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Gauge-Higgs Grand Unification

Yutaka Hosotani*
Department of Physics, Osaka University, Japan
E-mail: hosotani@phys.sci.osaka-u.ac.jp

Naoki Yamatsu
Department of Physics, Osaka University, Japan
E-mail: yamatsu@het.phys.sci.osaka-u.ac.jp

Gauge-Higgs grand unification is formulated. By extending $SO(5) \times U(1)_X$ gauge-Higgs electroweak unification, strong interactions are incorporated in $SO(11)$ gauge-Higgs unification in the Randall-Sundrum warped space. Quarks and leptons are contained in spinor and vector multiplets of $SO(11)$. Although the KK scale can be as low as 10 TeV, proton decay is forbidden by a conserved fermion number in the absence of Majorana masses of neutrinos.

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*Speaker.

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1. Gauge-Higgs unification

We are in search of a principle for the 125 GeV Higgs scalar boson, which regulates Higgs couplings, explains how the electroweak (EW) gauge symmetry breaking takes place, and solves the gauge-hierarchy problem. One possible, promising answer is the gauge-Higgs unification.

In the gauge-Higgs EW unification one starts with gauge theory, say, in 5 dimensions. 4-dimensional components of gauge potential A_M contain 4D gauge fields such as photon γ, W and Z bosons. The extra dimensional component transforms as a scalar under 4-dimensional Lorentz transformations, and its zero mode is identified with the 4D Higgs scalar field. When the extra dimensional space is not simply connected, there arises an Aharonov-Bohm (AB) phase θ_H along the extra dimension. The 4D Higgs field is a 4D fluctuation mode of the AB phase. The value of θ_H is not determined at the classical level, but is dynamically determined at the quantum level. In non-Abelian gauge theory it may lead to spontaneous gauge symmetry breaking. It is called the Hosotani mechanism.[1]-[5]

The most notable feature of gauge-Higgs unification by the Hosotani mechanism is that the finite mass of the Higgs boson is generated quantum mechanically, independent of a cutoff scale. Further the interactions of the Higgs boson with itself and other fields are governed by the gauge principle so that the model is very restrictive and predictive.

2. $SO(5) \times U(1)$ EW unification

There is a realistic model of gauge-Higgs EW unification. It is the $SO(5) \times U(1)_X$ gauge-Higgs EW unification in the Randall-Sundrum (RS) warped space.[6]-[21] The metric of the RS space is given by $ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$, where $\eta_{\mu\nu} = \text{diag}\ (-1,1,1,1)$, $\sigma(y) = \sigma(-y) = \sigma(y+2L)$, and $\sigma(y) = ky$ for $0 \le y \le L$. $z_L = e^{kL} \gg 1$ is called the warp factor. The bulk part $0 < y < L$ is an AdS space with a cosmological constant $\Lambda = -6k^2$, which is sandwiched by the Planck brane at $y = 0$ and the TeV brane at $y = L$. The RS is an orbifold; spacetime points (x^μ, y), $(x^\mu, -y)$ and $(x^\mu, y+2L)$ are identified.

$SO(5) \times U(1)_X$ gauge theory is defined in RS. Although physical quantities must be single-valued, the gauge potential $A_M(x,y)$ may not be. It satisfies

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix}(x, y_j - y) = P_j \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix}(x, y_j + y) P_j^{-1}\ , (y_0, y_1) = (0, L) \tag{2.1}$$

where $P_j \in SO(5)$ up to sign and $P_j^2 = 1$. Note that $A_M(x, y+2L) = UA_M(x,y)U^{-1}$ where $U = P_1P_0$. The set $\{P_0, P_1\}$ defines orbifold boundary conditions. One chooses

$$P_0 = P_1 = \begin{pmatrix} I_4 & \\ & -1 \end{pmatrix}\ , \tag{2.2}$$

which breaks $SO(5)$ to $SO(4) \simeq SU(2)_L \times SU(2)_R$. Quark- and lepton-multiplets are introduced in the bulk in the vector representation of $SO(5)$, whereas dark fermion multiplets in the spinor representation. In addition to them, brane scalar field Φ and brane fermions fields are introduced on the Planck brane. The brane scalar field $\Phi(x)$ spontaneously breaks $SU(2)_R \times U(1)_X$ to $U(1)_Y$, reducing the original symmetry $SO(5)\times U(1)_X$ to the standard model (SM) symmetry $SU(2)_L \times U(1)_Y$.

The zero modes of $A_y(x,y)$ reside in the $SO(5)/SO(4)$ part, $A_y^{(a5)}$ ($a = 1 \sim 4$) in the standard notation. They transform as an $SO(4)$ vector, or an $SU(2)_L$ doublet. Three out of the four components are absorbed by W and Z gauge bosons. The unabsorbed component becomes the neutral Higgs boson $H(x)$ of mass 125 GeV. The Higgs boson appears as an AB phase;

$$
\begin{aligned}
&e^{i\hat{\theta}(x)} = P\exp\left\{ ig_A \int_0^{2L} dy A_y(x,y) \right\} , \\
&\hat{\theta}(x) = \theta_H + \frac{H(x)}{f_H} \quad , \quad f_H = \frac{2}{g_A}\sqrt{\frac{k}{z_L^2-1}} \sim \frac{2m_{\rm KK}}{\pi g_w \sqrt{kL}} .
\end{aligned}
\tag{2.3}
$$

Here g_w is the 4D $SU(2)_L$ gauge coupling constant. The KK mass scale is given by $m_{\rm KK} = \pi k/(z_L - 1)$. The gauge invariance guarantees that physics is periodic in θ_H with a period 2π.

The model is successful, being consistent with data at low energies for $\theta_H \lesssim 0.1$. The values of the parameters of the model are determined such that the observed m_Z, g_w, $\sin\theta_W$, m_H and quark/lepton masses are reproduced. There remain two relevant free parameters, z_L and n_F (the number of dark fermion multiplets). With z_L and n_F given, the effective potential $V_{\rm eff}(\theta_H)$ is evaluated. The value of θ_H is determined dynamically by the location of the global minimum of $V_{\rm eff}(\theta_H)$.

Quite remakable is the fact that most of the important physical quantities such as the Higgs couplings, the KK scale $m_{\rm KK}$, and the KK spectrum of gauge bosons and quarks/leptons are approximately determined as functions of θ_H, independent of detailed values of (z_L, n_F). There hold universality relations.[19, 20] For instance, the mass $m_{Z^{(1)}}$ of the first KK Z, depicted in Fig. 1, and the KK scale $m_{\rm KK}$ satisfy

$$
\begin{aligned}
&m_{Z^{(1)}} \sim 1044\,{\rm GeV}\, (\sin\theta_H)^{-0.808} , \\
&m_{\rm KK} \sim 1352\,{\rm GeV}\, (\sin\theta_H)^{-0.786} .
\end{aligned}
\tag{2.4}
$$

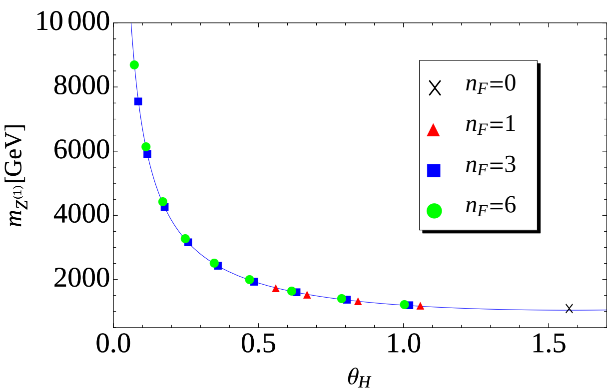

Figure 1: θ_H vs $m_{Z^{(1)}}$ for $m_H = 126\,{\rm GeV}$ with n_F degenerate dark fermions.

Similarly the Higgs cubic and quartic self-couplings are given by

$$\lambda_3/\mathrm{GeV} \sim 26.7\cos\theta_H + 1.42(1+\cos 2\theta_H)\ ,$$
$$\lambda_4 \sim -0.0106 + 0.0304\cos 2\theta_H + 0.00159\cos 4\theta_H\ . \tag{2.5}$$

These numbers should be compared with $\lambda_3^{\rm SM} = 31.5\,\mathrm{GeV}$ and $\lambda_4^{\rm SM} = 0.0320$ in SM.

The model gives definitive prediction for Z' events at LHC. The e^+e^- or $\mu^+\mu^-$ signals through virtual production of $\gamma^{(1)}, Z^{(1)}, Z_R^{(1)}$ should be detected at the 14 TeV LHC. The predicted cross section is shown in Fig. 2. The widths are large in the gauge-Higgs unification, as the gauge couplings of the first KK modes are large for right-handed quarks and leptons.

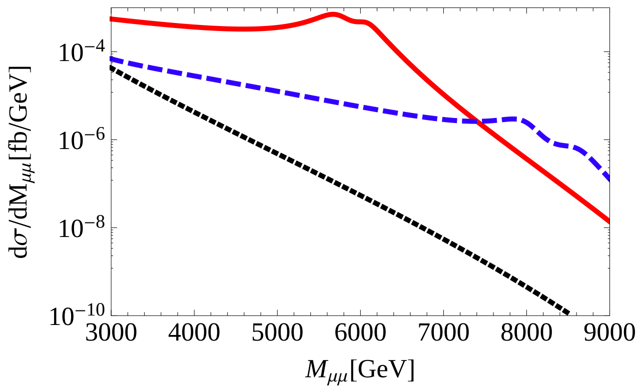

Figure 2: The differential cross section for $pp \to \mu^+\mu^- X$ at the 14 TeV LHC for $\theta_H = 0.114$ (red solid curve) and for $\theta_H = 0.073$ (blue dashed curve) . The nearly straight black line represents the SM background.

3. $SO(11)$ grand unification

What is next? It is most natural to extend the gauge-Higgs unification scenario to incorporate strong interactions. We would like to have a theory with gague group $\mathscr{G}$, which reduces to the SM symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ at low energies, and in which 4D Higgs boson of $m_H = 125\,\mathrm{GeV}$ appears as a part of the extra-dimensional component of gauge fields. There have been several attempts along this line. $SU(6)$ gauge-Higgs unification was considered on $M^4 \times (S^1/Z_2)$.[22, 23, 24] However it necessarily yields an extra $U(1)$, and dynamical EW symmetry breaking can be achieved only with extra matter fields resulting in exotic particles at low energies. $SU(5) \times SU(5)$ unification model has been proposed.[25] There is an approach from the composite Higgs scenario.[26] None of these models is satisfactory with the phenomenology below the EW scale.

$SO(5) \times U(1)_X$ gauge-Higgs unification in RS space is a good, realistic model at low energies. One might expect that in gauge-Higgs grand unification the gauge group $\mathscr{G}$ reduces to $SU(3)_C \times SO(5) \times U(1)_X$ at some scale, and further to $SU(3)_C \times SU(2)_L \times U(1)_Y$ at a lower scale. This approach, however, turns out not to work.

We propose $SO(11)$ gauge-Higgs grand unification in the RS space.[27] In the bulk region $(0 \le y \le L)$ there are, in addition to the $SO(11)$ gauge fields A_M, fermion multiplets in the $SO(11)$

spinor representation, $\Psi_{\mathbf{32}}$, and in the $SO(11)$ vector representation, $\Psi_{\mathbf{11}}$. On the Planck brane ($y=0$) a scalar field in the $SO(10)$ spinor representation, $\Phi_{\mathbf{16}}$, is introduced. The symmetry breaking pattern in this scenario is

$$\begin{aligned} SO(11) &\to SO(4)\times SO(6) \simeq SU(2)_L\times SU(2)_R\times SU(4) \quad \text{by BC} \\ &\to SU(3)_C\times SU(2)_L\times U(1)_Y \qquad \text{by } \langle\Phi_{\mathbf{16}}\rangle \\ &\to SU(3)_C\times U(1)_{\rm EM} \qquad \text{by } \theta_H \ . \end{aligned} \tag{3.1}$$

The first step of the symmetry breaking in (3.1) is achieved by the orbifold boundary condition, which is given, in the form of (2.1), with

$$P_0 = \begin{pmatrix} I_{10} & \\ & -1 \end{pmatrix} \ , \quad P_1 = \begin{pmatrix} I_4 & \\ & -I_7 \end{pmatrix} \ . \tag{3.2}$$

P_0 and P_1 break $SO(11)$ to $SO(10)$ at the Planck brane and to $SO(4)\times SO(7)$ at the TeV brane, respectively. With these two combined the symmetry is reduced to $SO(4)\times SO(6)\simeq SU(2)_L\times SU(2)_R\times SU(4)$. At this stage $A_\mu(x,y)$ has zero modes (4D massless gauge fields) in $SO(4)\times SO(6)$, whereas $A_y(x,y)$ has zero modes only in the $A_y^{(a11)}$ ($a=1\sim 4$) components, which become the 4D Higgs doublet.

On the Planck brane $SO(10)$ gauge invariance is maintained. The brane scalar field $\Phi_{\mathbf{16}}$ spontaneously breaks $SO(10)$ to $SU(5)$. The resultant symmetry is $SU(5)\cap[SO(4)\times SO(6)]$, namely $SU(3)_C\times SU(2)_L\times U(1)_Y$.

The last step in (3.1) is induced by the Hosotani mechanism. The EW symmetry breaking takes place by dynamics of the AB phase θ_H associated with $A_y^{(411)}$ in (2.3). In this scheme generators of $U(1)_Y$ and $U(1)_{\rm EM}$ are given, in terms of $SO(11)$ generators, by

$$\begin{aligned} Q_Y &= \tfrac{1}{2}(T_{12}-T_{34}) - \tfrac{1}{3}(T_{56}+T_{78}+T_{9\,10}) \ , \\ Q_{\rm EM} &= T_{12} - \tfrac{1}{3}(T_{56}+T_{78}+T_{9\,10}) \ . \end{aligned} \tag{3.3}$$

It follows that the Weinberg angle is the same as in the $SU(5)$ or $SO(10)$ GUT;

$$g'_Y = \sqrt{\frac{3}{5}}\, g_w \ , \quad e = \sqrt{\frac{3}{8}}\, g_w \ , \quad \sin^2\theta_W = \frac{3}{8} \ . \tag{3.4}$$

4. Quarks and leptons

We introduce fermions $\Psi_{\mathbf{32}}$ and $\Psi_{\mathbf{11}}$ in the bulk. Quarks and leptons in SM are contained mostly in $\Psi_{\mathbf{32}}$. To see it explicitly, we take the following representation of $SO(11)$ Clifford algebra $\{\Gamma_j,\Gamma_k\} = 2\delta_{jk} I_{32}$ ($j,k = 1\sim 11$);

$$\begin{aligned} \Gamma_{1,2,3} &= \sigma^{1,2,3}\otimes\sigma^1\otimes\sigma^1\otimes\sigma^1\otimes\sigma^1, \\ \Gamma_{4,5} &= \sigma^0\otimes\sigma^{2,3}\otimes\sigma^1\otimes\sigma^1\otimes\sigma^1, \\ \Gamma_{6,7} &= \sigma^0\otimes\sigma^0\otimes\sigma^{2,3}\otimes\sigma^1\otimes\sigma^1, \\ \Gamma_{8,9} &= \sigma^0\otimes\sigma^0\otimes\sigma^0\otimes\sigma^{2,3}\otimes\sigma^1, \end{aligned}$$

$$\Gamma_{10,11} = \sigma^0 \otimes \sigma^0 \otimes \sigma^0 \otimes \sigma^0 \otimes \sigma^{2,3}, \tag{4.1}$$

where $\sigma^0 = I_2$ and $\sigma^{1,2,3}$ are Pauli matrices. The $SO(11)$ generators are given by $T^{\rm sp}_{jk} = -\frac{1}{2} i \Gamma_j \Gamma_k$ ($j \neq k$). The upper and lower half components of $\Psi_{\bf 32}$ correspond to **16** and $\overline{\bf 16}$ of $SO(10)$. The orbifold boundary condition matrices in the spinorial representation are given by $P_0^{\rm sp} = \Gamma_{11}$ and $P_1^{\rm sp} = I_2 \otimes \sigma^3 \otimes I_8$. $\Psi_{\bf 32}$ and $\Psi_{\bf 11}$ satisfy

$$\begin{aligned} \Psi_{\bf 32}(x, y_j - y) &= -P_j^{\rm sp} \gamma^5 \Psi_{\bf 32}(x, y_j + y) , \\ \Psi_{\bf 11}(x, y_j - y) &= \eta_j^{\bf 11} P_j \gamma^5 \Psi_{\bf 11}(x, y_j + y) , \end{aligned} \tag{4.2}$$

where $\eta_j^{\bf 11} = +1$ or -1. The bulk action for the fermions takes the form

$$\int d^5x \sqrt{-\det G} \Big\{ \overline{\Psi}_{\bf 32} \mathscr{D}(c_{\bf 32}) \Psi_{\bf 32} + \overline{\Psi}_{\bf 11} \mathscr{D}(c_{\bf 11}) \Psi_{\bf 11} \Big\} \tag{4.3}$$

where $\mathscr{D}(c) = \gamma^A {e_A}^M D_M - c\sigma'(y)$ and $D_M = \partial_M + \frac{1}{8}\omega_{MBC}[\gamma^B, \gamma^C] - igA_M$. The generators of $SU(2)_L$ and $SU(2)_R$ are given by

$$\Big[T_L^a, T_R^a \Big] = \tfrac{1}{2} \sigma^a \otimes \left[\begin{pmatrix} 1 & \\ & 0 \end{pmatrix}, \begin{pmatrix} 0 & \\ & 1 \end{pmatrix} \right] \otimes I_8 . \tag{4.4}$$

The content of $\Psi_{\bf 32}$ is given by

$$\Psi_{\bf 32} = \begin{pmatrix} \Psi_{\bf 16} \\ \Psi_{\overline{\bf 16}} \end{pmatrix} , \quad \Psi_{\bf 16} = \begin{pmatrix} \ell \\ \hat{q}_1 \\ q_3 \\ \hat{q}_2 \\ q_1 \\ \hat{\ell} \\ q_2 \\ \hat{q}_3 \end{pmatrix} , \quad \Psi_{\overline{\bf 16}} = \begin{pmatrix} \hat{q}'_3 \\ q'_2 \\ \hat{\ell}' \\ q'_1 \\ \hat{q}'_2 \\ q'_3 \\ \hat{q}'_1 \\ \ell' \end{pmatrix} ,$$

$$\begin{aligned} &\ell = \begin{pmatrix} \nu \\ e \end{pmatrix}, \; q_j = \begin{pmatrix} u_j \\ d_j \end{pmatrix}, \; \ell' = \begin{pmatrix} \nu' \\ e' \end{pmatrix}, \; q'_j = \begin{pmatrix} u'_j \\ d'_j \end{pmatrix}, \\ &\hat{\ell} = \begin{pmatrix} \hat{e} \\ \hat{\nu} \end{pmatrix}, \; \hat{q}_j = \begin{pmatrix} \hat{d}_j \\ \hat{u}_j \end{pmatrix}, \; \hat{\ell}' = \begin{pmatrix} \hat{e}' \\ \hat{\nu}' \end{pmatrix}, \; \hat{q}'_j = \begin{pmatrix} \hat{d}'_j \\ \hat{u}'_j \end{pmatrix}, \end{aligned} \tag{4.5}$$

A field with hat has an opposite charge to the corresponding one without hat. u_j (u'_j) and $\hat{u}_j$ ($\hat{u}'_j$), for instance, have $Q_{\rm EM} = +\frac{2}{3}$ and $-\frac{2}{3}$, respectively. With the orbifold boundary condition (4.2), zero modes of $\Psi_{\bf 32}$ appear in

$$\ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} , \quad q_{jL} = \begin{pmatrix} u_{jL} \\ d_{jL} \end{pmatrix} , \quad \ell'_R = \begin{pmatrix} \nu'_R \\ e'_R \end{pmatrix} , \quad q'_{jR} = \begin{pmatrix} u'_{jR} \\ d'_{jR} \end{pmatrix} . \tag{4.6}$$

All of the SM fermions, but nothing else, appear in $\Psi_{\bf 32}$ as zero modes.

The content of $\Psi_{\mathbf{11}}$ is

$$\Psi_{\mathbf{11}} = \left[\begin{pmatrix} \hat{E} & N \\ \hat{N} & E \end{pmatrix}; (D_j, \hat{D}_j);\ S \right] . \tag{4.7}$$

N, E, and D_j have the same electric charges as ν, e, and d_j, respectively. S is an $SO(10)$ singlet, and is neutral. With given $(\eta_0^{\mathbf{11}}, \eta_1^{\mathbf{11}})$ in the boundary condition (4.2), zero modes are found in

$$\begin{aligned} &(+,+): \begin{pmatrix} \hat{E}_R & N_R \\ \hat{N}_R & E_R \end{pmatrix},\ S_L \qquad (-,-): \begin{pmatrix} \hat{E}_L & N_L \\ \hat{N}_L & E_L \end{pmatrix},\ S_R \\ &(+,-): \quad D_{jR}, \hat{D}_{jR} \qquad\qquad (-,+): \quad D_{jL}, \hat{D}_{jL} \end{aligned} \tag{4.8}$$

5. KK spectrum

To see whether or not the EW symmetry is dynamically broken, one need to know all KK mass spectra which depend on θ_H. In the gauge field sector the W tower, the Z tower, and Y boson tower have θ_H-dependent spectra. The spectra are found from zeros of several equations given by

$$\begin{aligned} &W \text{ tower:} \quad 2S(1;\lambda_n)C'(1;\lambda_n) + \lambda_n \sin^2\theta_H = 0\ , \\ &Z \text{ tower:} \quad 5S(1;\lambda_n)C'(1;\lambda_n) + 4\lambda_n \sin^2\theta_H = 0\ , \\ &Y \text{ tower:} \quad 2S(1;\lambda_n)C'(1;\lambda_n) + \lambda_n(1+\cos^2\theta_H) = 0\ . \end{aligned} \tag{5.1}$$

Here $C(z;\lambda) = \frac{1}{2}\pi\lambda z z_L F_{1,0}(\lambda z, \lambda z_L)$ and $S(z;\lambda) = -\frac{1}{2}\pi\lambda z F_{1,1}(\lambda z, \lambda z_L)$ where $F_{\alpha,\beta}(u,v) = J_\alpha(u)Y_\beta(v) - Y_\alpha(u)J_\beta(v)$. $z = e^{ky}$ and $C' = dC/dz$. The mass is given by $m_n = k\lambda_n$. The lowest modes of W and Z towers are W and Z bosons. Their masses are found to be

$$m_W \sim \frac{\sin\theta_H}{\pi\sqrt{kL}}\, m_{\rm KK}\ , \quad m_Z \sim \frac{m_W}{\cos\theta_W}\ , \quad \sin^2\theta_W = \frac{3}{8}\ . \tag{5.2}$$

Among A_y, the components $[\tilde{A}_y^{a4}, \tilde{A}_y^{a11}]$ $(a = 1 \sim 3, 5 \sim 10)$ have θ_H-dependent spectra given by

$$S(1;\lambda_n)C'(1;\lambda_n) + \lambda_n \begin{pmatrix} \sin^2\theta_H \\ \cos^2\theta_H \end{pmatrix} = 0 \quad \text{for } a = \begin{cases} 1 \sim 3\ , \\ 5 \sim 10. \end{cases} \tag{5.3}$$

In the absence of brane interactions the spectrum of the $\Psi_{\mathbf{32}}$ tower is found to be

$$S_L(1;\lambda_n, c_{\mathbf{32}}) S_R(1;\lambda_n, c_{\mathbf{32}}) + \begin{pmatrix} \sin^2\frac{1}{2}\theta_H \\ \cos^2\frac{1}{2}\theta_H \end{pmatrix} = 0 \tag{5.4}$$

where the upper component is for ℓ, ℓ', q_j, q'_j and the lower component for $\hat{\ell}, \hat{\ell}', \hat{q}_j, \hat{q}'_j$. Here $S_{L/R}(z;\lambda,c) = \mp\frac{1}{2}\pi\lambda\sqrt{z z_L} F_{c\pm\frac{1}{2}, c\pm\frac{1}{2}}(\lambda z, \lambda z_L)$. For $\Psi_{\mathbf{11}}$ the 4th and 11th components mix, and their spectrum is given by

$$S_L(1;\lambda_n, c_{\mathbf{11}}) S_R(1;\lambda_n, c_{\mathbf{11}}) + \begin{pmatrix} \sin^2\theta_H \\ \cos^2\theta_H \end{pmatrix} = 0 \tag{5.5}$$

for $\eta_0^{\mathbf{11}}\eta_1^{\mathbf{11}} = \pm 1$. To get the observed quark/lepton spectrum, one must take account of brane interactions among $\Psi_{\mathbf{32}}$, $\Psi_{\mathbf{11}}$ and $\Phi_{\mathbf{16}}$.

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6. EW symmetry breaking

One need to evaluate $V_{\rm eff}(\theta_H)$ to find if the EW symmetry breaking takes place by the Hosotani mechanism. Full analysis must be waited for until parameters in the brane interactions are fixed to reproduce the observed quark-lepton spectrum. Here we point out that even in the absence of fermions the EW symmetry breaking occurs in the $SO(11)$ gauge-Higgs unification.

In pure gauge theory $V_{\rm eff}(\theta_H)$ is evaluated with (5.1) and (5.3). See Fig. 3. It has the global minimum at $\theta_H = \pm\frac{1}{2}\pi$, and the EW gauge symmetry is dynamically broken. This has never happened in the gauge-Higgs EW unification models. The symmetry breaking is caused, because in the current model there are six Y towers with the spectrum in (5.1) where the lowest modes have the smallest mass for $\cos\theta_H = 0$.

The minimum at $\theta_H = \frac{1}{2}\pi$, however, is not acceptable phenomenologically, as it leads to a stable Higgs boson due to the H parity.[16, 18] Desirable value of $\theta_H \lesssim 0.1$ can be achieved by including fermion multiplets $\Psi_{\mathbf{32}}$ and $\Psi_{\mathbf{11}}$ and brane interactions.

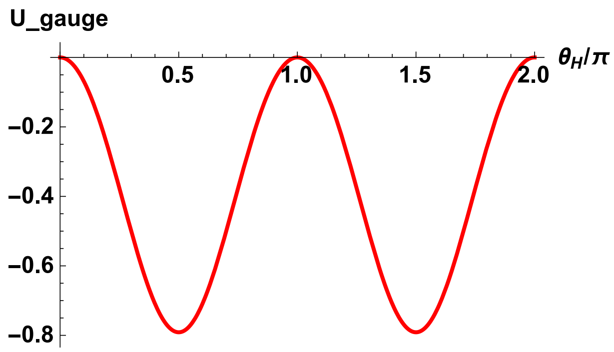

Figure 3: $U = (4\pi)^2 (kz_L^{-1})^{-4} V_{\rm eff}(\theta_H)$ in pure gauge theory is plotted in the $\xi = 0$ gauge. $V_{\rm eff}(\theta_H)$ with a minimum at $0 < \theta_H < \frac{1}{2}\pi$ is achieved with the inclusion of fermions and brane interactions.

7. Energy scales in gauge-Higgs grand unification

There are several energy scales in the gauge-Higgs grand unification in RS.

$$\begin{aligned}
&(1)\ \text{Size of the 5th dimension: } E_{\rm size} = \frac{\pi}{L}\\
&(2)\ \text{GUT scale: } m_{\rm GUT}\\
&(3)\ \text{KK scale: } m_{\rm KK} = \frac{\pi k}{z_L - 1} \sim \pi k e^{-kL} \sim \frac{\sqrt{kL}}{\sin\theta_H}\, m_W\\
&(4)\ \text{EW scale: } m_{\rm EW}\\
&(5)\ \text{QCD scale: } \Lambda_{QCD}
\end{aligned} \tag{7.1}$$

In the flat $M^4 \times (S^1/Z_2)$, $E_{\rm size} = m_{\rm KK} = 1/R$, but in the RS space, $E_{\rm size} \gg m_{\rm KK} \gg m_{\rm EW}$.

The GUT scale $m_{\rm GUT}$ is defined by the gauge coupling unification. In the gauge-Higgs unification KK modes of gauge fields and fermions are excited above $m_{\rm KK}$ in GUT multiplets, which does not necessarily change the GUT unification scale so much. The preliminary study indicates that $m_{\rm KK} \ll m_{\rm GUT}$. It is possible to have $m_{\rm GUT} \sim E_{\rm size}$.

8. Forbidden proton decay

If KK excited states of X and Y bosons show up above $m_{\rm KK}$, one may worry about large decay rate of protons. In 4D $SU(5)$ or $SO(10)$ GUT, for instance, proton decay proceeds through X and Y boson exchange. The masses of X and Y bosons are $O(m_{\rm GUT})$ where $m_{\rm GUT} \sim 10^{15}\,$GeV. In the gauge-Higgs unification $m_{\rm KK}$ is much lower. For $\theta_H \sim 0.1$, $m_{\rm KK} \sim 10\,$TeV so that X and Y boson exchange may lead to rapidly decaying protons.

Remarkably the proton decay is forbidden in the $SO(11)$ gauge-Higgs grand unification, provided Majorana mass terms for neutrinos are absent. Notice that all quarks and leptons are contained in $\ell, q_j.\ell', q'_j$ in $\Psi_{\bf 32}$ in (4.5) and (4.6). All of them have a fermion number $N_\Psi = +1$. In the presence of brane interactions on the Planck brane, $\Psi_{\bf 32}$ and $\Psi_{\bf 11}$ mix, but still the fermion number N_Ψ is conserved. Proton has $N_\Psi = +3$, which cannot decay to, say, $e^+\pi^0$ that has $N_\Psi = -1$.

This should be contrasted to 4D GUT. In 4D $SU(5)$ GUT, $\Psi_{\bar{\bf 5}}$ and $\Psi_{\bf 10}$ contains (ℓ_L, d^c_{jL}) and $(q_{jL}, u^c_{jL}, e^c_L)$, respectively so that gauge interactions lead, for instance, to $u \to u^c + X$ and $d + X \to e^+$, which results in $uud \to uu^c e^+$, namely proton decay $p \to \pi^0 e^+$. Such transitions do not take place in the $SO(11)$ gauge-Higgs grand unification as a consequence of the N_Ψ conservation. In 4D $SO(10)$ GUT, $\Psi_{{\bf 16}L}$ contains all quarks and leptons, which are obtained from the $\Psi_{\bf 16}$ content in (4.5) by replacing $\ell, q_j, \hat{\ell}, \hat{q}_j$ by $\ell_L, q_{jL}, \ell^c_L, q^c_{jL}$. Consequently gauge interactions induce proton decay as in 4D $SU(5)$ GUT. In the $SO(11)$ theory the zero modes ℓ^c_L, q^c_{jL} reside in the $\Psi_{\overline{\bf 16}}$ part of $\Psi_{\bf 32}$. In the $SO(11)$ gauge-Higgs unification, the number of components of spinor representation is doubled, compared to that in $SO(10)$ theory, from 16 to 32, but the orbifold boundary conditions reduce the number of chiral zero modes to 16.

If Majorana mass terms were introduced for neutrinos on the Planck brane, the N_Ψ fermion number would not be conserved. It would give rise to proton decay at the higher loop level. Its rate would be suppressed if Majorana masses were sufficiently large.

9. Summary

Grand unification is necessary to explain the observed charge quantization in quarks and leptons. We have formulated the $SO(11)$ gauge-Higgs grand unification in which the 4D Higgs boson of $m_H = 125\,$GeV appears as a part of gauge fields in 5 dimensions. It generalizes the $SO(5) \times U(1)_X$ gauge-Higgs EW unification. Dynamical EW gauge symmetry breaking is achieved by the Hosotani mechanism. The $SO(11)$ structure appears above $m_{\rm KK}$, which can be as low as 10 TeV. Nevertheless the stability of protons is guaranteed by the conservation of the new fermion number N_Ψ.

There remain many problems to be clarified. First of all, one has to determine the parameters of the model, including brane interactions, such that the observed mass spectrum of quarks and lepton is reproduced, and the EW symmetry breaking is indeed spontaneously broken. Gauge coupling unification need to be examined by solving RGE. The scenario of the gauge-Higgs unification is promising.

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