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Heavy and light spectroscopy near the physical point, Part II: Tetraquarks

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Having introduced the ensembles and basic spectrum in Part I of this conference contribution, we focus on results for a doubly heavy tetraquark candidate, $qq'\bar{Q}\bar{Q}$. Based on phenomenological observations regarding heavy baryon systems, we motivate two possible lattice interpolating operators: a diquark-antidiquark and a meson-meson. We show that these operators exhibit good behaviour both in terms of lattice QCD and their phenomenological interpretation. In particular we study the $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ flavor combinations and analyze their binding. At the physical point, at finite lattice spacing, we find strong indications for binding of these tetraquark candidates. We comment on possible finite volume effects and search windows for experimental confirmation.

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1. Introduction and phenomenological observations

The search for exotic hadrons is a focus of particle physics both in theory and experiment. An example of an exotic hadron could be a four quark bound state, a tetraquark. We consider such a hadron with two light and two heavy quarks, i.e. with flavor content $qq'\bar{Q}\bar{Q}$ where q,q' are the light flavor combinations $qq' = ud$ or us. In the limit of infinitely heavy quarks $m_Q \to \infty$ the attractive nature of the color Coulomb potential guarantees a bound ground state of such a tetraquark. Whether a binding is realized away from this limit, as in nature, for charm and bottom quarks, is subject to non-perturbative effects and only lattice QCD calculations can give a rigorous answer to this question. Previous studies have attempted to measure this binding [1–16], however in [17] [1] we report on the, to our knowledge, first such calculation close to the physical point away from the static limit in the $qq'\bar{b}\bar{b}$ sector. The study benefits from the use of wall sources and a GEVP analysis. To handle the bottom quark in our calculation we use lattice NRQCD. Note, at this conference the initiation and progress of a similar program has been reported [18].

There are indications from the observed spectrum that there should be tetraquark bound states of the $qq'\bar{b}\bar{b}$-type [2]. Additionally these bound states should be strong interaction stable due to the following considerations: Firstly, the mass ratios $(B^* - B)/(\Xi^*_{bb} - \Xi_{bb})$ and $(B^*_s - B_s)/(\Omega^*_{bb} - \Omega_{bb})$ are close to unity; this is indicative of the b-quark mass being large enough to be close to the heavy quark limit. In this limit the heavy quark spin decouples and a heavy antidiquark behaves like a single heavy quark. For quarks that are sufficiently heavy, the observed heavy baryon spectrum gives an idea of the possible binding energies through the splittings of the spin 0 and spin 1 diquark component baryons with the same flavor content and a comparison to the corresponding spin averages. In particular, we have $\Sigma_b - \Lambda_b \approx 194$ MeV and $\Xi'_b - \Xi_b \approx 162$ MeV [20], i.e. the masses lie ~ 145 MeV below and ~ 48 MeV above the corresponding spin average in the $qq' = ud$ case, and ~ 106 MeV below and ~ 35 MeV above, for $qq' = us$. In the so-called "good diquark" spin 0 configuration [21] there is therefore an opportunity for binding energies in the same ballpark.

2. Operators, correlators and a GEVP

Our first operator has the favorable diquark-antidiquark structure noted above, with $\bar{b}\bar{b}$ color 3_c, spin 1 and light quark flavor-spin-color $(\bar{3}_F, 0, \bar{3}_c)$ [17]:

$$D(x) = (u^\alpha_a(x))^T (C\gamma_5)^{\alpha\beta} q^\beta_b(x) \times \bar{b}^\kappa_a(x)(C\gamma_i)^{\kappa\rho}(\bar{b}^\rho_b(x))^T\,, \tag{2.1}$$

where $q = d$ or s and C is the charge conjugation matrix. Though there can be relative orbital momentum, the ground state should have none, yielding a $J^P = 1^+$ state. In general, $D(x)$ will also couple to any pair of conventional mesons with the same quantum numbers (the lowest lying being BB^* for $q = d$ and B_sB^* for $q = s$). Combining such a pair of heavy-light mesons leads us to consider a meson-meson operator of the form

$$M(x) = \bar{b}^\alpha_a(x)\gamma_5^{\alpha\beta} u^\beta_a(x)\, \bar{b}^\kappa_b(x)\gamma_i^{\kappa\rho} d^\rho_b(x) - \bar{b}^\alpha_a(x)\gamma_5^{\alpha\beta} d^\beta_a(x)\, \bar{b}^\kappa_b(x)\gamma_i^{\kappa\rho} u^\rho_b(x)\,, \tag{2.2}$$

[1] In this conference we presented these findings and some textual overlap with our publication [17] is to be expected.
[2] See e.g. [19] as well as [17] and references therein for a list of model calculations on this topic.

for the $\bar{3}_F$, $I = 0$ channel, and the analogous operator with $B_s B^*$ structure for the $\bar{3}_F$ isodoublet channel.

To study possible tetraquark binding, we compute the ratio of the four quark operators above with the corresponding two single meson correlation functions; i.e. the pseudoscalar (P) and vector (V) meson correlators, $C_{PP}(t)$ and $C_{VV}(t)$,

$$G_{\mathscr{O}_1\mathscr{O}_2}(t) = \frac{C_{\mathscr{O}_1\mathscr{O}_2}(t)}{C_{PP}(t)C_{VV}(t)} \ , \tag{2.3}$$

where $\mathscr{O}_{1,2} = D, M$ are the diquark-antidiquark and meson-meson operators, respectively. At large Euclidean times the binding correlator is expected to grow as $e^{-\Delta E t}$ with ΔE being the negative ground state binding energy.

The operators (Eqs. 2.1 and 2.2) overlap with the same ground and excited states, though with different relative strengths. Including possible operator mixing, we define the matrix of binding correlation functions,

$$F(t) = \begin{pmatrix} G_{DD}(t) & G_{DM}(t) \\ G_{MD}(t) & G_{MM}(t) \end{pmatrix} \ . \tag{2.4}$$

Using the variational method the binding can be extracted by solving the GEVP,

$$F(t)\nu = \lambda(t)F(t_0)\nu \ , \tag{2.5}$$

with the eigenvectors ν and the binding energy determined directly from the eigenvalues $\lambda(t)$ via,

$$\lambda(t) = A\, e^{-\Delta E(t-t_0)} = (1+\delta)e^{-\Delta E(t-t_0)} \ . \tag{2.6}$$

From a 2×2 matrix two eigenvalues can be extracted; one corresponds to the ground state and the other to a mixture of all excited state contaminations.

3. Numerical Setup

We use dynamical $n_f = 2+1$ Wilson-Clover [22] gauge field configurations generated by the PACS-CS collaboration [23], with a partially-quenched valence strange quark tuned to obtain the physical K mass at the physical point. An overview of the ensembles can be found in Tab. 1. The basic spectrum of [23] was reproduced in this work. In the valence sector we use Coulomb gauge-fixed wall sources, whereby the FACG algorithm of [24] was used to fix the configurations to Coulomb gauge to an accuracy of $\Theta < 10^{-14}$. We set sources at multiple time positions and compute propagators for light and strange quarks using a modified deflated SAP-solver [25].

To calculate bottom quark propagators, at the same source positions, we use our own NRQCD code [26, 27] implementing the NRQCD lattice action with the Hamiltonian [30, 31]

$$\begin{aligned} H = &-\frac{\Delta^{(2)}}{2M_0} - c_1\frac{(\Delta^{(2)})^2}{8M_0^3} + \frac{c_2}{U_0^4}\frac{ig}{8M_0^2}(\tilde{\boldsymbol{\Delta}}\cdot\tilde{\boldsymbol{E}} - \tilde{\boldsymbol{E}}\cdot\tilde{\boldsymbol{\Delta}}) - \frac{c_3}{U_0^4}\frac{g}{8M_0^2}\boldsymbol{\sigma}\cdot(\tilde{\boldsymbol{\Delta}}\times\tilde{\boldsymbol{E}} - \tilde{\boldsymbol{E}}\times\tilde{\boldsymbol{\Delta}}) \\ &- \frac{c_4}{U_0^4}\frac{g}{2M_0}\boldsymbol{\sigma}\cdot\tilde{\boldsymbol{B}} + c_5\frac{a^2\Delta^{(4)}}{24M_0} - c_6\frac{a(\Delta^{(2)})^2}{16nM_0^2} \ , \end{aligned} \tag{3.1}$$

Label	E_H	E_M	E_L
Extent	$32^3 \times 64$	$32^3 \times 64$	$32^3 \times 64$
a^{-1} [GeV] [28]	2.194(10)	2.194(10)	2.194(10)
κ_l	0.13754	0.13770	0.13781
κ_s	0.13640	0.13640	0.13640
am_π	0.18928(36)	0.13618(46)	0.07459(54)
am_K	0.27198(28)	0.25157(30)	0.23288(25)
$m_\pi L$	6.1	4.4	2.4
M_Υ [GeV]	9.528(79)	9.488(71)	9.443(76)
Configurations	400	800	195
Measurements	800	800	3078

Table 1: Overview of our ensemble parameters: $am_{\pi,K}$ are from global cosh/sinh fits to a shared mass and common amplitudes over the C_{PP}, $C_{A_tA_t}$, and C_{A_tP} correlators using both wall-local and wall-wall data. Fit ranges were chosen so χ^2/dof is close to 1. This analysis leads values for am_π with uncertainties improved by a factor of ~ 6 relative to those of [23]. Throughout the strange quark is tuned to its physical value in the valence sector with $\kappa_s^{\mathrm{val}} = 0.13666$. These configurations use the Iwasaki gauge action [29] with $\beta = 1.9$ and non-perturbative clover coefficient $c_{SW} = 1.715$.

with the tadpole-improvement coefficient U_0 set via the fourth root of the plaquette and tree-level values $c_i = 1$. A tilde denotes tree-level improvement and the c_5, c_6 terms remove the remaining $\mathcal{O}(a)$ and $\mathcal{O}(a^2)$ errors. This setup is known to account for relativistic effects at the few percent level while capturing the relevant heavy-light quark physics [20,27,32]. For details on mass tuning, as well as the meson and baryon spectrum in our calculation we refer to our companion paper [33].

4. Numerical results

Our results for the ground (red) and excited state (blue) binding energies are shown in Fig. 1. For comparison, results obtained from the single-operator diquark-antidiquark (grey dashes) and meson-meson (grey crosses) analyses are also included. These illustrate that both operators couple well to the ground state. We also see, as t/a increases, that the second GEVP eigenvalue approaches the relevant two-meson PV threshold in both channels, strongly supporting an interpretation of the corresponding lowest eigenvalues as corresponding to genuine tetraquarks.

4.1 Finite volume effects and an interpretation as attractive meson-meson interaction

Since only one set of volumes per ensemble is available for study, the possibility that the observed binding is due to an attractive meson-meson interaction generated by the finite volume of the ensemble must be considered. In this case the observed binding would disappear as the volume is enlarged and there would be no bound tetraquark state.

To estimate the approximate shift such an interaction would produce, we invoke the finite volume formalism of [34]. Here, for a system of two particles, e.g. the $\pi\pi$-system, with $I = 0$ and $I = 1$ explicit energy shifts due to finite volumes were derived:

$$\Delta E_{n=0} \approx -\frac{4\pi a_0}{\mu L^3}\left[1 + c_1\frac{a_0}{L} + c_2\left(\frac{a_0}{L}\right)^2\right] \ , \tag{4.1}$$

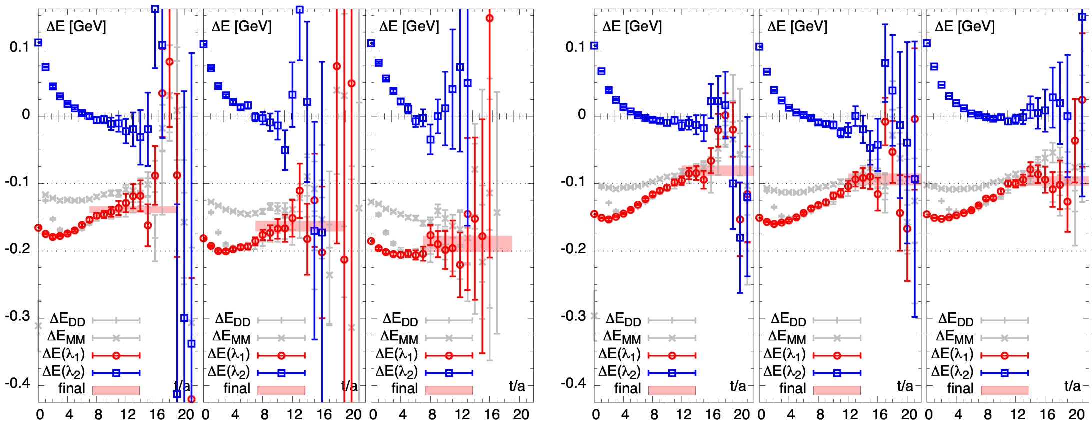

Figure 1: $ud\bar{b}\bar{b}$ (left panel) and $\ell s\bar{b}\bar{b}$ (right panel) tetraquark effective binding energies, computed using the log-derivative of the correlators or GEVP eigenvalues. Red circles (blue squares) represent the bindings relative to the BB^* (B_sB^*) threshold of the first and second GEVP eigenvalues, respectively. Red bands denote the final fit results. Grey dashes and grey crosses indicate the bindings obtained from the corresponding diquark-diquark and meson-meson single-operator analyses. Within the panels from left to right: E_H ($m_\pi L \simeq 6.1$, $m_\pi \simeq 415$ MeV), E_M ($m_\pi L \simeq 4.4$, $m_\pi \simeq 299$ MeV) and E_L ($m_\pi L \simeq 2.4$, $m_\pi \simeq 164$ MeV).

where $c_1 = -2.837297$ and $c_2 = 6.375183$, $\mu = m_1 \cdot m_2/(m_1+m_2)$ is the reduced mass, while a_0 is the scattering length, and

$$\Delta E_{n=1} \approx -\frac{12\tan(\delta_0)}{\mu L^2}\left[1 + c_1' \tan(\delta_0) + c_2' \tan^2(\delta_0)\right] , \tag{4.2}$$

where $c_1' = -0.061367$ and $c_2' = -0.354156$, while δ_0 is the scattering phase shift. Using these relations the finite volume energy shifts for a hypothetical BB-system, whereby we choose $\mu = m_B \cdot m_{B^*}/(m_B + m_{B^*})$, can be estimated by dialing through reasonable scattering lengths and phase shifts that lead to an attractive interaction. In particular we scan $a_0 = 0.3 \ldots 1.2$ fm and $\delta_0 = 15 \ldots 45°$. Even for our smallest physical volume $m_\pi L = 2.4$ we observe that the largest shifts are at the $\Delta E \approx -10$ MeV level for both the ground and threshold energies. This is not exactly the system studied in our calculation, since the corresponding two meson system would be BB^*, and we do not know the scattering parameters, but we see no reason to expect the shift to be an order of magnitude larger in this case. As such the volume effects are predicted to be small, compared to the phenomenological expectations of the binding in Sec. 1, and an interpretation of our results as reflecting an attractive meson-meson interaction rather than a true tetraquark bound state thus appears to us as ruled out by the bindings with magnitude ≥ 100 MeV that we observe.

4.2 Chiral and volume extrapolations

To estimate the binding energy we perform a single exponential fit, Eq. 2.6, to the first eigenvalue $\lambda(t)$, plotted as effective energy in Fig. 1, and accept those that satisfy $\chi^2/dof \leq 1$. For our final results we choose the longest such fit range in t/a; these are $7 \to 19$ and $12 \to 25$ for the $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ channels, respectively.

When the strange quark masses on all ensembles have been tuned to the physical value, as was done here, the leading order chiral behavior of this binding is linear in m_π^2 [35]. Consequently

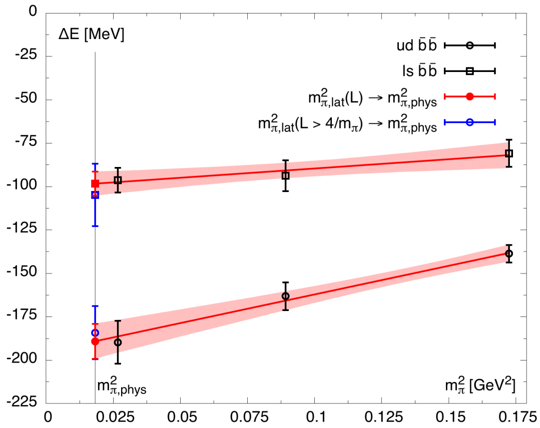

Figure 2: Chiral extrapolations of the $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ binding energies. Red lines and points show the extrapolations using all three ensembles, the blue points those using E_H and E_M.

we use a linear extrapolation to determine our tetraquark bindings at the physical value of m_π. We estimate our finite volume and chiral systematics by performing two extrapolations: In the first we use only those ensembles with the two largest $m_\pi L$, i.e. E_H and E_M. In the second we use all three ensembles. Then we take half the difference of the resulting central values as our systematic error. These extrapolations are shown in Fig. 2, with the filled red symbols giving the physical point results for the three-ensemble fits and the open blue symbols the corresponding results for the fits employing only E_H and E_M. The results of both extrapolations are in good agreement, implying that finite volume errors are under control and indeed small, as predicted in Sec. 4.1. For the binding energies at physical m_π we find:

$$\Delta E_{ud\bar{b}\bar{b}} = -189(10)(3)\ \text{MeV} \qquad \text{and} \qquad \Delta E_{\ell s\bar{b}\bar{b}} = -98(7)(3)\ \text{MeV} \ . \tag{4.3}$$

Light quark cut-off effects are at the $\mathcal{O}(a^2)$-level and hence expected to be small, while the NRQCD Hamiltonian in Eq. 3.1 is $\mathcal{O}(a^2)$ improved.

5. Conclusion

We predict the existence of strong- and electromagnetic-interaction stable $qq'\bar{b}\bar{b}$ tetraquarks with $ud\bar{b}\bar{b}$ and $\ell s\bar{b}\bar{b}$ flavour content. For these states we found binding energies of 189(10)(3) MeV and 98(7)(3) MeV, corresponding to physical states of mass 10.415(10) and 10.594(8) GeV, respectively. With the effect of a finite volume attractive meson-meson interaction estimated to be an order of magnitude smaller than the measured bindings, we expect these to be genuinely bound states. Still, a future finite volume analysis is desirable.

With such deep binding energies these states should decay only weakly, with ordinary heavy meson decay products emitted from a displaced vertex. For example experimentally fully reconstructable modes for the decay of the $ud\bar{b}\bar{b}$ tetraquark are $B^+\bar{D}^0$ and $J/\Psi B^+ K^0$, while they are $J/\Psi B_s K^+$ and $J/\Psi B^+\phi$ for the $us\bar{b}\bar{b}$ case.

In the future we plan to extend our calculation to include $\bar{Q}\bar{Q} = \bar{c}\bar{b}$ and other heavy flavor combinations. If they are bound, they may be more accessible experimentally.

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