
0.05/0.05	7.97	15.35	17.77	8.06	8.02	17.80	18.96	17.80	18.91
0.2/0.2	3.52	8.64	11.97	3.63	3.60	12.26	14.28	12.26	13.54
0.02/0.2	6.35	16.50	18.82	6.45	6.43	19.15	19.85	19.15	19.29
0.2/0.02	4.81	10.43	13.54	4.91	4.93	13.03	15.07	13.03	15.18

Note: In the combination of de-noising, 3*3 said both before and after use the same filter; 5*5 said in front of the 3*3 filter, 5*5 filter was used in high frequency coefficient.

Table 1 : Performance of Different Filters (Unit dB)

According to Table 1, the effect of the combined algorithms is better than the traditional median filter and the wavelet de-noising. The SNR value of Db3 wavelet is better than that of Db6 wavelet. The SNR value is higher when the median filter is 5*5 in high frequency coefficient part. Taking the image added with Gaussian noise of 0.05 and salt & pepper of 0.05, the SNR value of combined algorithm reaches 18.96dB that increases by 3.61dB than traditional median filter of 3*3, and 10.9dB than the single wavelet de-noising.

4.2 Standard Gray Image De-noising

Moreover, we conduct an experiment on Lena standard test image in Matlab image library. The image added noise of different intensities is processed with median filter, wavelet transform and hybrid de-noising algorithm. The SNR shown in Table 2 is used to evaluate the results. Fig. 2 shows the results of different algorithms. As the results in Fig. 2, the single de-noising method cannot preserve the edge information of image.

Figure 2 : Results of Different De-noising Algorithms of Lena: (a) Original Image; (b) Noisy Image; (c)Median Filter; (d) Wavelet Transform De-noising; (e) Wavelet Transform and 3*3 Median Filter ; (f)Wavelet Transform and 5*5 Median Filter($\sigma = 0.02$, $\rho = 0.02$)

Noise intensity σ / density ρ	No de-noising	Median filter		Wavelet transform		Combining wavelet transform and median filtering			
						Db3		Db6	
		3*3	5*5	Db3	Db6	3*3	5*5	3*3	5*5
0.05/0.05	6.89	13.97	16.21	6.90	6.91	16.25	17.35	16.25	16.78
0.2/0.2	2.68	7.31	10.44	2.82	2.80	11.05	13.20	11.05	12.63
0.02/0.2	5.70	15.09	17.01	5.74	5.77	17.29	17.87	17.29	18.13
0.2/0.02	3.72	9.04	11.94	3.80	3.83	12.45	14.21	12.45	14.16

Note: As shown in Table 1

Table 2: SNR of Different Filters for Lena (Unit dB)

As shown in Table 2, the de-noising effect of combined algorithm is better than others. The median filter of 5*5 is better than 3*3 in combination with the wavelet basis of Db3 and high frequency. Taking the identical noise in the former experiment, the SNR value of combined

algorithm reaches 17.35dB, increased by 24.2% than the traditional median filter of 3*3, and much larger than the single wavelet de-noising.

5. Conclusion

In order to solve the problem that single de-noising method cannot effectively remove hybrid noise, this paper proposes a method in combination with the median filter and the wavelet transform. Firstly, the image is processed with the median filter. Then an improved threshold function and median filter are used for the high-frequency coefficients obtained by the wavelet decomposition; finally, the de-noising image is obtained by wavelet reconstruction for all wavelet coefficients. Experimental results show that the proposed method can make up for the shortcomings of single de-noising method, and this method can not only eliminate the interference of hybrid noise, but also reserve the edge profile and details of image. In addition, this method can make full use of its advantages; thus it is feasible.

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