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Large muon $(g-2)$ from TeV-scale MSSM with infinite tanβ

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The muon anomalous magnetic moment a_μ is studied in the infinite tanβ limit of the MSSM. Since the muon mass arises completely from loop effects, large corrections to a_μ are expected in comparison to the usual case with moderately high tanβ. Due to the qualitatively different parameter dependence, the gap between the experimental value and the Standard Model prediction can be filled only in parameter volumes in which a mass hierarchy suppresses the chargino loop in favour of the neutralino contribution. Two such possibilities are found to have either large Higgsino mass or large muon sneutrino mass. Supersymmetric particles even at the TeV scale can lead to the best fit of a_μ.

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The muon anomalous magnetic moment, $a_\mu \equiv (g_\mu - 2)/2$, has long been a prime observable sensitive to hypothetical virtual states, and hence a major indirect probe to new physics. Currently, there is an interesting discrepancy between the experimental and the Standard Model (SM) values of a_μ [1]:

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = (28.7 \pm 8.0) \times 10^{-10}, \tag{1}$$

which amounts to more than 3σ. This has called for many attempts to fill the gap with various new physics contributions.

In this respect, one noteworthy property of a_μ is that it is correlated with loop corrections to the muon mass m_μ. This is easy to see by comparing the schematic diagrams of a_μ and the muon self energy Σ_μ, shown in figure 1. Manifestly, attaching a photon line to the Σ_μ graph on the right

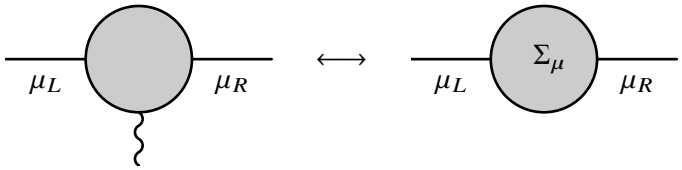

Figure 1: Diagrammatic similarity between a_μ (left) and the muon self energy (right).

results in the a_μ diagram on the left. This means that large loop corrections to m_μ generically imply large contributions to a_μ.

For this reason, there have been significant interests in models where m_μ arises solely from loop effects. This idea may be realized for instance within a popular model such as the minimal supersymmetric standard model (MSSM). In this model, the tree-level muon mass is given by

$$m_\mu^{\rm tree} = y_\mu v_d, \tag{2}$$

the product of y_μ, the muon Yukawa coupling, and v_d, the vacuum expectation value (VEV) of the down-type Higgs. Obviously, this leads to the following two options to eliminate the tree-level muon mass: (a) $y_\mu = 0$ or (b) $v_d = 0$. There are already studies which employ the former approach while using the non-holomorphic [2] or the holomorphic [3] smuon trilinear coupling for the radiative generation of m_μ. We consider the latter option focusing on the supersymmetric contributions to a_μ [4] which shall be the subject of this presentation.

The supersymmetric contributions to a_μ at the one-loop level are well-known in the MSSM [5]. They arise from the chargino-sneutrino and the neutralino-smuon diagrams which depend on the following five mass parameters: the Higgsino mass μ, the bino mass M_1, the wino mass M_2 as well as the soft masses of the left- and the right-handed smuons, m_L and m_R. In a simplified case where all these five parameters are equal to $M_{\rm SUSY}$, the above contributions can be approximated by

$$a_\mu^{\rm SUSY,1L} \approx 13 \times 10^{-10}\ {\rm sign}(\mu) \tan\beta \left(\frac{100\ {\rm GeV}}{M_{\rm SUSY}}\right)^2. \tag{3}$$

One can notice the following properties of $a_\mu^{\rm SUSY,1L}$: (a) it is suppressed by the second power of $M_{\rm SUSY}$, the new physics scale, (b) it is proportional to $\tan\beta$, (c) its sign is determined by the sign

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of μ. For a usual value of $\tan\beta \lesssim 60$, the best fit of (1) is achieved only if $M_{\rm SUSY} \lesssim 500$ GeV. Now that the Large Hadron Collider is putting more and more stringent lower bounds on the supersymmetric particle masses, one may consider how to fit (1) even for higher $M_{\rm SUSY}$.

To this end, a straightforward attempt would be to push $\tan\beta$ up beyond the usual range. This has been however a less explored possibility. The fear is mostly based on the perturbativity of the down-type Yukawa couplings. As they are proportional to $\tan\beta$ under the tree-level approximation, one might naively expect them to grow nonperturbatively large for too high $\tan\beta$. It is however well-known that the down-type fermion masses can receive significant loop corrections which pick up the up-type Higgs VEV v_u [6]. This effect may be expressed in the form,

$$m_f = y_f\, v_d + y_f\, v_u\, \Delta_f^{\rm red}, \tag{4}$$

where m_f is the pole mass of f standing for a charged lepton or down-type quark, y_f is its Yukawa coupling, and $\Delta_f^{\rm red}$ is a finite quantity calculable from the self energy diagrams. This implies that m_f might be correct even if v_d is unconventionally small or even vanishes. Indeed, it has been shown that such a possibility can meet various phenomenological constraints [7, 8].

With $\Delta_f^{\rm red}$ from (4) taken into account, the supersymmetric contribution to a_μ becomes [9]

$$a_\mu^{\rm SUSY} = \frac{a_\mu^{\rm SUSY,1L}}{1+\tan\beta\Delta_\mu^{\rm red}} \equiv \frac{y_\mu v_u}{m_\mu} a_\mu^{\rm red}, \tag{5}$$

where $a_\mu^{\rm SUSY,1L}$, appearing on the left-hand side of (3), is the supersymmetric one-loop contribution to a_μ without resummation of $\tan\beta$-enhanced terms. One can understand the first equality above in the following way. Due to the chiral symmetry, both $a_\mu^{\rm SUSY}$ and $a_\mu^{\rm SUSY,1L}$ contain a factor of y_μ which is rewritten in terms of the other quantities appearing in (4). This replacement is done however to differing accuracies, i.e. the $\Delta_f^{\rm red}$ term in (4) is neglected for $a_\mu^{\rm SUSY,1L}$ whereas it is included for $a_\mu^{\rm SUSY}$. The above resummation formula then follows. Another quantity $a_\mu^{\rm red}$ is defined in (5), in terms of which the infinite $\tan\beta$ limit of $a_\mu^{\rm SUSY}$ reads simply

$$\lim_{\tan\beta\to\infty} a_\mu^{\rm SUSY} = \frac{a_\mu^{\rm red}}{\Delta_\mu^{\rm red}}, \tag{6}$$

which follows from (4) and (5).

In this limit, one can again consider a simplified case where the five mass parameters are the same to obtain an expression analogous to (3). This results in

$$\lim_{\tan\beta\to\infty} a_\mu^{\rm SUSY} \approx -72\times10^{-10}\left(\frac{1\ {\rm TeV}}{M_{\rm SUSY}}\right)^2. \tag{7}$$

For the same new physics scale $M_{\rm SUSY}$, the magnitude of $a_\mu^{\rm SUSY}$ above does greatly exceed that of (3) valid for moderate $\tan\beta$. Unfortunately, the sign turns out to be wrong which, being independent of the μ sign, seems unfixable. This is because the diagrams of both $a_\mu^{\rm red}$ and $\Delta_\mu^{\rm red}$ pick up a Higgsino mass due to the Peccei-Quinn symmetry and therefore the μ sign cancels out of (6).

It is still early to give up. One can get a hint on how to overcome this sign problem by closer inspection of the structures of $a_\mu^{\rm red}$ and $\Delta_\mu^{\rm red}$ appearing in the fraction of (6). Schematically, the

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contributions to the numerator and the denominator look like

$$\lim_{\tan\beta\to\infty} a_\mu^{\rm SUSY} = \frac{a_\mu^{\rm red}}{\Delta_\mu^{\rm red}} \sim \frac{+(\chi^\pm \text{ term}) + (\chi^0 \text{ terms})}{-(\chi^\pm \text{ term}) + (\chi^0 \text{ terms})}, \tag{8}$$

i.e. both $a_\mu^{\rm red}$ and $\Delta_\mu^{\rm red}$ consist of the chargino terms and the neutralino terms whose diagrams are shown in figure 2 in the mass insertion approximation. The last fraction above is intended

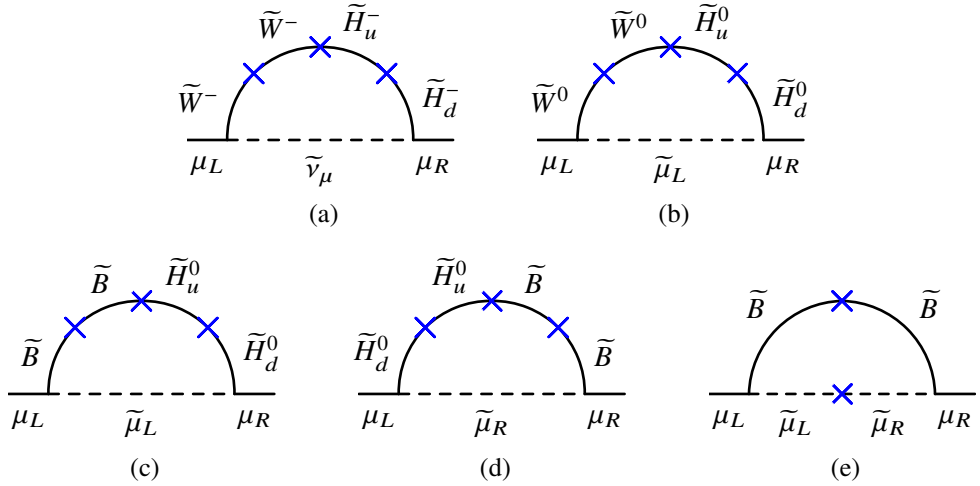

Figure 2: Mass-insertion diagrams for $\Delta_\mu^{\rm red}$ and $a_\mu^{\rm red}$. For the latter, an external photon couples to any of the charged particles in the loop.

to mean that the numerical values of the chargino terms in $a_\mu^{\rm red}$ and $\Delta_\mu^{\rm red}$ have opposite signs to each other whereas the neutralino terms have the same sign. When the five mass parameters are of similar sizes, the chargino terms tend to dominate in both $a_\mu^{\rm red}$ and $\Delta_\mu^{\rm red}$ thereby resulting in negative $a_\mu^{\rm SUSY}$. A way to turn around the sign would then be to let the neutralino terms dominate or equivalently to suppress the chargino terms in both $a_\mu^{\rm red}$ and $\Delta_\mu^{\rm red}$. For this, one may suppress any of the propagators forming the loop of the chargino graph in figure 2(a), as long as some of the neutralino diagrams survive. This leads to the following three solutions: (a) "large-μ limit" which suppresses the Higgsino propagators as shown in figure 3 while leaving diagram 3(e) unsuppressed, (b) "$\tilde{\mu}_R$-dominance" which suppresses the sneutrino propagator by raising m_L as shown in figure 4 while maintaining the contribution from figure 4(d), (c) suppression of the wino propagators as in figure 5 [as well as the $\tilde{\mu}_R$ propagators to isolate diagram 5(c)] by raising $|M_2|$ and m_R. It turns out that solution (c) requires extreme mass hierarchies to work in practice and is therefore less interesting than the other two options.

As explained above, $a_\mu^{\rm SUSY}$ is invariant under a μ sign flip. Furthermore, a simultaneous sign flip of both M_1 and M_2 leaves $a_\mu^{\rm SUSY}$ in (6) invariant, as one can understand from each diagram in figure 2 which picks up the sign of either M_1 or M_2. Taking advantage of these symmetries, we shall assume in what follows that μ and M_1 are positive without loss of generality while leaving the M_2 sign free.

With the above qualitative observations in mind, one can examine in more detail how the sign of $a_\mu^{\rm SUSY}$ depends on the mass parameters. Two plots are shown in figure 6 employing the mass insertion approximation under which the sign of $a_\mu^{\rm SUSY}$ is fully determined by the ratios of the

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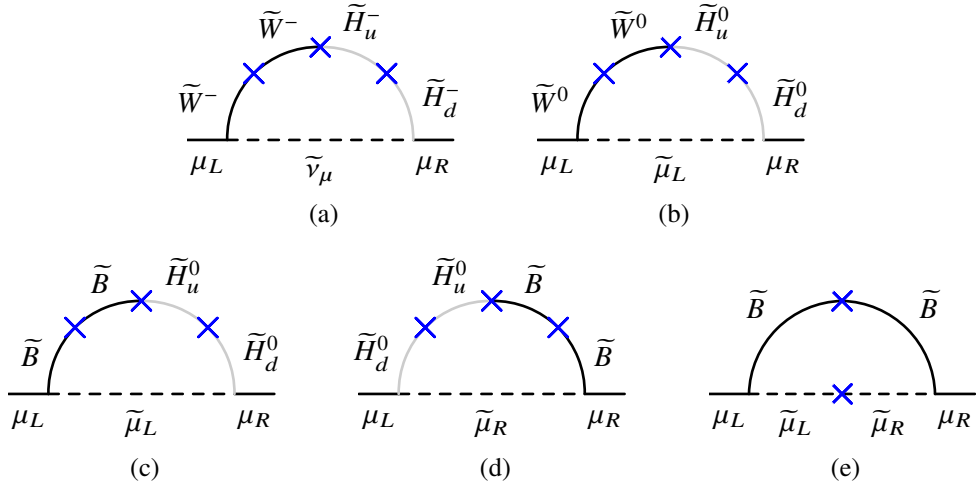

Figure 3: Same diagrams as in figure 2 in the "large-μ limit". Grey lines represent mass-suppressed propagators.

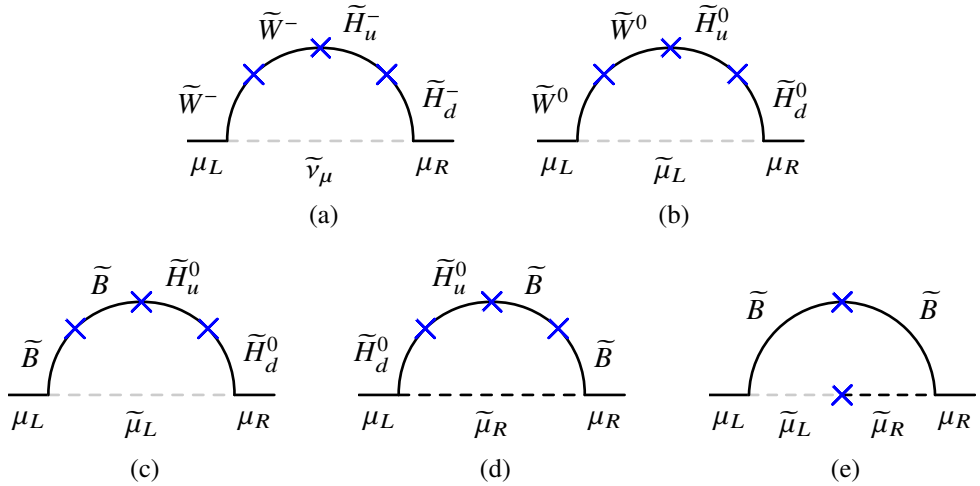

Figure 4: Same diagrams as in figure 2 in the case of "$\tilde{\mu}_R$-dominance". Grey lines represent mass-suppressed propagators.

five mass parameters. In figure 6(a), the origin at which the mass parameters are equal leads to negative $a_\mu^{\rm SUSY}$ as already seen in (7). In the same plot, one finds indeed the two types of regions where $a_\mu^{\rm SUSY}$ is positive as expected from the aforementioned mass hierarchies: the white regions on the right and around the upper border correspond to the "large-μ limit" and "$\tilde{\mu}_R$-dominance", respectively. The less interesting solution (c) is visible as a small white area around the left part of the bottom border of figure 6(b), in which $|M_2|$ and m_R are much larger than the other three mass parameters.

Given a set of the mass parameter ratios leading to the desired sign of $a_\mu^{\rm SUSY}$, its size can be adjusted to fit (1) by choosing an overall mass scale. Obviously, the higher the mass scale is, the smaller $a_\mu^{\rm SUSY}$ becomes. We parametrize the scale by $M_{\rm SUSY,min}$, the smallest of $\{\mu, M_1, |M_2|, m_L, m_R\}$. The results are presented in figure 7 in which one finds regions with colours corresponding to

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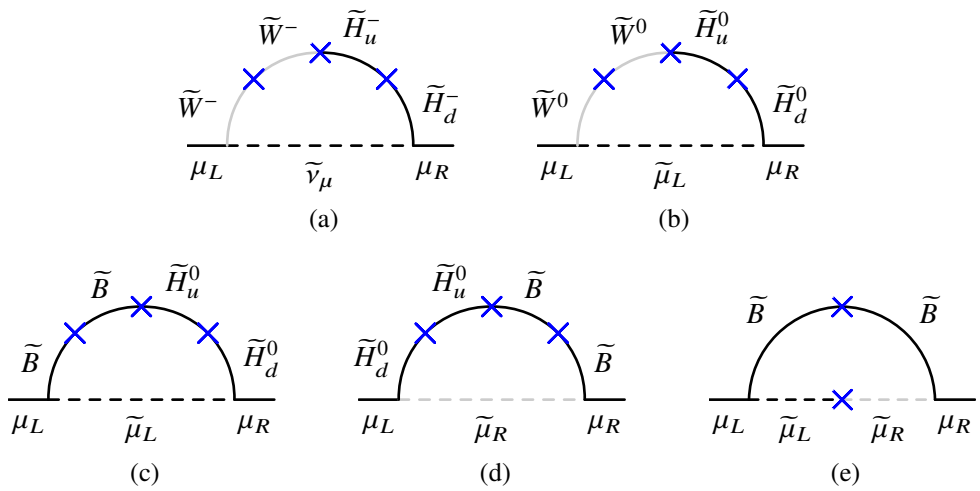

Figure 5: Same diagrams as in figure 2 in the case where $M_1, \mu, m_L \ll |M_2|, m_R$. Grey lines represent mass-suppressed propagators.

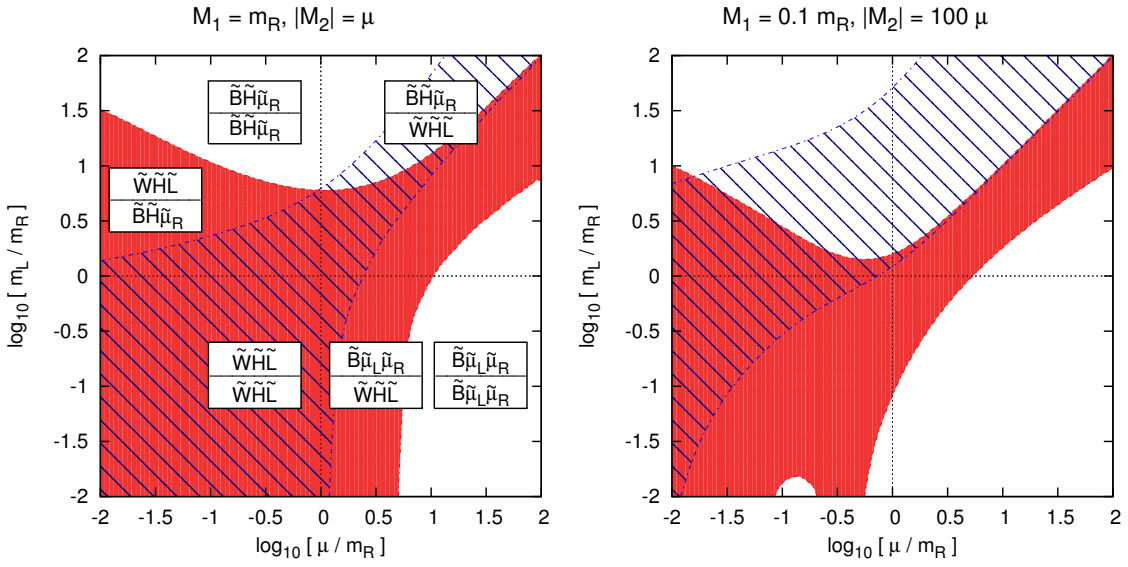

Figure 6: Sign of $a_\mu^{\rm SUSY}$ on the plane of $(\mu/m_R, m_L/m_R)$ for the two signs of M_2. The remaining two mass ratios are fixed as shown above each plot. The sign of $a_\mu^{\rm SUSY}$ in each region is: + in white, sign$(-M_2)$ in red, sign$(+M_2)$ in hatched, − in overlap.

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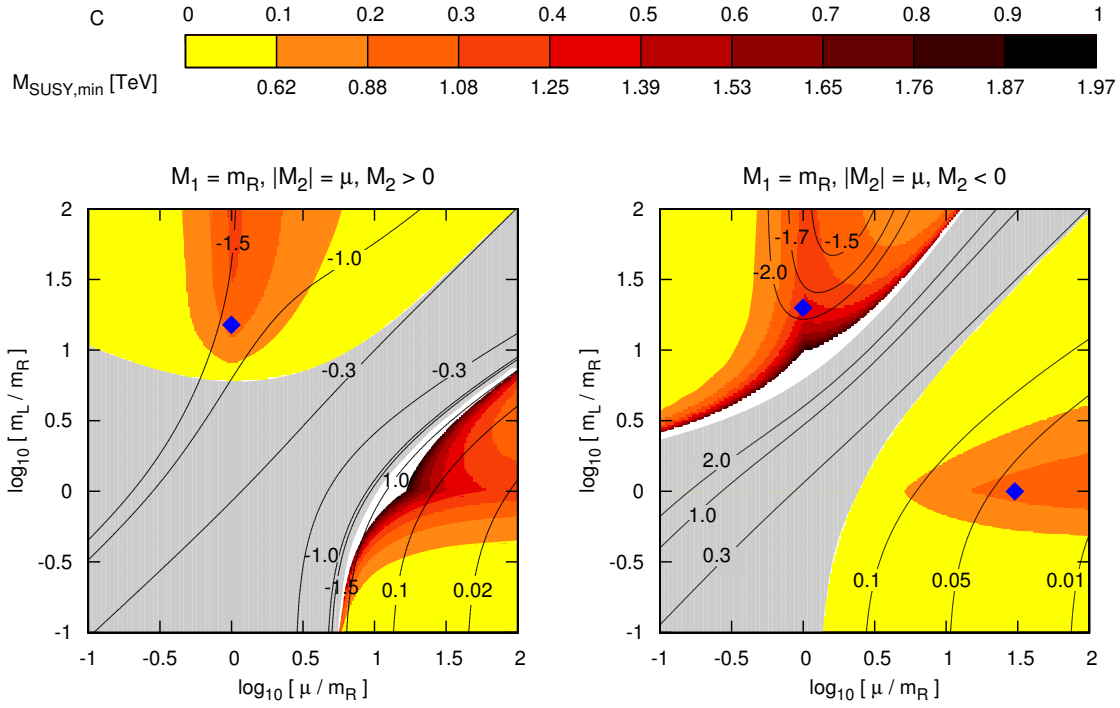

(a) (b)

Figure 7: Values of $M_{\rm SUSY,min}$ for the best fit of $a_\mu^{\rm SUSY}$ are represented on the plane of $(\mu/m_R, m_L/m_R)$ as the gradation of colour, for (a) positive and (b) negative M_2. The grey regions lead to negative $a_\mu^{\rm SUSY}$. The contour lines indicate y_μ. In the white regions, $|y_\mu|$ is nonperturbatively large. The blue diamonds are the specimen points from table 1.

$M_{\rm SUSY,min}$ around 1 TeV or higher. This reveals a promising possibility that supersymmetric particles at the TeV-scale or higher can explain (1), which is the motivation to consider the infinite $\tan\beta$ limit in this work.

For this scenario to be viable, it must meet all relevant constraints. The lepton flavour violating process $\mu \to e\gamma$ can be suppressed by assuming small enough slepton mixing between the first two generations. The correlation between $\mu \to e\gamma$ and a_μ [10] still holds for $\tan\beta \to \infty$. One can also satisfy B-physics constraints [8]. The most dangerous decay mode is $B^+ \to \tau^+\nu$ to which the charged Higgs exchange contribution can be suppressed enough by raising $M_{H^\pm}$ to a few TeV. Pollution to $B_s \to \mu^+\mu^-$ and $B \to X_s\,\gamma$ can be suppressed with vanishing A_t and flavour-violating squark mass insertions. For the former and the latter processes, it further helps to raise the heavy Higgs and the squark masses, respectively. One can make the lightest Higgs mass and decays SM-like by staying in the decoupling regime [11].

Moreover, we impose the following constraints: (a) charginos and smuons are heavier than 100 GeV, (b) y_μ is perturbative, (c) our vacuum is stable or long-lived on the cosmological timescale. To evaluate the false vacuum lifetime, we use the method from ref. [12]. Each of these requirements excludes some regions depicted in figure 8. The plots show that great parts of the "large-μ" region for negative M_2 as well as the "$\tilde{\mu}_R$-dominance" region for either sign of M_2

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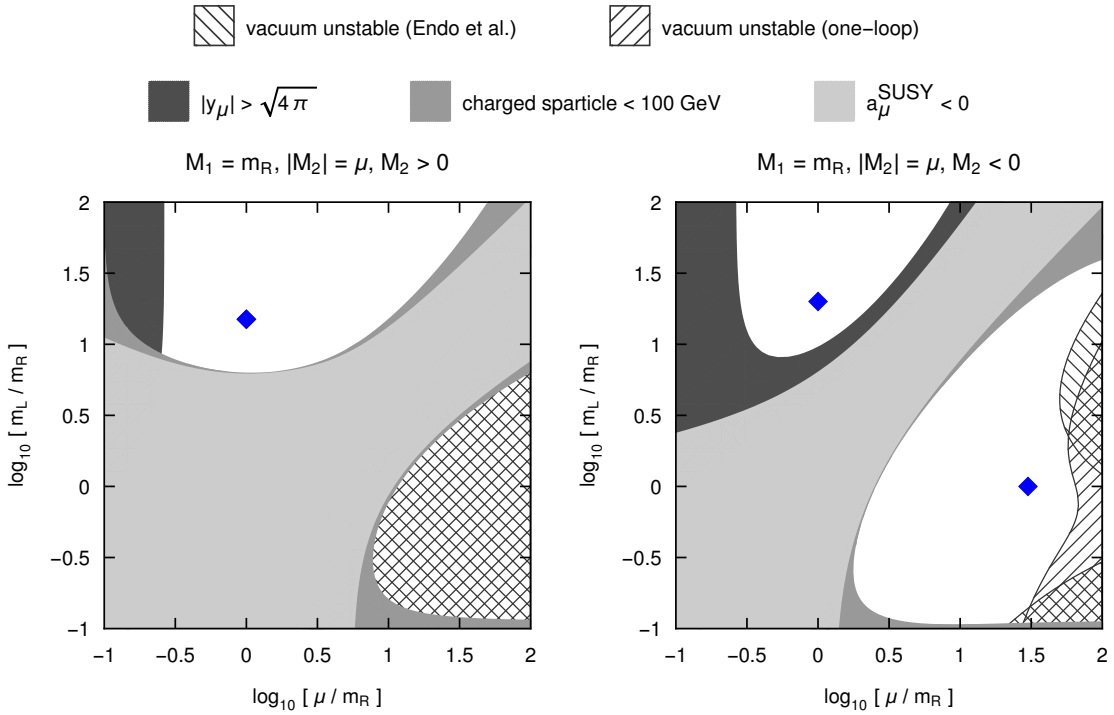

(a) (b)

Figure 8: Regions on the plane of $(\mu/m_R, m_L/m_R)$ excluded by the constraints indicated in the legend, for (a) positive and (b) negative M_2. The blue diamonds are the specimen points from table 1.

survive the constraints. This means that there are indeed parameter volumes in which TeV-scale supersymmetric particles can account for the measured value of a_μ. We also check that y_τ and y_b, the tau and the bottom Yukawas, can be perturbative and consistent with the vacuum metastability. Even though m_τ, bigger than m_μ, might cause a worry about too large $|y_\tau|$, there is room for perturbative radiative generation of m_τ since $(g_\tau - 2)$ does not need to be explained. The even larger bottom quark pole mass can also be perturbatively generated thanks to the gluino-loop contribution in addition to the other types of loops shared by the tau self energy.

The concrete values of the five mass parameters at selected points from figures 7 and 8 are listed in table 1. The first two points belong to the "large-μ limit" with negative M_2, and the rest to the "$\tilde{\mu}_R$-dominance" regime with either sign of M_2.

To find maximal $M_{\mathrm{SUSY,min}}$ which can fit a_μ, we explore the full five-dimensional parameter space, relaxing the equalities, $M_1 = m_R$ and $|M_2| = \mu$, which we assumed for the sake of planar presentation of the preceding plots. The resulting maximum $M_{\mathrm{SUSY,min}}$ is plotted in figure 9. The plots reveal that the relaxation of the above mass equalities allows $M_{\mathrm{SUSY,min}}$ to be slightly higher than before.

For a compact summary, we derive a formula resembling (7),

$$a_\mu^{\mathrm{SUSY}} \approx 37 \times 10^{-10} \left(\frac{1\ \mathrm{TeV}}{M_{\mathrm{SUSY}}} \right)^2 , \tag{9}$$

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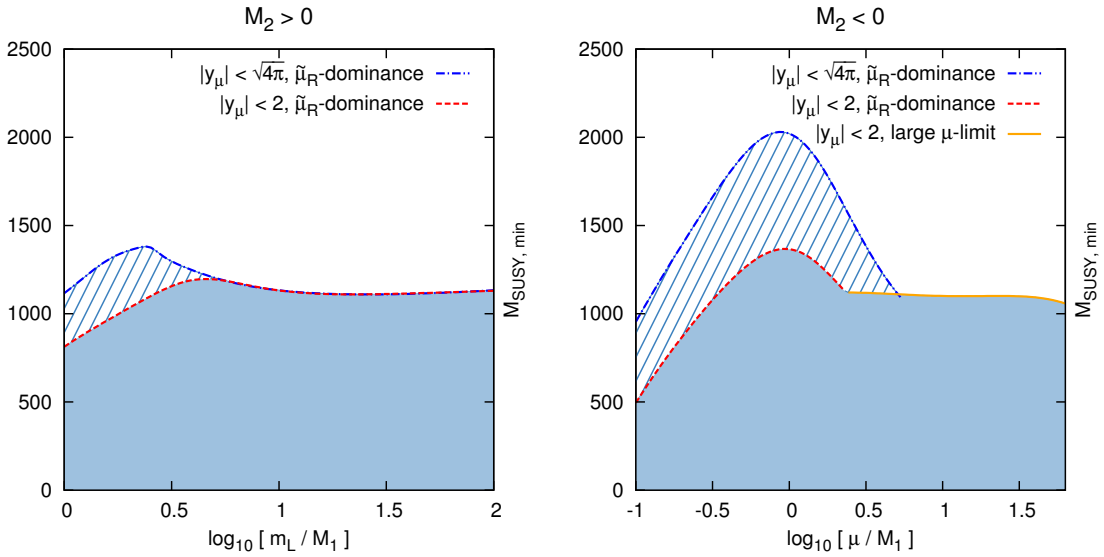
μ	M_1	M_2	m_L	m_R	$a_\mu^{\rm SUSY}/10^{-9}$	y_μ	Characteristic
30	1	−30	1	1	2.80	0.04	large-μ
15	1	−1	1	1	3.01	0.09	large-μ
1	1	1	15	1	2.64	−1.37	$\tilde{\mu}_R$-dominance
1	1	30	30	1	2.77	−1.18	$\tilde{\mu}_R$-dominance
1.3	1.3	−1.3	26	1.3	2.90	−1.89	$\tilde{\mu}_R$-dominance

Table 1: Specimen points leading to a reasonable fit of a_μ. Masses are in TeV.



(a) (b)

Figure 9: Maximum $M_{\rm SUSY,min}$ that can fit a_μ as a function of (a) m_L/M_1 for $M_2 > 0$ and (b) μ/M_1 for $M_2 < 0$. Each line style indicates to which regime the point belongs as well as the cutoff on $|y_\mu|$. The vacuum metastability is required.

which applies if either $|\mu| \gg |M_1| = m_L = m_R \equiv M_{\rm SUSY}$ or $m_L \gg |\mu| = |M_1| = m_R \equiv M_{\rm SUSY}$. These are two mass hierarchies representative of the "large-μ limit" and the "$\tilde{\mu}_R$-dominance", respectively.

In summary, we considered the infinite $\tan\beta$ limit of the MSSM as a possibility to account for the discrepancy between the experimental value and the SM prediction of a_μ even if the supersymmetric particles are as heavy as 1 TeV or higher. The motivation was the observation that fully radiative muon mass generation would imply large new physics effects on a_μ. We found two successful types of mass hierarchies, the "large-μ limit", and the "$\tilde{\mu}_R$-dominance", which allowed us to achieve the goal. We took into account phenomenological constraints from collider searches, flavour and Higgs physics, as well as theoretical constraints from perturbativity and vacuum metastability. For more details of the analysis, we refer the reader to ref. [4].

For those a_μ enthusiasts, we add a note on GM2Calc, a calculator of the MSSM contributions

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to a_μ [13]. It can approximate the infinite $\tan\beta$ limit as it works for arbitrarily high $\tan\beta$.

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