Family symmetries and CP

I. de Medeiros Varzielas[*]

University of Southampton
E-mail: `ivo.de@soton.ac.uk`

CP-odd invariants, independent of basis and valid for any choice of CP transformation are a powerful tool in the study of CP. They are particularly convenient to study the CP properties of models with family symmetries. After interpreting the consequences of adding specific CP symmetries to a Lagrangian invariant under $\Delta(27)$, I use the invariant approach to systematically study Yukawa-like Lagrangians with an increasing field content in terms of $\Delta(27)$ representations. Included in the Lagrangians studied are models featuring explicit CP violation with calculable phases (referred to as explicit geometrical CP violation) and models that automatically conserve CP, despite having all the $\Delta(27)$ representations.

---

[*]Speaker.

1. Introduction

This contribution to the proceedings of Planck 2015 follows closely the layout of the seminar I presented in the conference. I include here an expanded discussion of situations with multiple $\Delta(27)$ singlets and triplets, studied recently in [1, 2, 3].

1.1 Why study CP?

Flavour is an unsolved problem in the Standard Model (SM) and the same can be said of CP phenomena, which are currently not well understood. When combined these constitute the flavour and CP problems of the SM but also of extensions like Supersymmetry.

The Baryon Asymmetry of the Universe can not be quantitatively accounted for in the SM, and the experimental bound on CP violation in the strong sector is extremely small. In the SM there is CP violation only in association with the Yukawa couplings of the quark sector, although experimental verification of CP violation in the lepton sector may soon be achieved by the increasingly precise neutrino oscillation experiments.

It is very timely to consider what are the most promising solutions to these kind of problems and possibly make predictions of what would be the observed phases in the PMNS leptonic mixing matrix. A recent ambitious example is the $A_4 \times SU(5) \times$ CP model studied in [4, 5], which simultaneously solves the strong CP problem, predicts all the CP phases of the PMNS and, through leptogenesis, links this prediction with the Baryon Asymmetry of the Universe.

Given that there are good reasons to study CP, I now consider how one may do so.

1.2 The invariant approach

The invariant approach (IA) to CP is not new [6]. It starts by splitting the Lagrangian into $\mathscr{L}_{CP}$, that automatically conserves CP (e.g. kinetic terms, gauge interactions) and $\mathscr{L}_{rem.}$, the remaining part:

$$\mathscr{L} = \mathscr{L}_{CP} + \mathscr{L}_{rem.}. \tag{1.1}$$

Subsequently:

- Impose the most general CP transformations (that leave $\mathscr{L}_{CP}$ invariant).

- Apply them and see if it restricts $\mathscr{L}_{rem.}$.

The possibility of (explicit) CP violation only exists in the Lagrangian if the most general CP transformations constrain the Lagrangian (i.e. restrict $\mathscr{L}_{rem.}$).

The IA is powerful because:

- Gets results just from the Lagrangian.

- Independent of basis.

- Shows relevant quantities for physical processes.

A review of the IA for SM leptons is present in [1, 2].

2. Invariant approach and family symmetries

As shown by its innovative application to cases with family symmetries [1], the IA proves to be particularly useful because the CP-odd invariants (CPIs) can be constructed directly from the Lagrangian, without knowledge of the family symmetry, and then used together with the specific structures enforced by the family symmetry on e.g. the Yukawa couplings. This becomes clearer when discussing specific examples, focused on trilinear terms which I refer to as Yukawa-like couplings, as most cases I consider here are meant as fermion-fermion-scalar terms.

2.1 Discrete groups

An interesting example of the use of CPIs with discrete groups arises from applying a relevant SM lepton sector CPI, I_1, constructed similarly to the quark sector CPI in [6]. Defining the Hermitian combinations $H_\nu \equiv m_\nu m_\nu^\dagger$ and $H_l \equiv m_l m_l^\dagger$ [1]:

$$I_1 \equiv \text{Tr} \left[H_\nu, H_l \right]^3 . \tag{2.1}$$

It turns out that I_1 is useful to analyse a Lagrangian with A_4 family symmetry determining the mass structures to be [1]:

$$m_\nu = \alpha \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \beta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \beta + \gamma \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \delta \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2.2}$$

in a basis where m_l is diagonal. The structures β, γ, δ each correspond to contractions to a different singlet of A_4 (β corresponds to the trivial singlet). With H_l diagonal, I_1 is

$$I_1 = 6i(m_\mu^2 - m_e^2)(m_\tau^2 - m_e^2)(m_\tau^2 - m_\mu^2)\text{Im}(H_\nu^{21} H_\nu^{13} H_\nu^{32}). \tag{2.3}$$

CP conservation requires $I_1 = 0$ and since there are no mass degeneracies the relevant quantity is:

$$\text{Im}(H_\nu^{21} H_\nu^{13} H_\nu^{32}) = -\text{Im}(\beta\delta^* + \gamma\beta^* + \delta\gamma^*)\text{Re}(R) \tag{2.4}$$

where R is a rather complicated expression,

$$\begin{aligned} R = {} & 27|\alpha|^4 - 6|\alpha|^2|\beta + \gamma + \delta|^2 + |\gamma\delta|^2 + |\delta\beta|^2 + |\beta\gamma|^2 \\ & + 4|\beta|^2(\gamma\delta^*) + 4|\gamma|^2(\delta\beta^*) + 4|\delta|^2(\beta\gamma^*) \\ & + -6\alpha^{*2}(\beta^2 + \gamma^2 + \delta^2 - \beta\gamma - \delta\beta - \gamma\delta) \\ & + 2\beta^{*2}(\gamma^2 + \delta^2 + \gamma\delta) + 2\gamma^{*2}(\delta^2 + \delta\beta) + 2\delta^{*2}\beta\gamma. \end{aligned}$$

The conclusion is a result known previously in the literature, that this type of A_4 model automatically conserves CP, in the presence of only 1 singlet (this corresponds effectively to having 2 out of β, γ, δ equal to zero and therefore $I_1 = 0$). This brief A_4 example also serves to show that the IA is useful beyond the $\Delta(27)$ cases which I focus on here.

3

2.2 $\Delta(27)$

In the following sections some knowledge of $\Delta(27)$ is useful. I define $\omega \equiv e^{i2\pi/3}$, c (for cyclic) and d (for diagonal) as the relevant generators ($\omega^3 = 1$, $c^3 = d^3 = 1$). The irreducible representations are 1 or 3 dimensional - singlets and triplets. Generators act on singlets by multiplying with a phase: $c1_{ij} = \omega^i 1_{ij}$ and $d1_{ij} = \omega^j 1_{ij}$, where $i, j = 0, 1, 2$ for a total of 9 singlets. In a suitable basis the generators act on a 3_{01} triplet $A = (a_1, a_2, a_3)_{01}$ or a 3_{02} triplet $\bar{B} = (\bar{b}^1, \bar{b}^2, \bar{b}^3)_{02}$ as:

$$c_{3_{0j}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad c_{3_{01}} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} a_2 \\ a_3 \\ a_1 \end{pmatrix}, \tag{2.5}$$

$$d_{3_{01}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad d_{3_{02}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}. \tag{2.6}$$

The nomenclature for the generators represents their action on triplets. d distinguishes 3_{01} and 3_{02} according to their subscripts, which are the powers of ω on the first two diagonal entries of the respective matrix. I refer to 3_{01} as the triplet representation and to 3_{02} as the anti-triplet representation. c cyclically permutes the components equally for triplet and anti-triplet.

Singlet product leads to a singlet transforming with the sum of the indices (modulo 3): $1_{ij} \times 1_{kl}$ transforms as $1_{(i+k)(j+l)}$. The product of triplet with anti-triplet is the sum of all nine singlets, including the trivial singlet

$$[A\bar{B}]_{00} \equiv (a_1\bar{b}^1 + a_2\bar{b}^2 + a_3\bar{b}^3)_{00}, \tag{2.7}$$

which is the $SU(3)$ invariant contraction, and the 8 non-trivial singlets

$$[A\bar{B}]_{01} \equiv (a_1\bar{b}^3 + a_2\bar{b}^1 + a_3\bar{b}^2)_{01}, \tag{2.8}$$

$$[A\bar{B}]_{02} \equiv (a_1\bar{b}^2 + a_2\bar{b}^3 + a_3\bar{b}^1)_{02}, \tag{2.9}$$

$$[A\bar{B}]_{10} \equiv (a_1\bar{b}^1 + \omega^2 a_2\bar{b}^2 + \omega a_3\bar{b}^3)_{10}, \tag{2.10}$$

$$[A\bar{B}]_{11} \equiv (\omega a_1\bar{b}^3 + a_2\bar{b}^1 + \omega^2 a_3\bar{b}^2)_{11}, \tag{2.11}$$

$$[A\bar{B}]_{12} \equiv (\omega^2 a_1\bar{b}^2 + \omega a_2\bar{b}^3 + a_3\bar{b}^1)_{12}, \tag{2.12}$$

$$[A\bar{B}]_{20} \equiv (a_1\bar{b}^1 + \omega a_2\bar{b}^2 + \omega^2 a_3\bar{b}^3)_{20}, \tag{2.13}$$

$$[A\bar{B}]_{21} \equiv (\omega^2 a_1\bar{b}^3 + a_2\bar{b}^1 + \omega a_3\bar{b}^2)_{21}, \tag{2.14}$$

$$[A\bar{B}]_{22} \equiv (\omega a_1\bar{b}^2 + \omega^2 a_2\bar{b}^3 + a_3\bar{b}^1)_{22}. \tag{2.15}$$

2.2.1 $\Delta(27)$ and adding CP

I will now study the CP properties of a specific $\Delta(27)$ invariant Lagrangian with a triplet A, anti-triplet $\bar{B}$, and singlets C, D (respectively 3_{01}, 3_{02}, 1_{10}, 1_{01}):

$$\mathcal{L}_{CD} = y_c(A\bar{B})_{20}C_{10} + y_d(A\bar{B})_{02}D_{01} + H.c.. \tag{2.16}$$

Now I add a specific CP transformation, like the trivial CP transformation CP_1 acting A, $\bar{B}$, C and D as:

$$CP_1 A = A^* = (a^{1*}, a^{2*}, a^{3*})_{02}, \tag{2.17}$$

$$CP_1 \bar{B} = \bar{B}^* = (\bar{b}_1^*, \bar{b}_2^*, \bar{b}_3^*)_{01}, \tag{2.18}$$

$$CP_1 C_{10} = C_{20}^*, \tag{2.19}$$

$$CP_1 D_{01} = D_{02}^*. \tag{2.20}$$

Note that A^*, $\bar{B}^*$, C^*, D^* are respectively 3_{02}, 3_{01}, 1_{20}, 1_{02} (reflected by the indices and labels).

Imposing invariance under CP_1 on $\mathcal{L}_{CD}$, the y_c term which CP_1 transforms to:

$$\rightarrow y_c(a^{1*}\bar{b}_1^* + \omega a^{2*}\bar{b}_2^* + \omega^2 a^{3*}\bar{b}_3^*)_{20} C_{20}^*, \tag{2.21}$$

should become the *H*.*c*., with y_c^*:

$$y_c^*(a^{1*}\bar{b}_1^* + \omega^2 a^{2*}\bar{b}_2^* + \omega a^{3*}\bar{b}_3^*)_{10} C_{20}^*. \tag{2.22}$$

Beyond the coefficient being conjugated, the expressions are different (noted by their labels). Instead, CP_1 transforms the y_d into:

$$\rightarrow y_d(a^{1*}\bar{b}_2^* + a^{2*}\bar{b}_3^* + a^{3*}\bar{b}_1^*)_{01} D_{02}^*, \tag{2.23}$$

which compares to its *H*.*c*. with y_d^*:

$$y_d^*(a^{1*}\bar{b}_2^* + a^{2*}\bar{b}_3^* + a^{3*}\bar{b}_1^*)_{01} D_{02}^*. \tag{2.24}$$

Apart from conjugating y_d the expressions are the same. This reveals that the CP_1 transformed expression is not invariant under $\Delta(27)$ for arbitrary y_c (adding the subscripts will not make a trivial singlet). $\mathcal{L}_{CD}$ is only invariant under both $\Delta(27)$ and CP_1 if $y_c = 0$ (and y_d to be real) or conversely, keeping $y_c \neq 0$ explicitly violates either $\Delta(27)$ or CP_1.

Although imposing a specific CP transformation can force coefficients to vanish, this does not imply CP violation occurs if those coefficients do not vanish. $\mathcal{L}_{CD}$ with arbitrary y_c and y_d is actually CP conserving. More considerations on adding CP to family symmetries and $\Delta(27)$ in particular can be found in [2], where changes of basis are considered.

2.2.2 $\Delta(27)$ **just singlets**

To illustrate how the IA proceeds, I start with Yukawa-like terms without $\Delta(27)$ triplets. I name singlets under $\Delta(27)$ h_{ij}, the label means it is a 1_{ij}. With h_{00}, h_{01}, h_{10}, the Yukawa-like terms are [3]:

$$\begin{aligned}\mathcal{L}_{III} =& z_{00} h_{00} h_{00} h_{00} + z_{01} h_{01} h_{01} h_{01} + z_{10} h_{10} h_{10} h_{10} \\ &+ y_{00} h_{00} h_{00} h_{00}^\dagger + y_{01} h_{00} h_{01} h_{01}^\dagger + y_{10} h_{00} h_{01} h_{01}^\dagger + H.c..\end{aligned} \tag{2.25}$$

The next step is to consider the most general CP transformation for each field, each singlet transforms with its own phase p_{ij}

$$h_{ij} \rightarrow e^{ip_{ij}} h_{ij}^*. \tag{2.26}$$

For $\mathcal{L}_{III}$ to remain invariant under these transformations leads to a set of necessary and sufficient conditions for CP conservation

$$z_{00}e^{i3p_{00}} = z_{00}^*, \quad z_{01}e^{i3p_{01}} = z_{01}^*, \quad z_{10}e^{i3p_{10}} = z_{10}^*, \tag{2.27}$$

$$y_{00}e^{ip_{00}} = y_{00}^*, \quad y_{01}e^{ip_{00}} = y_{01}^*, \quad y_{10}e^{ip_{00}} = y_{10}^*. \tag{2.28}$$

By combining conditions that cancel dependence on the CP transformations one obtains CPIs. A CPI with y_{01} and y_{10} requires canceling the dependence on p_{00}, as in $\mathrm{Im}[y_{01}y_{10}^*]$

$$y_{01}y_{10}^\dagger = (y_{01}y_{10}^\dagger)^* \to \mathrm{Im}[y_{01}y_{10}^*] = 0, \tag{2.29}$$

where y_{ij} are complex numbers ($y_{ij}^\dagger = y_{ij}^*$). The CPI vanishing is necessary (but often not sufficient) for CP conservation, and in this case constrains the relative phase between the couplings.

I generalise the field content to include all 9 $\Delta(27)$ singlets h_{ij}. Imposing a Z_3 symmetry where each h_{ij} transforms equally can reduce the allowed terms. There are 9 Yukawa-like terms like $z_{00}h_{00}h_{00}h_{00}$ (one for each singlet) but I focus on the mixed terms like $y_1 h_{00}h_{01}h_{02}$, of which there are 12 combinations [3]:

$$\begin{aligned}
\mathcal{L}_{IX} = &\, y_1 h_{00}h_{01}h_{02} + y_2 h_{00}h_{10}h_{20} + y_3 h_{00}h_{11}h_{22} + y_4 h_{00}h_{12}h_{21} + \\
&\, y_5 h_{01}h_{10}h_{22} + y_6 h_{01}h_{11}h_{21} + y_7 h_{01}h_{12}h_{20} + \\
&\, y_8 h_{02}h_{10}h_{21} + y_9 h_{02}h_{11}h_{20} + y_{10}h_{02}h_{12}h_{22} + \\
&\, y_{11}h_{10}h_{11}h_{12} + y_{12}h_{20}h_{21}h_{22} + H.c.. \tag{2.30}
\end{aligned}$$

The CP conservation condition for each coupling depends on the 3 phases of the respective singlets:

$$y_1 e^{i(p_{00}+p_{01}+p_{02})} = y_1^*, \quad y_2 e^{i(p_{00}+p_{10}+p_{20})} = y_2^*, \quad y_6 e^{i(p_{01}+p_{11}+p_{21})} = y_6^*, \tag{2.31}$$

$$y_{10} e^{i(p_{02}+p_{12}+p_{22})} = y_{10}^*, \quad y_{11} e^{i(p_{10}+p_{11}+p_{12})} = y_{11}^*, \quad y_{12} e^{i(p_{20}+p_{21}+p_{22})} = y_{12}^*. \tag{2.32}$$

It is possible to combine several of the mixed couplings to form a CPI. An example is

$$\mathrm{Im}[y_1 y_2^* y_6^* y_{10}^* y_{11}y_{12}], \tag{2.33}$$

so this particular combination of couplings has to be real for CP to be conserved. Other combinations can be built from the couplings in $\mathcal{L}_{IX}$.

2.2.3 $\Delta(27)$ **pair of triplets**

The next case study for the IA are Yukawa-like terms with 2 $\Delta(27)$ triplets (the case with 1 $\Delta(27)$ triplet can be found in [2, 3]). An interesting Lagrangian is similar to $\mathcal{L}_{CD}$:

$$\mathcal{L}_{3s} = y_{00}(L\nu^c)_{00}h_{00} + y_{01}(L\nu^c)_{02}h_{01} + y_{10}(L\nu^c)_{20}h_{10} + H.c., \tag{2.34}$$

where there are now 3 singlets, and L, ν^c are the triplet and anti-triplet respectively. The general CP transformations are associated with unitary transformations:

$$h_{00} \to e^{ip_{00}}h_{00}^*; \quad h_{01} \to e^{ip_{01}}h_{01}^*; \quad h_{10} \to e^{ip_{10}}h_{10}^*; \tag{2.35}$$

$$L \to U_L^T L^*; \quad \nu^c \to U_\nu \nu^{c*}. \tag{2.36}$$

Identifying the Yukawa matrices associated with each h_{ij} as Y_{ij}, CP invariance requires

$$U_L Y_{ij} U_\nu e^{ip_{ij}} = Y_{ij}^*. \tag{2.37}$$

The relevant CPI is [1]:

$$I_{3s} \equiv \text{ImTr}\,(Y_{00}Y_{01}^\dagger Y_{10}Y_{00}^\dagger Y_{01}Y_{10}^\dagger). \tag{2.38}$$

Note that this invariant applies to the Lagrangian even in the absence of $\Delta(27)$. However, $\Delta(27)$ invariance imposes additionally $Y_{00} = y_{00}\text{I}$ (proportional to the identity matrix) and

$$Y_{01} = y_{01} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}; \quad Y_{10} = y_{10} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}. \tag{2.39}$$

If one calculates the CPI for the $\Delta(27)$ invariant Lagrangian one obtains:

$$I_{3s} = \text{Im}(3\omega^2 |y_{00}|^2 |y_{01}|^2 |y_{10}|^2), \tag{2.40}$$

where the only phase present is ω^2. The IA reveals a case of geometrical CP violation, i.e. where CP is violated but the arbitrary phases of the couplings (in this case the y_{ij}) are irrelevant.

Note that this type of invariant can only be built with 3 or more Yukawa matrices, which is a hint that cases with 2 singlets automatically conserve CP - as is the case for $\mathcal{L}_{CD}$ and is shown in [2]. In fact there is explicit geometrical CP violation for Lagrangians of type $\mathcal{L}_{3s}$ with almost any combination of 3 $\Delta(27)$ singlets [2] - the exceptions are when choosing one of the 12 combinations of 3 singlets that make up an invariant term in $\mathcal{L}_{IX}$, in such cases the Lagrangian conserves CP automatically. One such example is:

$$\mathcal{L}_{3s1} = y_{00}(L\nu^c)_{00}h_{00} + y_{01}(L\nu^c)_{02}h_{01} + y_{02}(L\nu^c)_{01}h_{02} + H.c., \tag{2.41}$$

associating with each h_{ij} as Y_{ij} and using the matrices imposed by $\Delta(27)$ invariance:

$$I_{3s1} \equiv \text{ImTr}\,(Y_{00}Y_{01}^\dagger Y_{02}Y_{00}^\dagger Y_{01}Y_{02}^\dagger) = 0. \tag{2.42}$$

The CP symmetries present in these 12 special cases are discussed in [2].

Any choice of 4 or more singlets includes combinations of 3 that allow CP violation. By adding any other singlet to the set h_{00}, h_{01}, h_{02} in $\mathcal{L}_{3s_1}$, we have a singlet h_{ij} with $i \neq 0$. In general there is no vanishing of the I_{3s}-type CPIs involving Y_{ij} with Y_{00}, Y_{01}, Y_{02}:

$$\text{ImTr}\,(Y_{00}Y_{01}^\dagger Y_{ij}Y_{00}^\dagger Y_{01}Y_{ij}^\dagger), \tag{2.43}$$

$$\text{ImTr}\,(Y_{01}Y_{02}^\dagger Y_{ij}Y_{01}^\dagger Y_{02}Y_{ij}^\dagger), \tag{2.44}$$

$$\text{ImTr}\,(Y_{02}Y_{00}^\dagger Y_{ij}Y_{02}^\dagger Y_{00}Y_{ij}^\dagger). \tag{2.45}$$

2.2.4 $\Delta(27)$ **three triplets and beyond**

The next step is to investigate Yukawa-like Lagrangians in the presence of 3 $\Delta(27)$ triplets. Considering Higgs doublets $h_u \sim 1_{10}$ and $h_d \sim 1_{01}$ (the notation is slightly different from the notation in other sections), and a Z_2 symmetry that ensures u^c couple only to h_u and d^c couple only to h_d, the Lagrangian is:

$$\mathscr{L}_{2HDM} = y_u (Qu^c)_{20} h_u + y_d (Qd^c)_{02} h_d + H.c., \tag{2.46}$$

where Q is a triplet and d^c, u^c are anti-triplets of $\Delta(27)$. Again I take Yukawa matrices Y_u, Y_d corresponding to the terms and apply the IA. The general CP transformations are denoted as

$$h_u \to e^{ip_u} h_u^*, \quad h_d \to e^{ip_d} h_d^*, \tag{2.47}$$

$$Q \to U_Q^T Q^*, \quad u^c \to U_u u^{c*}, \quad d^c \to U_d d^{c*}. \tag{2.48}$$

CP invariance demands, similarly to the SM case [6]:

$$U_Q Y_u U_u e^{ip_u} = Y_u^*, \tag{2.49}$$

$$U_Q Y_d U_d e^{ip_d} = Y_d^*. \tag{2.50}$$

so I use the Hermitian combinations $H_{u,d} \equiv Y_{u,d} Y_{u,d}^\dagger$

$$U_Q H_u U_Q^\dagger = H_u^*, \quad U_Q H_d U_Q^\dagger = H_d^*, \tag{2.51}$$

concluding $\mathrm{Tr}\,[H_u, H_d]^3 = 0$ [6] is necessary and sufficient for CP conservation. As $\Delta(27)$ imposes

$$Y_u = y_u \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \tag{2.52}$$

$$Y_d = y_d \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \tag{2.53}$$

CP is automatically conserved for any y_u, y_d. In order to enable CP violation through an I_{3s}-type CPIs not vanishing requires at least 3 singlets coupling to one of the sectors, meaning that it is now possibly to have up to 6 distinct singlets and automatically conserve CP.

Beyond 3 triplets, more and more singlets can be included while the Lagrangian automatically conserves CP. The final generalisation I consider is to add another anti-triplet x^c. If the sectors are separated by an Abelian symmetry (like the Z_2 discussed for $\mathscr{L}_{2HDM}$), there are 3 sectors of $\Delta(27)$ singlets that I denote $h_{d_{ij}}$, $h_{u_{kl}}$, $h_{x_{mn}}$. Using the IA and considering how CPIs can be constructed we extend the previous results to conclude that the relevant CPIs are of I_{3s}-type for each sector (due to the different U_d, U_u, U_x matrices):

$$\mathscr{L}_{4Q} = Y_{d_{ij}} (Qd^c) h_{d_{ij}} + Y_{u_{kl}} (Qu^c) h_{u_{kl}} + Y_{x_{mn}} (Qx^c) h_{x_{mn}} + H.c., \tag{2.54}$$

$$Q \to U_Q^T Q^*, \quad d^c \to U_d d^{c*}, \quad u^c \to U_u u^{c*}, \quad x^c \to U_x x^{c*}. \tag{2.55}$$

It is interesting that at 4 triplets (in this case 1 triplet and 3 anti-triplets) we have reached a situation where CP can be automatically conserved even with fields transforming as each of the 9 $\Delta(27)$ singlets. One example is $h_{d_{00}}, h_{d_{01}}, h_{d_{02}}, h_{u_{10}}, h_{u_{11}}, h_{u_{12}}, h_{x_{20}}, h_{x_{21}}, h_{x_{22}}$ [3].

3. Conclusions

The invariant approach is very useful with family symmetries, and the examples I described serve to demonstrate this. One of the advantages of the method is that it does not depend on the group when the CP-odd invariants are constructed. $\Delta(27)$ as a family symmetry has rich interplay with CP, which was also revealed through the examples that were explored.

I showed several Lagrangians, from cases with only 1-dimensional representations of $\Delta(27)$ (singlets), to Yukawa-like terms involving $\Delta(27)$ triplet and anti-triplet, and progressing to three and more $\Delta(27)$ triplets.

The number and type of representations fundamentally affects the CP properties of the Lagrangian. For those with only singlets, the invariant approach reveals the relevant physical phases, which turn out to be relative phases of the complex couplings. For the two triplet case (with one sector), CP is automatically conserved for Yukawa-like terms involving any 2 $\Delta(27)$ singlets and for 12 special combinations out the total 84 combinations of 3 singlets (the other cases are examples of explicit geometrical CP violation). The same type of conclusion holds independently for each sector, and therefore with 3 sectors it is even possible to have all 9 $\Delta(27)$ singlets present while automatically conserving CP.

Acknowledgments

References

[1] G. C. Branco, I. de Medeiros Varzielas and S. F. King, Phys. Rev. D **92** (2015) 3, 036007 [arXiv:1502.03105 [hep-ph]].

[2] I. de Medeiros Varzielas, J. Phys. Conf. Ser. **631** (2015) 1, 012020 [arXiv:1503.02633 [hep-ph]].

[3] G. C. Branco, I. de Medeiros Varzielas and S. F. King, Nucl. Phys. B **899** (2015) 14 [arXiv:1505.06165 [hep-ph]].

[4] F. Björkeroth, F. J. de Anda, I. de Medeiros Varzielas and S. F. King, JHEP **1506** (2015) 141 [arXiv:1503.03306 [hep-ph]].

[5] F. Björkeroth, F. J. de Anda, I. de Medeiros Varzielas and S. F. King, arXiv:1505.05504 [hep-ph].

[6] J. Bernabeu, G. C. Branco and M. Gronau, Phys. Lett. B **169** (1986) 243.