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Status report on ε_K with lattice QCD inputs

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We report the current status of ε_K, the indirect CP violation parameter in the neutral kaon system, evaluated using the lattice QCD inputs. We use lattice QCD to fix $\hat{B}_K$, ξ_0, ξ_2, $|V_{us}|$, $m_c(m_c)$, and $|V_{cb}|$. Since Lattice 2015, FLAG updated $\hat{B}_K$, exclusive V_{cb} has been updated with new lattice data in the $\bar{B} \to D\ell\nu$ decay channel, and RBC-UKQCD has updated ξ_0 and ξ_2. Our preliminary results show that the standard model evaluation of ε_K with exclusive $|V_{cb}|$ (lattice QCD inputs) has 3.2σ tension with the experimental value, while that of ε_K with inclusive $|V_{cb}|$ (heavy quark expansion) shows no tension.

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1. Introduction

This paper is a follow-up and update of our previous paper [1, 2]. In the standard model, the indirect CP violation parameter of the neutral kaon system ε_K is

$$\begin{aligned}\varepsilon_K &\equiv \frac{\mathscr{A}(K_L \to \pi\pi(I=0))}{\mathscr{A}(K_S \to \pi\pi(I=0))} \\ &= e^{i\theta}\sqrt{2}\sin\theta\left(C_\varepsilon \hat{B}_K X_{\text{SD}} + \frac{\xi_0}{\sqrt{2}} + \xi_{\text{LD}}\right) + \mathscr{O}(\omega\varepsilon') + \mathscr{O}(\xi_0\Gamma_2/\Gamma_1)\,, \qquad (1.1)\end{aligned}$$

where C_ε is a well-known coupling, and X_{SD} is the short distance contribution from the box diagrams. Master formulas for C_ε, X_{SD}, ξ_0, and ξ_{LD} are given in Ref. [1].

Since Lattice 2015, there have been major updates of lattice QCD inputs such as V_{cb}, $\hat{B}_K$, ξ_0, and ξ_2. Hence, it is time to update the current status of ε_K.

2. Input parameter $|V_{cb}|$

Decay mode	$\lvert V_{cb}\rvert$	Ref.
$\bar{B} \to D^*\ell\bar{\nu}$	39.04(49)(53)(19)	[3]
$\bar{B} \to D\ell\bar{\nu}$	40.7(10)(2)	[4]
ex-combined	39.62(60)	this paper
$\bar{B} \to X_c\ell\bar{\nu}$	42.00(64)	[5]
Decay mode	$\lvert V_{ub}\rvert$	Ref.
$\bar{B} \to \pi\ell\bar{\nu}$	3.70(14)	[6, 7]
$\bar{B} \to X_u\ell\bar{\nu}$	4.45(16)(22)	[8]
Decay mode	$\lvert V_{ub}/V_{cb}\rvert$	Ref.
$\Lambda_b \to \Lambda_c\ell\bar{\nu}$	0.083(4)(4)	[9]

Table 1: Results of $|V_{cb}|$ and $|V_{ub}|$.

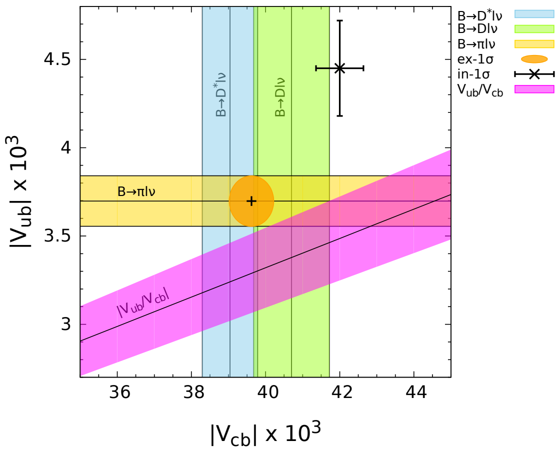


Figure 1: $|V_{cb}|$ versus $|V_{ub}|$.

Let us begin with V_{cb}. In Table 1, we summarize updated results for $|V_{cb}|$ and $|V_{ub}|$. In Ref. [4], DeTar has collected the results for the $\bar{B} \to D\ell\bar{\nu}$ decay mode at non-zero recoil from both lattice QCD [10, 11] and the experiments of Babar [12] and Belle [13] to make a combined fit of all of them. This result corresponds to the green band in Fig. 1. We combine the results of Refs. [4] ($\bar{B} \to D\ell\bar{\nu}$) and [3] ($\bar{B} \to D^*\ell\bar{\nu}$) to obtain the uncorrelated weighted average, which corresponds to the "ex-combined" result in Table 1. This value is shown as an orange circle in Fig. 1. The black cross represents results of inclusive $|V_{cb}|$ and $|V_{ub}|$. The inclusive results are about 3σ away from those of the exclusive decays as well as the LHCb results of $|V_{ub}/V_{cb}|$ (the magenta band in Fig. 1).

3. Input parameter ξ_0

There are two independent methods to determine ξ_0 in lattice QCD: One is the indirect method,

Input	Method	Value	Ref.
ξ_0	indirect	$-1.63(19)\times 10^{-4}$	[14]
ξ_0	direct	$-0.57(49)\times 10^{-4}$	[15]
$\xi_{\rm LD}$	—	$(0\pm 1.6)\,\%$	[16]

(a) Long Distance Effects

Collaboration	Value	Ref.
FLAG-2016	0.7625(97)	[17]
SWME-2014	0.7379(47)(365)	[18]
RBC-UK-2016	0.7499(24)(150)	[19]

(b) $\hat{B}_K$

Table 2: Input parameters: ξ_0, $\xi_{\rm LD}$ and $\hat{B}_K$

and the other is the direct method. The parameter ξ_0 is connected with ε'/ε and ξ_2 as follows,

$$\xi_0 = \frac{{\rm Im}A_0}{{\rm Re}A_0}, \qquad \xi_2 = \frac{{\rm Im}A_2}{{\rm Re}A_2}, \qquad {\rm Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \frac{\omega}{\sqrt{2}|\varepsilon_K|}(\xi_2 - \xi_0)\,. \tag{3.1}$$

In the indirect method, we determine ξ_0 from the experimental values of ${\rm Re}(\varepsilon'/\varepsilon)$, ε_K, ω, and the lattice QCD input ξ_2 using Eq. (3.1). Recently, RBC-UKQCD reported new results for ξ_2 in Ref. [14]. The results for ξ_0 using the indirect method are summarized in Table 2(a).

Recently, RBC-UKQCD also reported new lattice QCD results for ${\rm Im}A_0$ calculated using domain wall fermions [15]. Using the experimental value of ${\rm Re}A_0$, we can determine ξ_0 directly from ${\rm Im}A_0$. RBC-UKQCD also reported the S-wave $\pi-\pi$ (I=0) scattering phase shift $\delta_0 = 23.8(49)(12)$ [15]. This value is 3.0σ lower than the conventional determination of δ_0 in Refs. [20] (KPY-2011) and [21, 22] (CGL-2001). The values for δ_0 are summarized in Table 3. In Fig. 2, we show the results of KPY-2011. They used a singly subtracted Roy-like equation to do the interpolation around $\sqrt{s} = m_K$ (kaon mass). Their fitting to the experimental data works well from the threshold to $\sqrt{s} = 800\,{\rm MeV}$.

Collaboration	δ_0	Ref.
RBC-UK-2016	23.8(49)(12)°	[15]
KPY-2011	39.1(6)°	[20]
CGL-2001	39.2(15)°	[21, 22]

Table 3: Results of δ_0

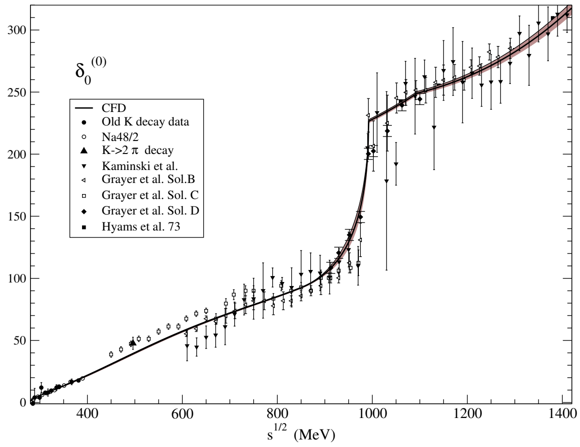
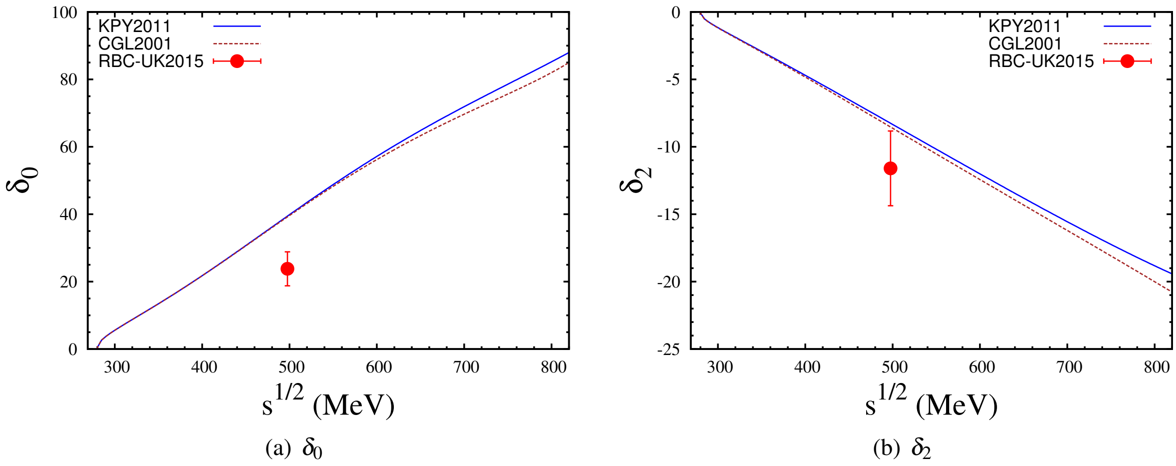


Figure 2: Experimental results of δ_0

In Fig. 3(a), we show the fitting results of both KPY-2011 and CGL-2001 as well as the RBC-UKQCD result. There is essentially no difference between KPY-2011 and CGL-2001 in the region near $\sqrt{s} = m_K$. Here, we observe the 3.0σ gap between RBC-UKQCD and KPY-2011. In contrast, in the case of δ_2 (S-wave, I=2), there is no difference between RBC-UKQCD and KPY-2011 within statistical uncertainty.



(a) δ_0 (b) δ_2

Figure 3: S-wave $\pi - \pi$ scattering phase shifts with $I = 0$ and $I = 2$.

Considering all aspects, we conclude that the direct calculation of ImA_0 and ξ_0 by RBC-UKQCD in Ref. [15] may have unresolved issues. Hence, we use the indirect method to determine ξ_0 in this paper.

Regarding $\xi_{\rm LD}$, the long distance effect in the dispersive part, there has been an on-going attempt to calculate it on the lattice [23]. However, this attempt [24], at present, belongs to the category of exploratory study rather than to that of precision measurement. Hence, we use the rough estimate of $\xi_{\rm LD}$ in Ref. [23] in this paper, which is given in Table 2(a).

4. Input parameter $\hat{B}_K$

In Table 2(b), we present results for $\hat{B}_K$ calculated in lattice QCD with $N_f = 2+1$ flavors. Here, FLAG-2016 represents the global average over the results of BMW-2011 [25], Laiho-2011 [26], RBC-UK-2016 [19], and SWME-2016 [27], which is reported in Ref. [17]. SWME-2014 represents the $\hat{B}_K$ result reported in Ref. [18]. RBC-UK-2016 represents that reported in Ref. [19].

The results of SWME-2016 are obtained using fitting based on staggered chiral perturbation theory (SChPT) in the infinite volume limit, while those of SWME-2014 are obtained using fitting based on SChPT with finite volume corrections included at the NLO level. In this paper, we use the FLAG-2016 result of $\hat{B}_K$.

5. Other input parameters

For the Wolfenstein parameters λ, $\bar{\rho}$, and $\bar{\eta}$, both CKMfitter and UTfit updated their results in Refs. [28, 29], while the angle-only-fit has not been updated since 2015. The results are summarized in Table 4(a).

For the QCD corrections η_{cc}, η_{ct}, and η_{tt}, we use the same values as in Ref. [1], which are given in Table 4(b). Other input parameters are the same as in Ref. [1] except for the charm quark mass $m_c(m_c)$, which are summarized in Table 4(c). For the charm quark mass, we use the HPQCD results of $m_c(m_c)$ reported in Ref. [30].

	CKMfitter	UTfit	AOF [31]
λ	$0.22548(68)$ /[28]	$0.22497(69)$ /[29]	$0.2253(8)$ /[32]
$\bar{\rho}$	$0.145(13)$ /[28]	$0.153(13)$ /[29]	$0.139(29)$ /[33]
$\bar{\eta}$	$0.343(12)$ /[28]	$0.343(11)$ /[29]	$0.337(16)$ /[33]

(a) Wolfenstein parameters

Input	Value	Ref.
η_{cc}	$1.72(27)$	[1]
η_{tt}	$0.5765(65)$	[34]
η_{ct}	$0.496(47)$	[35]

(b) QCD corrections

Input	Value	Ref.
G_F	$1.1663787(6) \times 10^{-5}$ GeV^{-2}	[32]
M_W	$80.385(15)$ GeV	[32]
$m_c(m_c)$	$1.2733(76)$ GeV	[30]
$m_t(m_t)$	$163.3(2.7)$ GeV	[36]
θ	$43.52(5)^\circ$	[32]
m_{K^0}	$497.614(24)$ MeV	[32]
ΔM_K	$3.484(6) \times 10^{-12}$ MeV	[32]
F_K	$156.2(7)$ MeV	[32]

(c) Other input parameters

Table 4: Input parameters

6. Results for ε_K with lattice QCD inputs

In Fig. 4, we show the results for ε_K evaluated directly from the standard model with the lattice QCD inputs described in the previous sections. In Fig. 4(a), the blue curve represents the theoretical evaluation of ε_K with the FLAG $\hat{B}_K$, AOF for the Wolfenstein parameters, and exclusive V_{cb} that corresponds to ex-combined in Table 1. Here the red curve represents the experimental value of ε_K. In Fig. 4(b), the blue curve represents the same as in 4(a) except for using the inclusive V_{cb} in Table 1. Our preliminary results are, in units of 1.0×10^{-3},

$$|\varepsilon_K| = 1.69 \pm 0.17 \qquad \text{for exclusive } V_{cb} \text{ (lattice QCD)} \tag{6.1}$$

$$|\varepsilon_K| = 2.10 \pm 0.21 \qquad \text{for inclusive } V_{cb} \text{ (QCD sum rules)} \tag{6.2}$$

$$|\varepsilon_K| = 2.228 \pm 0.011 \qquad \text{(experimental value)} \tag{6.3}$$

This indicates that there is 3.2σ tension in the exclusive V_{cb} channel (lattice QCD) and no tension in the inclusive V_{cb} channel (heavy quark expansion; QCD sum rules).

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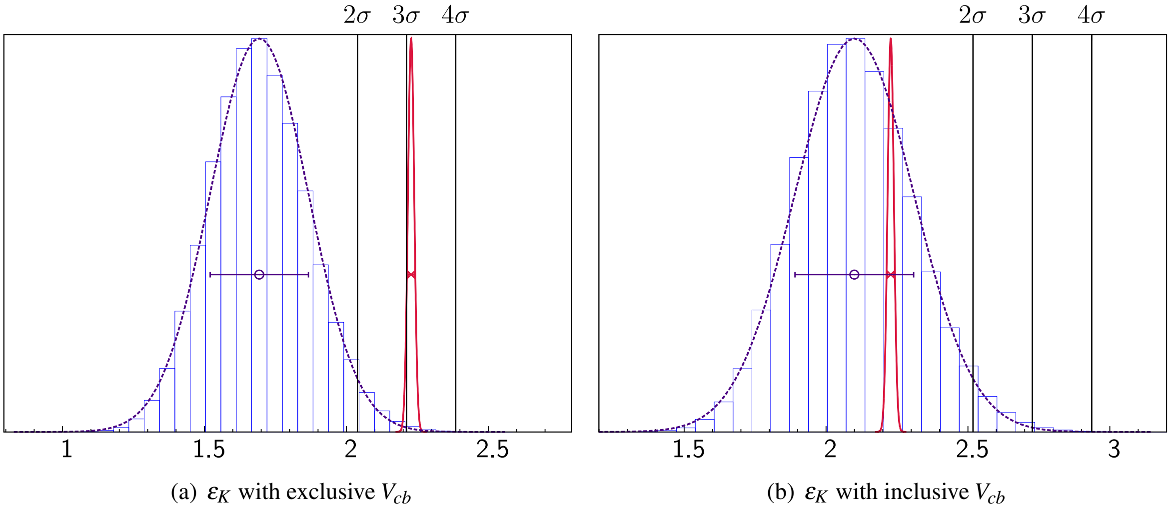

(a) ε_K with exclusive V_{cb} (b) ε_K with inclusive V_{cb}

Figure 4: ε_K with exclusive V_{cb} (left) and inclusive V_{cb} (right). Here, we use the FLAG-2016 $\hat{B}_K$ and AOF for the Wolfenstein parameters. The red curve represents the experimental value of ε_K and the blue curve the theoretical value evaluated directly from the standard model.

(No. KSC-2014-G3-003). Computations were carried out on the DAVID GPU clusters at Seoul National University.

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