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A discrete Anatomy of the Neutrino mass Matrix

Nicholas D. Vlachos[*†]
Dept. of Theoretical Physics University of Thessaloniki Thessaloniki 541 24 Greece
E-mail: vlachos@physics.auth.gr

By expressing the charged lepton and neutrino mass matrices as linear combinations of elements of a single finite group we obtain constraints on the resulting mixing matrix and a non zero value for the θ_{13} mixing angle.

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1. On the discrete origin of the neutrino mass

It is well known that neutrino oscillations are tightly connected to the existence of non-zero neutrino masses and the mixing in the leptonic sector. Recent neutrino data are in accordance with two large mixing angles and a tiny value for the third one. In fact, adopting the parametrization

$$C = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{23}s_{12}s_{13} - c_{12}s_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \tag{1.1}$$

(where $c_{ij} \equiv \cos\theta_{ij}$ etc) for the leptonic mixing matrix, the 3σ range of the angles is given by

$$\begin{aligned} \sin^2\theta_{12} &= [0.26 - 0.36] \\ \sin^2\theta_{23} &= [0.34 - 0.66] \\ \sin^2\theta_{13} &= [0.017 - 0.031]\cdot \end{aligned} \tag{1.2}$$

It has been suggested [1]-[21] that the given structure of the mixing matrix indicates the existence of underlying symmetries. The simplest implementation of such groups into the lepton sector has predicted a tri-bimaximal (TB) mixing [1] with a strictly zero value for θ_{13}. However, recent experimental findings suggest a small non-zero value around $\theta_{13} \approx 9^0$. In the present work we explore possibilities related to more elaborate forms of mass textures, since effective models emerging from unified theories in higher dimensions involve a variety of continuous or discrete groups which act as family symmetries.

2. Formulation

It has been shown that small permutation groups like S_4, A_4 constitute good approximate symmetries which, under specific alignments of the vacuum expectation values of the various Higgs fields, generate $m_{\ell,\nu}$ matrices compatible with TB-mixing. One way for obtaining a non-zero value for θ_{13} is to consider deformations of the simple TB mass matrices. Then, we can explore the possibility of still having an underlying group structure in this new landscape. One way to achieve this is by expressing the (hermitean) mass matrices as linear combinations of appropriate representations of finite group elements. Candidate groups are all the finite groups or subgroups that possess a 3×3 matrix representation. If the mass matrices M are not hermitean but complex symmetric, the formalism applies to the hermitean combination MM^*.

2.1 Expanding mass matrices in terms of discrete group elements.

An interesting property of the simple TB mass matrix structure is that the diagonalising matrix is independent of the mass eigenvalues. This way, and using the Cayley Hamilton theorem [12], we may write a 3×3 Hermitean mass matrix M in the form

$$M = c_1 I + c_2 U + c_3 U^2 \tag{2.1}$$

where U is a 3×3 unitary matrix. This unitary matrix obviously constitutes a symmetry of M. Without loss of generality we impose the condition $\det U = 1$, so that U can always be brought in the form

$$D = \begin{pmatrix} e^{i\lambda} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\lambda} \end{pmatrix} \tag{2.2}$$

by means of a similarity transformation (modulo permutations of the eigenvalues). Observing that U and M can be simultaneously diagonalised, it follows that the mixing associated to the mass matrix M can be simply obtained by diagonalising the matrix U. The eigenmasses are given only in terms of the coefficients c_i, and the value of the phase λ,

$$\begin{aligned} m_1 - m_3 &= 2i\,(c_2 \sin\lambda + c_3 \sin 2\lambda) \\ m_1 + m_3 &= 2\,(c_1 + c_2 \cos\lambda + c_3 \cos 2\lambda) \\ m_2 &= c_1 + c_2 + c_3\cdot \end{aligned} \tag{2.3}$$

Hence, the mass eigenvalues problem is essentially disentangled from the diagonalising matrix.

3. Deforming the TB-mixing matrix

Assume now that the generators U of the mass matrices are elements of a discrete group. It follows that they must satisfy relations of the form $U^n = 1$ for some integer value of n. Their eigenvalues will be $e^{\frac{2\pi i}{n}}$, $e^{-\frac{2\pi i}{n}}$, 1 in some order and can be diagonalised by means of a unitary transformation to produce a diagonal matrix $D_{i,n}$ where the subscript i refers to the eigenvalues ordering. This way, for the charged leptons we have

$$U_l = V_l D_{i,n} V_l^\dagger \tag{3.1}$$

and for he neutrinos

$$U_\nu = V_\nu D_{j,m} V_\nu^\dagger\ . \tag{3.2}$$

Hence, the mixing matrix is

$$C = V_l^\dagger V_\nu\ . \tag{3.3}$$

If U_l and U_ν belong to the same group they must satisfy a relation of the form

$$(U_l U_\nu)^p = 1\ . \tag{3.4}$$

Also,

$$U_l U_\nu = V_l D_n V_l^\dagger V_\nu D_m V_\nu^\dagger = V_l D_n C D_m C^\dagger V_l^\dagger = V_l \mathscr{T} V_l^\dagger \tag{3.5}$$

where

$$\mathscr{T} = D_n C D_m C^\dagger\ \cdot \tag{3.6}$$

This way,

$$(U_l U_\nu)^p = V_l \mathscr{T}^p V_l^\dagger \tag{3.7}$$

Because of (3.4), for some integer p the left part of the above equation is the unit matrix, hence we must have $\mathscr{T}^p = 1$ for the same integer p. This implies that one eigenvalue of $\mathscr{T}$ must be 1

$D_{i,n}$	$D_{j,m}$	n	m	p
1	1	3	2	4
2	2	3	2	3
3	3	2	2	4
1	3	n	2	2
2	3	2	2	4
3	2	3	2	3
1	2	3	2	3

Table 1: Solutions for TB-mixing

while the other two must be $e^{\frac{2\pi i}{p}}$, $e^{-\frac{2\pi i}{p}}$respectively, since the determinant of $\mathscr{T}$ is 1. The trace of $\mathscr{T}$ equals $1+2\cos\frac{2\pi}{p}$. Therefore, we must seek solutions of the form

$$1+2\cos\frac{2\pi}{p} = \mathrm{Tr}\mathscr{T} \tag{3.8}$$

for given values of m and n. In the cases where $\mathrm{Tr}\mathscr{T}$ takes complex values there is no solution. Let us now define

$$D_{1,m} = \text{Diagonal}\left[1, e^{\frac{2\pi i}{m}}, e^{-\frac{2\pi i}{m}}\right] \tag{3.9}$$

$$D_{2,m} = \text{Diagonal}\left[e^{\frac{2\pi i}{m}}, 1, e^{-\frac{2\pi i}{m}}\right] \tag{3.10}$$

$$D_{3,m} = \text{Diagonal}\left[e^{\frac{2\pi i}{m}}, e^{-\frac{2\pi i}{m}}, 1\right]\cdot \tag{3.11}$$

Next, as a starting point we take the matrix C to be the TB-mixing matrix. This is readily derived from (1.1) for $\sin^2\theta_{12} = 1/3, \sin^2\theta_{23} = 1/2, \theta_{13} = 0$, and choosing the phases appropriately, C can be written as

$$C = \begin{pmatrix} \sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}\cdot$$

A subsequent search leads to a limited number of solutions depicted in Table 1. In this table, we use a five integer notation (i, j, n, m, p) where i, n refer to the matrix $D_{i,n}$ for the charged leptons, j, m refer to the matrix $D_{j,m}$ for the neutrinos, and p as in Eq.(3.4).

Note that the m value which refers to the neutrinos turns out always to be 2.

The lepton mass matrix is then written as

$$M_l = c_1 I + c_2 U_l + c_3 U_l^2 \tag{3.12}$$

$$U_l = V_l D_{i,n} V_l^\dagger \tag{3.13}$$

while the neutrino mass matrix is written as

$$M_\nu = d_1 I + d_2 U_\nu + d_3 U_\nu^2 \tag{3.14}$$

$$U_\nu = V_\nu D_{j,m} V_\nu^\dagger\ \cdot \tag{3.15}$$

In fact, $U_\nu^2 = 1$ since the only acceptable solutions are for $m = 2$ meaning that the coefficient d_3 does not exist. This reduction in the degrees of freedom creates a degeneracy in the neutrino mass spectrum. This way, for $D_{1,2}$ we get $m_2 = m_3$ for $D_{2,2}$ we get $m_1 = m_3$ and for $D_{3,2}$ we get $m_1 = m_2$ respectively. Thus, we can either have an exact discrete symmetry with a degenerate neutrino spectrum or a non degenerate spectrum with a broken symmetry. In the latter case one may choose a diagonal neutrino mass matrix i.e. $M_d = \text{Diagonal}[m_1, m_2, m_3]$ and construct the actual matrix through the relation

$$M_\nu = V_\nu M_d V_\nu^\dagger \cdot$$

The matrices V_l and V_ν are arbitrary unitary matrices connected by the relation

$$C = V_l^\dagger V_\nu \ . \tag{3.16}$$

Table 1 can be used to construct mass matrices that lead to an exact TB mixing. Models where $n = 2$ (third column values) lead to a degenerate spectrum for the charged leptons and are therefore unphysical. The model (1,1,3,2,4) gives an interesting group structure but, as subsequent calculations show it cannot be deformed to comply with data. In the following, we will attempt to generalise the TB mixing in order to accomodate a non-zero θ_{13} angle. To that end, let us now assume that the mixing matrix C takes the general form (1.1) and put for simplicity $\delta = 0$.

We may again construct the matrix $\mathscr{T}$ as before, using the new mixing matrix. As previously, we must require that

$$\operatorname{Im}\operatorname{Tr}\mathscr{T} = 0, \ \operatorname{Re}\operatorname{Tr}\mathscr{T} = 1 + 2\cos\frac{2\pi}{p} \cdot \tag{3.17}$$

The initial search for viable models was done numerically. No solutions were found for $m \neq 2$. Once a potentially working model is found, it can be elaborated analytically. In what follows, we use the previous classification in terms of the diagonal matrices $D_{i,n}, D_{j,m}$ and the three integers n, m, p .

4. Potentially viable Models

1. Case $(1,1,3,2,p)$. For this set of integers, θ_{12} and θ_{13} are related as follows:

$$\cos\theta_{12} = -\frac{2}{\sqrt{3}}\frac{\cos\frac{\pi}{p}}{\cos\theta_{13}}.$$

Computing θ_{12} as a function of p, we find that the only acceptable value is $p = 3$. In this case, we obtain the following relations:

$$\cos\theta_{12} = -\frac{2\cos\frac{\pi}{3}}{\sqrt{3}\cos\theta_{13}} \equiv -\frac{1}{\sqrt{3}\cos\theta_{13}}$$

$$\tan 2\theta_{23} = \frac{1}{2}\left(\frac{\sin\theta_{13}}{\tan\theta_{12}} - \frac{\tan\theta_{12}}{\sin\theta_{13}}\right).$$

We observe that, although θ_{12} lies within the acceptable range, this is not the case for θ_{23}. Therefore, this model is rejected.

2. Case $(2,2,3,2,p)$

   Here, θ_{12} is given by
   $$\sin^2\theta_{12} = \frac{1}{6\cos^2\theta_{13}}\left(1-2\cos\frac{2\pi}{p}\right).$$
   The only acceptable value is $p=3$ giving
   $$\sin\theta_{12}\,\cos\theta_{13} = -\frac{1}{\sqrt{3}}$$
   identical to $(1,1,3,2,p)$. However, the angle θ_{23} differs
   $$\tan 2\theta_{23} = -\frac{2\cot 2\theta_{13}}{\sqrt{3-\sec^2\theta_{13}}}.$$

3. Case $(3,2,3,2,p)$

   The resulting formula for θ_{12} is identical to the one in $(1,1,3,2,p)$
   $$\sin\theta_{12} = -\frac{2\cos\frac{\pi}{p}}{\sqrt{3}\cos\theta_{13}}.$$
   Hence, as above, the only acceptable value is $p=3$, leading to the constraint
   $$\sin\theta_{12} = -\frac{1}{\sqrt{3}\cos\theta_{13}}$$
   as in $(1,1,3,2,p)$ and $(2,2,3,2,p)$. For $p=3$ the angle θ_{23} is given by
   $$\tan 2\theta_{23} = -\frac{2\cot 2\theta_{13}}{\sqrt{3-\sec^2\theta_{13}}}.$$
   Comparing with previous cases, we observe that for $p=3$, as far as the mixing is concerned, this model is identical to the second one, therefore it is compatible with data. Setting $\sin\theta_{13}=s$ the mixing matrix becomes
   $$C = \begin{bmatrix} \sqrt{\frac{2}{3}-s^2} & -\frac{1}{\sqrt{3}} & s \\ \frac{\sqrt{3}}{2}s+\frac{1}{2}\sqrt{\frac{2}{3}-s^2} & \frac{1}{\sqrt{3}} & \frac{1}{2}s-\frac{\sqrt{3}}{2}\sqrt{\frac{2}{3}-s^2} \\ -\frac{\sqrt{3}}{2}s+\frac{1}{2}\sqrt{\frac{2}{3}-s^2} & \frac{1}{\sqrt{3}} & \frac{1}{2}s+\frac{\sqrt{3}}{2}\sqrt{\frac{2}{3}-s^2} \end{bmatrix}.$$
   Since the only allowed generalization requires that $n=3$, $m=2$, $p=3$, for the present approach, we are led to the conclusion that the only finite symmetry groups that can connect the charged lepton and the neutrino mass matrices are either the discrete group A_4 or a group containing an A_4 subgroup and possessing a 3-dimensional representation (i.e. S_4). Observe that in the adopted formalism the middle column of C remains unchanged i.e. given by the column vector $\{-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\}^T$. It turns out that the so constructed generalization of the TB mixing matrix utilizes the freedom of making linear transformations inside the degenerate neutrino subspace to create a non vanishing value for the θ_{13} angle. This subspace is obviously orthogonal to the $\{-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\}$ axis.

5. The mass spectrum

We have found two models with identical predictions with respect to the mixing. The models differ only in the structure of the charged lepton mass matrix implying different coefficients $c_{1,2,3}$ for the two cases. Namely, if the mass matrix eigenvalues are given by m_1,m_2,m_3, and we define $\Delta m_{ij} = m_i - m_j$, the coefficient functions are given by

$$\begin{aligned} c_1^2 &= -\frac{1}{8}\csc^2\frac{\pi}{n}\sec\frac{\pi}{n}\left[-2m_2\cos\frac{\pi}{n} + m_3 e^{\frac{3i\pi}{n}} + m_1 e^{-\frac{3i\pi}{n}}\right] \\ c_2^2 &= \frac{1}{4}\csc^2\frac{\pi}{n}\left[\cos\frac{2\pi}{n}(\Delta m_{12} - \Delta m_{23}) - i\sin\frac{2\pi}{n}\Delta m_{13}\right] \\ c_3^2 &= \frac{1}{8}\csc\frac{\pi}{n}\left[\csc\frac{\pi}{n}(\Delta m_{23} - \Delta m_{12}) + i\sec\frac{\pi}{n}\Delta m_{13}\right] \end{aligned} \tag{5.1}$$

for $D_{2,n}$ and

$$\begin{aligned} c_1^3 &= -\frac{1}{8}\csc^2\frac{\pi}{n}\sec\frac{\pi}{n}\left[-2m_3\cos\frac{\pi}{n} + m_2 e^{\frac{3i\pi}{n}} + m_1 e^{-\frac{3i\pi}{n}}\right] \\ c_2^3 &= \frac{1}{4}\csc^2\frac{\pi}{n}\left[\cos\frac{2\pi}{n}(\Delta m_{13} + \Delta m_{23}) - i\sin\frac{2\pi}{n}\Delta m_{12}\right] \\ c_3^3 &= \frac{1}{8}\csc\frac{\pi}{n}\left[-\csc\frac{\pi}{n}(\Delta m_{13} + \Delta m_{23}) + i\sec\frac{\pi}{n}\Delta m_{12}\right] . \end{aligned} \tag{5.2}$$

for $D_{3,n}$.

The results found show that for leptons $n = 3$ while for neutrinos we get $m = 2$. The corresponding coefficient functions are:

- Charged Leptons

$$\begin{aligned} c_1^2 &= \frac{1}{3}(m_1 + m_2 + m_3) \\ c_2^2 &= \frac{1}{6}(2m_2 - m_1 - m_3) - \frac{i}{2\sqrt{3}}(m_1 - m_3) \\ c_3^2 &= \frac{1}{6}(2m_2 - m_1 - m_3) + \frac{i}{2\sqrt{3}}(m_1 - m_3) \end{aligned} \tag{5.3}$$

and

$$\begin{aligned} c_1^3 &= \frac{1}{3}(m_1 + m_2 + m_3) \\ c_2^3 &= \frac{1}{6}(2m_3 - m_1 - m_2) - \frac{i}{2\sqrt{3}}(m_1 - m_2) \\ c_3^3 &= \frac{1}{6}(2m_3 - m_1 - m_2) + \frac{i}{2\sqrt{3}}(m_1 - m_2) \end{aligned} \tag{5.4}$$

- Neutrinos

For the neutrinos $m = 2$. The D_2 and the corresponding neutrino matrices are given by:

$$\begin{aligned} D_2 &= \text{Diagonal}\,[-1, 1, -1] \\ M &= \text{Diagonal}\,[m_1, m_2, m_3] \ \cdot \end{aligned} \tag{5.5}$$

In this case, it is clear that the neutrino mass spectrum turns out to be degenerate since ${D_2}^2 = 1$ implying $m_1 = m_3$. In order to establish a breaking pattern we must write

$$M = d_1 I + d_2 D_2 + R_2$$

where R_2 is the remainder term to be determined.

$$R_2 = \begin{bmatrix} m_1 - d_1 + d_2 & 0 & 0 \\ 0 & m_1 - d_1 - d_2 & 0 \\ 0 & 0 & m_3 - d_1 + d_2 \end{bmatrix} .$$

Each non vanishing element must be proportional to the same mass difference in order to have a breaking pattern. Since

$$(R_1)_{11} - (R_1)_{33} = m_1 - m_3$$

this mass difference is $m_1 - m_3$ if no special relations between neutrino masses are assumed. So we have

$$\begin{aligned} m_1 - d_1 + d_2 &= r_1 (m_1 - m_3) \\ m_2 - d_1 - d_2 &= r_2 (m_1 - m_3) \\ m_3 - d_1 + d_2 &= r_3 (m_1 - m_3) \end{aligned}$$

with $r_1 - r_3 = 1$. Solving for d_1, d_2 we get

$$\begin{aligned} d_1 &= \frac{1}{2}(m_1 + m_2) - \frac{1}{2}(r_1 + r_2)(m_1 - m_3) \\ d_2 &= -\frac{1}{2}(m_1 - m_2) + \frac{1}{2}(r_1 - r_2)(m_1 - m_3) \end{aligned}$$

for arbitrary r_1 and r_2. A different breaking pattern would require invariant relations between the neutrino masses. For instance if we require that

$$\begin{aligned} m_1 - d_1 + d_2 &= r_1 (m_1 - m_2) \\ m_2 - d_1 - d_2 &= r_2 (m_1 - m_2) \\ m_3 - d_1 + d_2 &= r_3 (m_1 - m_2) \end{aligned}$$

consistency implies that

$$r_3 = r_1 + \frac{m_3 - m_1}{m_1 - m_2}$$

independent of the masses i.e. $\frac{m_3 - m_1}{m_1 - m_2} = \mu$, and

$$d_1 = \frac{1}{2}(m_1 + m_2) - \frac{1}{2}(r_1 + r_2)(m_1 - m_2)$$

$$d_2 = \frac{1}{2}(r_1 - r_2 - 1)(m_1 - m_2) \cdot$$

6. Conclusions

In this work we have examined the structure of the lepton and neutrino mass matrices assuming that they can be expanded as polynomials of finite group elements which act as generators. This procedure has been proven useful for putting some order on the enormous number of possibilities given in the literature in a systematic and mathematically consistent way since it does not involve the mass eigenvalues. Various models are classified by means of three integer numbers which define the group. The calculations show that the number of finite groups that can reproduce current data by allowing a non zero value for the θ_{13} mixing angle is restricted. The groups allowed are either A_4 or a group containing an A_4 subgroup and possessing a 3-dimensional representation (i.e.S_4). Exact group symmetry introduces a degeneracy in the neutrino spectrum which has to lifted by means of an external breaking mechanism.

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