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What is a Natural SUSY scenario?

J. A. Casas, J. M. Moreno*, **K. Rolbiecki**
Instituto de Física Teórica, IFT-UAM/CSIC,
Universidad Autónoma de Madrid,
Cantoblanco, 28049 Madrid, Spain
E-mail: jesus.moreno@csic.es, alberto.casas@uam.es, krzysztof.rolbiecki@desy.de

S. Robles
Instituto de Física Teórica, IFT-UAM/CSIC & Dep. Física Teórica,
Universidad Autónoma de Madrid,
Cantoblanco, 28049 Madrid, Spain
E-mail: sandra.robles@uam.es

B. Zaldívar
Service de Physique Théorique, Université Libre de Bruxelles,
Boulevard du Triomphe, CP225, 1050 Brussels, Belgium
E-mail: bryan.zaldivar@ulb.ac.be

Being the naturalness problem one of the main motivations for supersymmetric theories, it is reasonable to explore supersymmetric phenomenology focusing on scenarios where the fine-tuning is as mild as possible. Models in which the fine-tuning is kept under control are known as "Natural SUSY" ones. We re-examine here this issue in the context of the MSSM including several improvements, such as the mixing of the fine-tuning conditions for different soft terms and the presence of potential extra fine-tunings that must be combined with the electroweak one. We analyze in detail the complete fine-tuning bounds for the unconstrained MSSM, defined at any high-energy (HE) scale. We show that Natural SUSY does *not* demand light stops. Regarding phenomenology, the most stringent upper bound from naturalness is the one on the gluino mass, which typically sets the present level fine-tuning at $\mathscr{O}(1\%)$. However, this result presents a strong dependence on the HE scale. E.g. if the latter is 10^7 GeV the level of fine-tuning is $\sim$ four times less severe. The most robust result of Natural SUSY is by far that Higgsinos should be rather light, certainly below 700 GeV for a fine-tuning of $\mathscr{O}(1\%)$ or milder. Finally, as an application, we evaluate the fine-tuning in the minimal Gauge-mediated SUSY breaking model

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1. The “standard" Natural SUSY

Naturalness arguments have been used since long ago [1] to constrain from above supersymmetric masses[1]. Already in the LHC era, they were re-visited in ref. [4] to formulate the so-called Natural SUSY scenario, that we summarize here. Assuming that the extra (supersymmetric) Higgs states are heavy enough, the Higgs potential can be written in the Standard Model (SM) way

$$V = m^2|H|^2 + \lambda|H|^4 \,, \tag{1.1}$$

where the SM-like Higgs doublet, H, is a linear combination of the two supersymmetric Higgs doublets, $H \sim \sin\beta H_u + \cos\beta H_d$. Then, the absence of fine-tuning can be expressed as the requirement of not-too-large contributions to the Higss mass parameter, m^2. Since the physical Higgs mass is $m_h^2 = 2|m^2|$, a sound measure[2] of the fine-tuning is [5]

$$\tilde{\Delta} = \left|\frac{\delta m^2}{m^2}\right| = \frac{2\delta m^2}{m_h^2} \,. \tag{1.2}$$

For large $\tan\beta$, the value of m^2 is given by $m^2 = |\mu|^2 + m_{H_u}^2$, so one immediately notes that both μ and m_{H_u} should be not-too-large in order to avoid fine-tuning. For the $\mu-$parameter this implies

$$\mu \lesssim 200\,\mathrm{GeV}\left(\frac{m_h}{120\,\mathrm{GeV}}\right)\left(\frac{\tilde{\Delta}^{-1}}{20\%}\right)^{-1/2} \,. \tag{1.3}$$

This sets a constraint on Higgsino masses. Constraints for other particles come from the radiative corrections to $m_{H_u}^2$. The most important contribution comes from the stops. Following ref. [4]

$$\delta m_{H_u}^2|_{\mathrm{stop}} = -\frac{3}{8\pi^2}y_t^2\left(m_{Q_3}^2 + m_{U_3}^2 + |A_t|^2\right)\log\left(\frac{\Lambda}{\mathrm{TeV}}\right), \tag{1.4}$$

where Λ denotes the scale of the transmission of SUSY breaking to the observable sector and the 1-loop leading-log (LL) approximation was used to integrate the renormalization-group equation (RGE). Then, the above soft parameters $m_{Q_3}^2$, $m_{U_3}^2$ and A_t are to be understood at low-energy, and thus they control the stop spectrum. This sets an upper bound on the stop masses. In particular one has

$$\sqrt{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2} \lesssim 600\,\mathrm{GeV}\frac{\sin\beta}{(1+x^2)^{1/2}}\left(\frac{\log(\Lambda/\mathrm{TeV})}{3}\right)^{-1/2}\left(\frac{\tilde{\Delta}^{-1}}{20\%}\right)^{-1/2} , \tag{1.5}$$

where $x = A_t/\sqrt{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}$. Eq. (1.5) imposes a bound on the lightest stop. Besides the stops, the most important contribution to m_{H_u} is the gluino one, due to its large 1-loop RG correction to the stop masses. Again, in the 1-loop LL approximation used in ref. [4], one gets

$$\delta m_{H_u}^2|_{\mathrm{gluino}} \simeq -\frac{2}{\pi^2}y_t^2\left(\frac{\alpha_s}{\pi}\right)|M_3|^2\log^2\left(\frac{\Lambda}{\mathrm{TeV}}\right) , \tag{1.6}$$

[1]For a partial list of references on naturalness in SUSY, see ref. [2, 3]

[2]This measure produces similar results to the somewhat standard parametrization of the fine-tuning, see eq.(3.3) below.

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where M_3 is the gluino mass. From the previous equation,

$$M_3 \lesssim 950\,\mathrm{GeV}\sin\beta \left(\frac{\log(\Lambda/\mathrm{TeV})}{3}\right)^{-1}\left(\frac{m_h}{125\,\mathrm{GeV}}\right)\left(\frac{\tilde{\Delta}^{-1}}{20\%}\right)^{-1/2}. \tag{1.7}$$

Altogether, the summary of the minimal requirements for a natural SUSY spectrum, as given in ref. [4], is: i) two stops and one (left-handed) sbottom, both below $500-700$ GeV, ii) two Higgsinos, *i.e.*, one chargino and two neutralinos, below $200-350$ GeV, iii) a not too heavy gluino, below 900 GeV $-$ 1.5 TeV.

In the next section we point out the weak points of the above arguments that support the ‘standard" Natural SUSY scenario. Some of them have been addressed in the literature after ref. [4].

2. The Natural SUSY scenario. A critical review

2.1 The dependence on the initial parameters

The one-loop LL approximation used to write eqs.(1.5, 1.6), from which the naturalness bounds were obtained, is too simplistic in two different aspects.

First, it is not accurate enough since the top Yukawa-coupling, y_t, and the strong coupling, α_s, are large and vary a lot along the RG running. As a result, the soft masses evolve greatly and cannot be considered as constant, even as a rough estimate. This effect can be incorporated by integrating numerically the RGE, which corresponds to summing the leading-logs at all orders [6, 7].

Second, and even more important, the physical squark, gluino and electroweakino masses are not initial parameters, but rather a low-energy consequence of the initial parameters at the high-energy scale. Notice that there is not one-to-one correspondence between the initial parameters and the physical quantities, since the former get mixed along their coupled RGEs. Consequently, it is not possible in general to determine individual upper bounds on the physical masses, not even on the initial parameters. Instead, one should expect to obtain contour-surfaces with equal degree of fine-tuning in the parameter-space and, similarly, in the “space" of the possible supersymmetric spectra. A second complication is that the results depend (sometimes critically) on what one considers as initial parameters.

From the previous discussion it turns out that the most rigorous way to analyze the fine-tuning is to determine the full dependence of the electroweak scale (and other potentially fine-tuned quantities) on the initial parameters, and then derive the regions of constant fine-tuning in the parameter space. These regions can be (non-trivially) translated into constant fine-tuning regions in the space of possible physical spectra. This goal is enormously simplified if one determines in the first place the analytical dependence of low-energy quantities on the high-energy initial parameters.

2.2 Fine-tunings left aside

In a MSSM scenario, there are two implicit potential fine-tunings that have to be taken into account to evaluate the global degree of fine-tuning. They stem from the need of having a physical Higgs mass consistent with $m_h^{\mathrm{exp}} \simeq 125$ GeV and from the requirement of rather large $\tan\beta$. Let us comment on them in order.

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Fine-tuning to get $m_h^{\rm exp} \simeq 125$ GeV

As is well known, the tree-level Higgs mass in the MSSM is given by $(m_h^2)_{\rm tree-level} = M_Z^2 \cos^2 2\beta$, so both large $\tan\beta$ values and radiative corrections are needed in order to reconcile it with the experimental value. A simplified expression of such corrections [8, 9, 10], useful for the sake of the discussion, is

$$\delta m_h^2 = \frac{3G_F}{\sqrt{2}\pi^2} m_t^4 \left(\log\left(\frac{\overline{m}_{\tilde t}^2}{m_t^2}\right) + \frac{X_t^2}{\overline{m}_{\tilde t}^2}\left(1 - \frac{X_t^2}{12\overline{m}_{\tilde t}^2}\right)\right) + \cdots , \tag{2.1}$$

with $\overline{m}_{\tilde t}$ the average stop mass and $X_t = A_t - \mu\cot\beta$. The X_t-contribution arises from the threshold corrections to the quartic coupling at the stop scale. This correction is maximized for $X_t = \sqrt{6}\overline{m}_{\tilde t}$. Notice that if the threshold correction were not present one would need heavy stops [11] for large $\tan\beta$, which is inconsistent with the requirements of Natural SUSY in its original formulation. However, taking X_t close to the “maximal" value, it is possible to obtain the correct Higgs mass with rather light stops, even in the $500-700$ GeV range; a fact frequently invoked in the literature to reconcile the Higgs mass with Natural SUSY.

On the other side, requiring $X_t \sim$ maximal entails an additional fine-tuning that has to be properly taken into account.

Fine-tuning to get large $\tan\beta$

The value of $\tan\beta \equiv \langle H_u\rangle/\langle H_d\rangle$ is given, at tree level, by

$$\frac{2}{\tan\beta} \simeq \sin 2\beta = \frac{2B\mu}{m_{H_d}^2 + m_{H_u}^2 + 2\mu^2} = \frac{2B\mu}{m_A^2} , \tag{2.2}$$

where m_A is the mass of the pseudoscalar Higgs state; all the quantities above are understood to be evaluated at the low-scale. Clearly, in order to get large $\tan\beta$ one needs small $B\mu$ at low-energy. However, even starting with vanishing B at M_X one gets a large radiative correction due to the RG running. Consequently, very large values of $\tan\beta$ are very fine-tuned[3], as they require a cancellation between the initial value of B and the radiative contributions. On the other hand, moderately large values may be non-fined-tuned, depending on the size of the RG contribution to $B\mu$ and the value of m_A. Hence, a complete analysis of the MSSM naturalness has to address this potential source of fine-tuning.

3. The electroweak fine-tuning of the MSSM

Natural SUSY in the MSSM requires large $\tan\beta$ values, otherwise the radiative corrections needed to reconcile the Higgs mass with its experimental value, would imply gigantic stop masses [11]. In this large limit, the minimization relation reads:

$$-\frac{1}{8}(g^2 + g'^2)v^2 = -\frac{M_Z^2}{2} = \mu^2 + m_{H_u}^2 . \tag{3.1}$$

The two terms on the r.h.s have opposite signs and their absolute values are typically much larger than M_Z^2, hence the potential fine-tuning associated to the electroweak breaking.

[3]The existence of this fine-tuning was first observed in ref. [12, 13] and has been discussed, from the Bayesian point of view in ref. [14].

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It is well-known that the radiative corrections to the Higgs potential reduce the fine-tuning [15]. This effect can be included taking into account that the effective quartic coupling of the SM-like Higgs runs from its initial value at the SUSY threshold, $\lambda(Q_{\rm threshold}) = \frac{1}{8}(g^2+g'^2)$, until its final value at the electroweak scale, $\lambda(Q_{EW})$. The effect of this running is equivalent to replacing $M_Z^2 \to m_h^2$ in eq.(3.1) above, i.e.

$$-\frac{m_h^2}{2} = \mu^2 + m_{H_u}^2 \ , \tag{3.2}$$

which is the expression from which we will evaluate the electroweak fine-tuning in the MSSM.

It is a common practice to quantify the amount of fine-tuning using the parametrization first proposed by Ellis et al. [16] and Barbieri and Giudice [1], which in our case reads

$$\frac{\partial m_h^2}{\partial \theta_i} = \Delta_{\theta_i}\frac{m_h^2}{\theta_i} \ , \qquad \Delta \equiv {\rm Max}\ |\Delta_{\theta_i}| \ , \tag{3.3}$$

where θ_i is an independent parameter that defines the model under consideration and Δ_{θ_i} is the fine-tuning parameter associated to it [4]. Typically θ_i are the initial (high-energy) values of the soft terms and the μ parameter. Nevertheless, for specific scenarios of SUSY breaking and transmission to the observable sector, the initial parameters might be particular theoretical parameters that define the scenario and hence determine the soft terms.

3.1 Generic expression for the fine-tuning

Clearly, in order to use the standard measure of the fine-tuning (3.3) it is necessary to write the r.h.s. of the minimization equation (3.2) in terms of the initial parameters. This in turn implies to write the low-energy values of $m_{H_u}^2$ and μ in terms of the initial, high-energy, soft-terms and $\mu-$term (for specific SUSY constructions, these parameters should themselves be expressed in terms of the genuine initial parameters of the model). Low-energy (LE) and high-energy (HE) parameters are related by the RG equations, which normally have to be integrated numerically. However, it is extremely convenient to express this dependence in an exact, analytical way. Fortunately, this can be straightforwardly done, since the dimensional and analytical consistency dictates the form of the dependence,

$$\begin{aligned} m_{H_u}^2(LE) &= c_{M_3^2}M_3^2 + c_{M_2^2}M_2^2 + c_{M_1^2}M_1^2 + c_{A_t^2}A_t^2 + c_{A_tM_3}A_tM_3 + c_{M_3M_2}M_3M_2 + \cdots \\ &\quad \cdots + c_{m_{H_u}^2}m_{H_u}^2 + c_{m_{Q_3}^2}m_{Q_3}^2 + c_{m_{U_3}^2}m_{U_3}^2 + \cdots \end{aligned} \tag{3.4}$$

$$\mu(LE) = c_\mu \mu \ , \tag{3.5}$$

where M_i are the $SU(3)\times SU(2)\times U(1)_Y$ gaugino masses, A_t is the top trilinear scalar coupling; and $m_{H_u}, m_{Q_3}, m_{U_3}$ are the masses of the H_u-Higgs, the third-generation squark doublet and the stop singlet respectively, all of them understood at the HE scale. The numerical coefficients, $c_{M_3^2}, c_{M_2^2}, \ldots$ are obtained by fitting the result of the numerical integration of the RGEs to eqs.(3.4, 3.5), a task that we perform carefully in the subsection 3.2.

The above equations (3.4, 3.5) replace the one-loop LL expressions (1.4, 1.6) used in the standard Natural-SUSY treatment. If one considers the initial values of the soft parameters and

[4] This definition of the fine-tuning is reasonable and can be rigorously justified using Bayesian methods, see ref. [17]

μ as the independent parameters that define the MSSM, then one can easily extract the associated fine-tuning by applying eq.(3.3) to (3.2), and replacing $m^2_{H_u}$ by the expression (3.4).

From eqs.(3.2, 3.4, 3.5)) it is easy to derive the $\Delta-$parameters (3.3) for any MSSM scenario. A common practice is to consider the (HE) soft terms and the $\mu-$term as the independent parameters, say

$$\Theta_\alpha = \left\{\mu, M_3, M_2, M_1, A_t, m^2_{H_u}, m^2_{H_d}, m^2_{U_3}, m^2_{Q_3}, \cdots\right\}, \tag{3.6}$$

which is equivalent to the so-called “Unconstrained MSSM".Then one easily computes Δ_{Θ_α}

$$\Delta_{\Theta_\alpha} = \frac{\Theta_\alpha}{m_h^2}\frac{\partial m_h^2}{\partial \Theta_\alpha} = -2\frac{\Theta_\alpha}{m_h^2}\frac{\partial m^2_{H_u}}{\partial \Theta_\alpha} . \tag{3.7}$$

E.g. Δ_{M_3} is given by

$$\Delta_{M_3} = -2\frac{M_3}{m_h^2}\left(2c_{M_3^2}M_3 + c_{A_tM_3}A_t + c_{M_3M_2}M_2 + \cdots\right) . \tag{3.8}$$

The identification $\frac{\partial m_h^2}{\partial \Theta_\alpha} \simeq -2\frac{\partial m^2_{H_u}}{\partial \Theta_\alpha}$ in eq.(3.7) comes from eq.(3.2) and thus is valid for all the parameters except μ, for which we simply have

$$\Delta_\mu = \frac{\mu}{m_h^2}\frac{\partial m_h^2}{\partial \mu} = -4c^2_\mu\frac{\mu^2}{m_h^2} = -4\left(\frac{\mu(LE)}{m_h^2}\right)^2 . \tag{3.9}$$

Note that for any other theoretical scenario, the Δs associated with the genuine initial parameters, say θ_i, can be written in terms of Δ_{Θ_α} using the chain rule

$$\Delta_{\theta_i} \equiv \frac{\partial \ln m_h^2}{\partial \ln \theta_i} = \sum_\alpha \Delta_{\Theta_\alpha}\frac{\partial \ln \Theta_\alpha}{\partial \ln \theta_i} = \frac{\theta_i}{m_h^2}\sum_\alpha \frac{\partial m_h^2}{\partial \Theta_\alpha}\frac{\partial \Theta_\alpha}{\partial \theta_i} . \tag{3.10}$$

3.2 The fit to the low-energy quantities

Fits of the kind of eq.(3.4) can be found in the literature, see e.g.[18]. However, though useful, they should be refined in several ways in order to perform a precise fine-tuning analysis. The most important improvement is a careful treatment of the various threshold scales. In particular, the initial MSSM parameters (i.e. the soft terms and the $\mu-$parameter) are defined at a high-energy (HE) scale, which is usually identified as M_X, i.e. the scale at which the gauge couplings unify. Although this is a reasonable assumption, it is convenient to consider the HE scale as an unknown; e.g. in gauge-mediated scenarios it can be in principle any scale. The low-energy (LE) scale at which one sets the SUSY threshold and the supersymmetric spectrum is computed, is also model-dependent. Because of this, we have divided the RG-running into two segments, $[M_{\rm EW}, M_{\rm LE}]$ and $[M_{\rm LE}, M_{\rm HE}]$. Besides this refinement, we have integrated the RG-equations at two-loop order, using `SARAH 4.1.0` [19].

The results of the fits for all the LE quantities for $\tan\beta = 10$ and $M_{\rm HE} = M_X$ are given in ref. [3]. The value quoted for each $c-$coefficient has been evaluated at $M_{\rm LE} = 1$ TeV. The dependence of the $c-$coefficients on $M_{\rm LE}$ is logarithmic and can be well approximated by

$$c_i(M_{\rm LE}) \simeq c_i(1\ \ {\rm TeV}) + b_i \ln\frac{M_{\rm LE}}{1\ \ {\rm TeV}} . \tag{3.11}$$

The value of the b_i coefficients is also given in ref. [3]. Certainly, the value of $M_{\rm LE} \sim \overline{m_{\tilde t}}$ is itself a (complicated) function of the initial soft parameters. Nevertheless, it is typically dominated by the (RG) gluino contribution, $M_{\rm LE} \sim \overline{m_{\tilde t}} \sim \sqrt{3}|M_3|$ for $M_{\rm HE} = M_X$. This represents an additional dependence of $m^2_{H_u}$ on M_3, which should be taken into account when computing Δ_{M_3}. Actually, this effect diminishes the fine-tuning associated to M_3 (which is among the most important ones) because the impact of an increase of M_3 in the value of $m^2_{H_u}$ becomes (slightly) compensated by the increase of the LE scale and the consequent decrease of the $c_{M_3^2}$ coefficient in eq.(3.4). We have incorporated this fact in the computations of the fine-tuning.

4. The naturalness bounds

Let us explore further the size and structure of the fine-tuning, and the corresponding bounds on the initial parameters, in the unconstrained MSSM, i.e. taking as initial parameters the HE values of the soft terms and the μ-term: $\Theta_\alpha = \left\{\mu, M_3, M_2, M_1, A_t, m^2_{H_u}, m^2_{H_d}, m^2_{U_3}, m^2_{Q_3}, \cdots\right\}$. For any of those parameters we demand

$$|\Delta_{\Theta_\alpha}| \lesssim \Delta^{\rm max} \,, \tag{4.1}$$

where Δ_{Θ_α} are given by eq.(3.7). Now, for the parameters that appear just once in eqs.(3.4, 3.5) the corresponding naturalness bound (4.1) is trivial and has the form of an upper limit on the parameter size. For dimensional reasons this is exactly the case for dimension-two parameters in mass units, e.g. for the squared stop masses

$$\left|\Delta_{m^2_{Q_3}}\right| = \left|-2\frac{m^2_{Q_3}}{m^2_h}\, c_{m^2_{Q_3}}\right| \lesssim \Delta^{\rm max} \quad\longrightarrow\quad m^2_{Q_3} \lesssim 1.36\, \Delta^{\rm max}\, m^2_h \tag{4.2}$$

where we have plugged $c_{m_{Q_3}} = -0.367$, which correspond to $M_{\rm HE} = M_X$ and $M_{\rm LE} = 1$ TeV (see Tables in ref. [3]). For $\Delta^{\rm max} = 100$, we get $m_{Q_3} \lesssim 1.46$ TeV, $m_{U_3} \lesssim 1.64$ TeV, substantially higher than the usual quoted bounds [2]. This is mainly due to the refined RG analysis and the use of the radiatively upgraded expression eq.(3.2), rather than eq.(3.1), to evaluate the fine-tuning.

On the other hand, for dimension-one parameters (except μ) the naturalness bounds (4.1) appear mixed. In particular, this is the case for the bounds associated to M_3, M_2, A_t. A detailed analysis is presented in ref. [3], where $|M_3| \lesssim 660$ GeV, $|M_2| \lesssim 1630$ GeV, $|A_t| \lesssim 2430$ GeV is obtained for $\Delta^{\rm max} = 100$. We also comment there the differences and improvements respect to the results previously obtained by Feng [2].

The next step is to incorporate the other fine-tunings left aside (to get $m_h^{\rm exp} \simeq 125$ GeV and to get large $\tan\beta$) and translate those bounds into limits on the physical supersymmetric spectrum[5]. We summarize bellow the most important characteristics of a Natural-SUSY scenario, as obtained in [3]:

- Concerning the electroweak fine-tuning of the unconstrained MSSM (i.e. the one required to get the correct electroweak scale), the most robust result is by far that Higgsinos should be rather light, certainly below 700 GeV for $\Delta < 100$. This result is enormously stable against changes in the HE scale since the $\mu-$parameter runs very little from HE to LE.

[5]Unfortunately, there is no a one-to-one correspondence between the physical masses and the soft-parameters and $\mu-$term at high-energy. The only approximate exception are the gaugino and Higgsino masses.

- The most stringent naturalness upper bound, from the phenomenological point of view, is the one on the gluino mass. If $M_{\rm HE} \simeq M_X$ one gets $M_{\tilde{g}} \lesssim 1.5$ TeV for $\Delta^{\rm max} = 100$.

- Light stop masses are *not* really a generic requirement of Natural SUSY. Actually, stops could be well beyond the LHC limits without driving the electroweak fine-tuning of the MSSM beyond 1%. Even more, in some scenarios, like universal scalar masses with $M_{\rm HE} = M_X$, stops above 1.5 TeV are consistent with a quite mild fine-tuning of $\sim 10\%$.

Notice that for specific scenarios of SUSY breaking, the initial parameters might be particular theoretical parameters that define the scenario and hence determine the soft terms. This dependence has to be taken into account in the evaluation of the fine-tuning. We illustrate this issue in the next section.

5. An example: fine-tuning analysis in Gauge-mediated SUSY breaking

Models with gauge-mediated supersymmetry breaking (GMSB) have become one of the most popular supersymmetric scenarios [20]. In these models SUSY is broken in a hidden sector and transmitted at loop-level to the visible sector via heavy chiral supermultiplets (messengers) that are charged under the standard gauge interactions.

If the messengers form complete $SU(5)$ representations, then gauge unification is preserved. Hence, a usual choice is that the messenger sector consists of N_5 copies of fundamental representations, $5+\bar{5}$. They couple to the superfield X which breaks SUSY in the hidden sector, thanks to a non-vanishing VEV of its auxiliary component, $\langle F_X \rangle \neq 0$. A second scale in the problem is provided by $M_{\rm mess}$, the masses of the fermionic components of the messengers after the breaking.

Gauginos and sfermions get masses at one-loop and two-loop respectively, namely:

$$M_i = \frac{\alpha_i}{4\pi}\Lambda N_5 + ..., \tag{5.1}$$

$$m^2_{\tilde{f}_k} = 2\Lambda^2 N_5 \sum_{i=1}^{3} C_i \left(\frac{\alpha_i}{4\pi}\right)^2 + ..., \tag{5.2}$$

where $\Lambda \propto F_X/M_{\rm mess}$ (the proportionality constant depends on the coupling between X and the messenger), C_i are the corresponding Casimir coefficients (see, e.g., [20]) and the dots stand for terms suppressed in the $\Lambda \ll M_{\rm mess}$ limit.

The above expressions are to be understood at the high scale, $M_{\rm HE}$, where the effects of SUSY breaking are transmitted to the observable sector, which coincides with the messenger mass, $M_{\rm HE} = M_{\rm mess}$. Altogether the minimal GMSB scenario has only 4 independent parameters $\{\Lambda, M_{\rm HE}, \mu, B\}$, being highly predictive and a well-motivated MSSM model. Notice that, unlike the constrained MSSM, the soft masses are different for particles with different quantum numbers, although they are independent of the family. This partial universality is enough to avoid dangerous FCNC effects, which an important success of GMSB.

Using the procedure detailed in previous sections, we can now compute the fine-tuning in Λ, μ and $M_{\rm HE}$ for a given value of $\tan\beta$ (that selects a particular value of B). The relevant fine-tunings are $\Delta_\Lambda, \Delta_{M_{\rm HE}}$, given that $\Delta_\Lambda \sim \Delta_\mu$, as one can easily check. The results are shown in Fig. 1, where we have taken $N_5 = 1$ and fixed $\tan\beta = 10$. Notice that Λ values in the 1000 - 3000 TeV range are required in order to have an acceptable value for m_H. These is due to the fact that for $A_t \sim 0$ large m_H corrections imply large stop masses, and hence large Λ values. This implies, as it is already known, an unacceptable fine-tuning for the minimal implementation of GMSB.

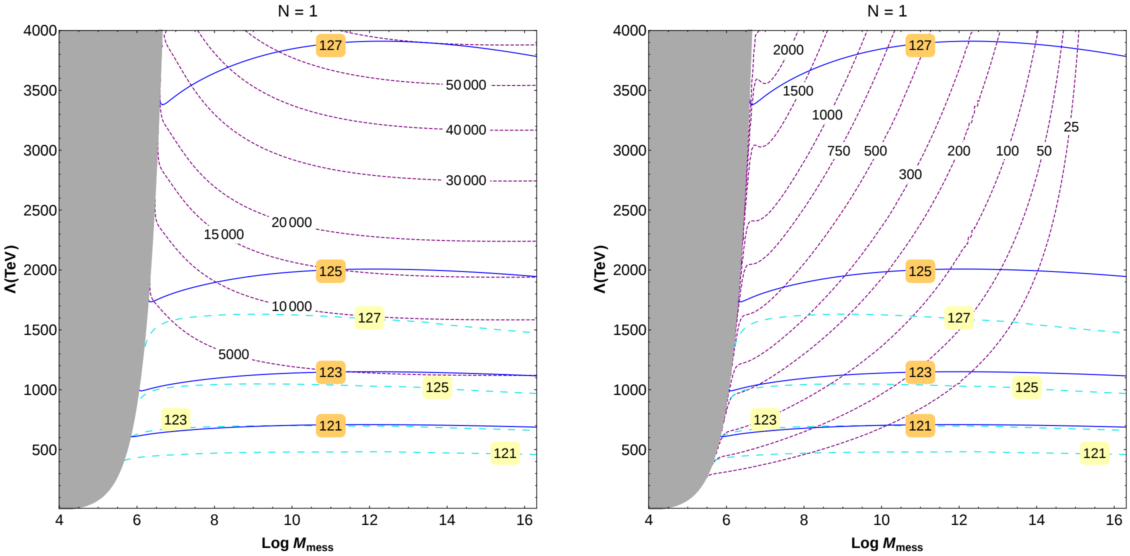

Figure 1: In the left side we show the contour lines of constant Δ_Λ (magenta) and m_H, evaluated using FeynHiggs [21] and SusyHD [22] (light and dark blue, respectively). The grey area is unphysical since it corresponds to negative squared masses for the scalar messengers. In the right side, $\Delta_{M_{\rm HE}}$ is instead shown.

6. Conclusions

In this work, we have addressed the supersymmetric fine-tuning in a systematic way, including the discussion of the measure of the fine-tuning, the mixing of the fine-tuning conditions, the method to extract fine-tuning bounds on the initial parameters and the low energy supersymmetric spectrum, as well as the role played by extra potential fine-tunings. Using the tables in [3], we have shown how evaluate the fine-tuning and the corresponding naturalness bounds for any theoretical model defined at any high-energy (HE) scale. Finally, we have analyzed in detail the complete fine-tuning bounds for the unconstrained MSSM, defined at any HE scale, including the impact that the experimental Higgs mass imposes on the soft terms. As a particular case, we have also evaluated the fine-tuning in the minimal Gauge Mediation Supersymmetry Model.

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References

[1] R. Barbieri and G. Giudice, *Upper Bounds on Supersymmetric Particle Masses*, *Nucl.Phys.* **B306** (1988) 63.

[2] J. L. Feng, *Naturalness and the Status of Supersymmetry*, *Ann.Rev.Nucl.Part.Sci.* **63** (2013) 351–382.

[3] J. A. Casas, J. M. Moreno, S. Robles, K. Rolbiecki, and B. Zaldivar, *What is a Natural SUSY scenario?*, *JHEP* **06** (2015) 070, [arXiv:1407.6966].

[4] M. Papucci, J. T. Ruderman, and A. Weiler, *Natural SUSY Endures*, *JHEP* **1209** (2012) 035.

[5] R. Kitano and Y. Nomura, *Supersymmetry, naturalness, and signatures at the LHC*, *Phys.Rev.* **D73** (2006) 095004, [hep-ph/0602096].

[6] C. Ford, D. Jones, P. Stephenson, and M. Einhorn, *The Effective potential and the renormalization group*, *Nucl.Phys.* **B395** (1993) 17–34, [hep-lat/9210033].

[7] M. Bando, T. Kugo, N. Maekawa, and H. Nakano, *Improving the effective potential*, *Phys.Lett.* **B301** (1993) 83–89, [hep-ph/9210228].

[8] H. E. Haber, R. Hempfling, and A. H. Hoang, *Approximating the radiatively corrected Higgs mass in the minimal supersymmetric model*, *Z.Phys.* **C75** (1997) 539–554, [hep-ph/9609331].

[9] J. Casas, J. Espinosa, M. Quiros, and A. Riotto, *The Lightest Higgs boson mass in the minimal supersymmetric standard model*, *Nucl.Phys.* **B436** (1995) 3–29, [hep-ph/9407389].

[10] M. S. Carena, J. Espinosa, M. Quiros, and C. Wagner, *Analytical expressions for radiatively corrected Higgs masses and couplings in the MSSM*, *Phys.Lett.* **B355** (1995) 209–221, [hep-ph/9504316].

[11] M. E. Cabrera, J. A. Casas, and A. Delgado, *Upper Bounds on Superpartner Masses from Upper Bounds on the Higgs Boson Mass*, *Phys. Rev. Lett.* **108** (2012) 021802, [arXiv:1108.3867].

[12] A. E. Nelson and L. Randall, *Naturally large tan BETA*, *Phys.Lett.* **B316** (1993) 516–520.

[13] L. J. Hall, R. Rattazzi, and U. Sarid, *The Top quark mass in supersymmetric SO(10) unification*, *Phys.Rev.* **D50** (1994) 7048–7065, [hep-ph/9306309].

[14] M. E. Cabrera, J. A. Casas, and R. Ruiz d Austri, *MSSM Forecast for the LHC*, *JHEP* **05** (2010) 043.

[15] B. de Carlos and J. Casas, *One loop analysis of the electroweak breaking in supersymmetric models and the fine tuning problem*, *Phys.Lett.* **B309** (1993) 320–328, [hep-ph/9303291].

[16] J. R. Ellis, K. Enqvist, D. V. Nanopoulos, and F. Zwirner, *Observables in Low-Energy Superstring Models*, *Mod.Phys.Lett.* **A1** (1986) 57.

[17] P. Ciafaloni and A. Strumia, *Naturalness upper bounds on gauge mediated soft terms*, *Nucl.Phys.* **B494** (1997) 41–53, [hep-ph/9611204].

[18] H. Abe, T. Kobayashi, and Y. Omura, *Relaxed fine-tuning in models with non-universal gaugino masses*, *Phys.Rev.* **D76** (2007) 015002, [hep-ph/0703044].

[19] F. Staub, *SARAH 4 : A tool for (not only SUSY) model builders*, *Comput. Phys. Commun.* **185** (2014) 1773–1790, [arXiv:1309.7223].

[20] G. F. Giudice and R. Rattazzi, *Theories with gauge mediated supersymmetry breaking*, *Phys. Rept.* **322** (1999) 419–499, [hep-ph/9801271].

[21] S. Heinemeyer, W. Hollik, and G. Weiglein, *FeynHiggs: A Program for the calculation of the masses of the neutral CP even Higgs bosons in the MSSM*, *Comput.Phys.Commun.* **124** (2000) 76–89.

[22] J. P. Vega and G. Villadoro, *SusyHD: Higgs mass Determination in Supersymmetry*, *JHEP* **07** (2015).

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