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Analysis of $\pi N \to \pi\pi N$ in Chiral Perturbation Theory

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The reaction $\pi N \to \pi\pi N$ is studied up to next-to-leading order in the frameworks of manifestly covariant and heavy-baryon chiral perturbation theory. In addition, the effects of explicit $\Delta(1232)$ degrees of freedom are analyzed. The relevant low-energy constants are determined from fits to low-energy total cross section data and predictions for total cross sections at higher energies as well as differential cross sections are made.

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PoS(CD15)086

1. Introduction

Chiral perturbation theory (χPT) was originally formulated by Weinberg [1] and later extended and applied to study the dynamics of the meson and baryon sector [2, 3, 4]. Since then, χPT has extensively been used in the single-baryon sector [5, 6, 7] and various extensions appeared over the years (see Ref. [7] for more details).

In this paper, the reaction $\pi N \to \pi\pi N$ is analyzed from threshold up to the Δ resonance region using various formulations of χPT. This process was of historical interest as a possible candidate to extract the $\pi\pi$ scattering lengths [10, 11, 12] and is nowadays an excellent testing ground for χPT due to the involvement of three pions in the initial and final states, the relatively high energies involved and the strong couplings to close-by resonances. This makes the reaction $\pi N \to \pi\pi N$ particularly well suited for studying the role of relativistic effects, unitarity and the Δ isobar.

All these arguments encourage a revision of the reaction $\pi N \to \pi\pi N$ in the framework of χPT. The reader is referred to Refs. [12, 13, 14, 15, 16] for previous analyses. This paper strongly borrows from Ref. [17].

2. Effective Lagrangian and Invariant Amplitudes

The so-called small scale expansion (SSE) or ε-expansion is employed troughout this work. Pion-nucleon dynamics is described at tree level by the covariant effective chiral Lagrangian consisting of the following pieces

$$\mathscr{L}_{\text{eff}} = \mathscr{L}^{(2)}_{\pi\pi} + \mathscr{L}^{(1)}_{\pi N} + \mathscr{L}^{(2)}_{\pi N} + \mathscr{L}^{(1)}_{\pi\Delta} + \mathscr{L}^{(2)}_{\pi\Delta} + \mathscr{L}^{(1)}_{\pi N\Delta} + \mathscr{L}^{(2)}_{\pi N\Delta}\,, \tag{2.1}$$

where the superscripts refer to the chiral dimension with the expansion parameter [8]

$$\varepsilon \in \left\{ \frac{q}{\Lambda_\chi}, \frac{M_\pi}{\Lambda_\chi}, \frac{\Delta}{\Lambda_\chi} \right\}, \tag{2.2}$$

and a full list of terms is given in detail in Ref. [19].

To study the role of relativistic effects, all calculations are also performed in the framework of heavy-baryon (HB) χPT. In this approach, in addition to Λ_χ, the nucleon mass m_N is treated as a hard scale as well and the expansion parameter in Eq. (2.2) is extended accordingly. The reader is referred to Refs. [8, 9] for more details on the HB expansion in the pion-nucleon-delta sector.

The necessary formulas to decompose the T-matrix for the reaction $\pi N \to \pi\pi N$ into invariant amplitudes and their relations to observables like total and differential cross sections are given in Refs. [14, 15] for both chiral approaches.

3. Tree-level contributions to the scattering amplitude

The diagrams contributing at order ε^1 and ε^2 in both frameworks are shown in Figure 1. While the leading-order diagrams are solely dependent on the well-established LECs F_π, g_A and h_A, the next-to-leading-order diagrams depend additionally on the LECs c_i from $\mathscr{L}^{(2)}_{\pi N}$ or b_i from $\mathscr{L}^{(2)}_{\pi N\Delta}$. The LECs c_i can be fixed from an analysis of pion-nucleon scattering.

PoS(CD15)086

It has to be emphasized that the Δ isobar, due to its appearance as an unstable particle, needs a special treatment in the resonance region $p^2 \sim m_\Delta^2$. In this region the Δ propagator in the amplitudes has to be dressed [18]. While this dressing is only necessary in the resonance region, we employed the dressed Δ propagator for all kinematical regions and in all diagrams.

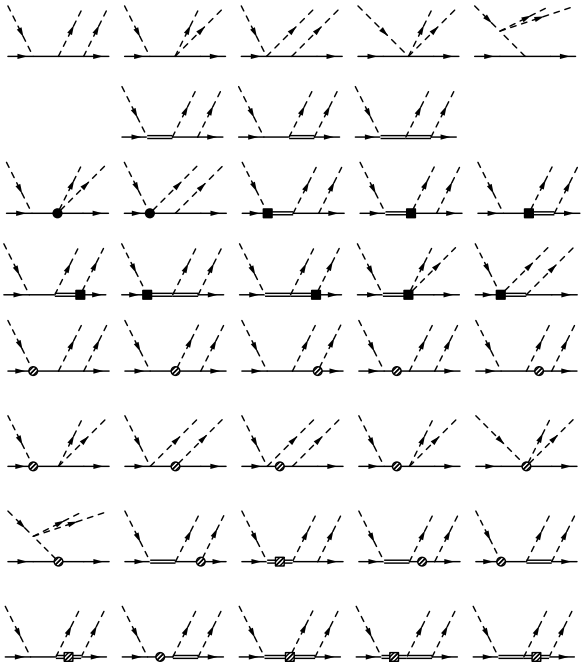

Figure 1: Leading-order and next-to-leading-order diagrams for the reaction $\pi N \to \pi\pi N$. Nucleons and pions are denoted by solid and dashed lines, respectively. Δ is denoted by a double solid line. The filled blob (filled square) denotes an insertion of the c_i- (b_i-) vertices. The additional next-to-leading-order graphs contributing in the HB framework are denoted by the shaded blob/square. Crossed diagrams are not shown.

4. Fitting Procedure

As explained in the previous section, the leading-order Lagrangian depends on various well-known LECs. Following values are used in the calculations: $M_\pi = 139.57$ MeV, $F_\pi = 92.4$ MeV, $m_N = 938.27$ MeV, $g_A = 1.26$, $\Delta = 294$ MeV, $\Gamma = 118$ MeV, $h_A = 1.34$ and $g_1 = 2.27$ (both large-N_c predictions). The LECs c_i entering the next-to-leading-order graphs are taken from pion-nucleon scattering [20, 21] and their values in the Δ-full and Δ-less approach are collected in Table 1. The values for the additional LECs b_3, b_4, b_5, b_6 entering at order ε^2 are unknown and their

Δ	c_1	c_2	c_3	c_4
KH	-0.95	1.90	-1.78	1.50
GW	-1.41	1.84	-2.55	1.87

$\not\Delta$	c_1	c_2	c_3	c_4
KH	-0.75	3.49	-4.77	3.34
GW	-1.13	3.69	-5.51	3.71

Table 1: LECs c_i from the pion-nucleon sector in a Δ-full (Δ) and Δ-less ($\not\Delta$) theory. All values are given in GeV^{-1}.

determination will be discussed in the following. Note that the LECs b_3 and b_6 can be absorbed

PoS(CD15)086

into the other LECs and are redundant in the static limit [25]. These shifts were performed in the HB as well as in the covariant amplitudes such that a meaningful comparison of both approaches is possible.

All fits were performed to all five physical channels simultaneously and only the total cross section data with $T_\pi < 250$ MeV was used. This data is taken from the compilation [22] and from [23, 24, 26]. The resulting values for the LECs b_i using the KH and GW sets of LECs c_i as input are listed in Table 2.

Fit	c_i	b_4+b_5	b_4-b_5	b_3+b_6	b_3-b_6	χ^2/dof
HB	KH	16.00 ± 0.37	-7.99 ± 5.72	——	——	9.63
	GW	15.99 ± 0.37	-8.42 ± 5.77	——	——	9.65
Cov	KH	4.97 ± 0.29	-17.71 ± 12.22	0.39 ± 0.86	1.60 ± 7.62	3.40
	GW	4.34 ± 0.29	-18.24 ± 10.77	0.70 ± 0.76	1.41 ± 6.38	3.47

Table 2: LECs determined from global fits to the total cross section data ($T_\pi < 250$ MeV) at next-to-leading-order using the KH and GW sets of LECs c_i as input. The values of LECs b_i are given in units of GeV^{-1}.

The fits show a strong anticorrelation between the LECs b_4 and b_5. While a reliable extraction of the linear combination b_4+b_5 is possible, the linear combination b_4-b_5 can only be given with a very large uncertainty of about 100%. The LECs b_3 and b_6 are strongly anticorrelated in the covariant approach as well.

The significance of relativistic effects in the reaction $\pi N \to \pi\pi N$ is very well demonstrated in the strong reduction of χ^2/dof from ~ 9.6 in the HB approach to ~ 3.4 in covariant χPT. A futher indication that the energies up to $T_\pi = 250$ MeV employed in the fit are probably beyond the applicability range of the HB approach at order ε^2, are the unnaturally large values of the linear combination b_4+b_5 in HBχPT, while the same linear combination in covariant χPT is of reasonably natural size.

5. Predictions

Total cross sections at higher energies and also other observables like differential cross sections can now be predicted with the LECs collected in Table 2. The predictions in both frameworks, HB and covariant χPT, will allow us to estimate the importance of relativistic effects in the reaction under consideration. Note that all predictions at next-to-leading-order are visualized by bands instead of lines. The bands are produced by taking both sets of LECs, denoted by KH and GW in Table 2. This allows us to give a rough uncertainty estimation associated with the input stemming from the the pion-nucleon system.

In Figure 2, the predictions for the total cross section with energies up to $T_\pi \simeq 400$ MeV are shown for four different approaches, covariant Δ-full, HB Δ-full, covariant Δ-less and HB Δ-less χPT. In total agreement with the observations made in the previous section, the covariant predictions agree way better with the data than the HB ones. The next-to-leading-order HB predictions without Δ underestimate the data, whereas the HB predictions with explicit Δ overestimate the data for $T_\pi > 300$ MeV. The explicit treatment of the Δ in the covariant case is mainly noticeable in the

PoS(CD15)086

upper two channels, whereas the covariant predictions for the other three channels are quite similar in both cases, Δ-full and Δ-less.

In Figure 3, the covariant predictions for $d\sigma/dM_{\pi\pi}^2$ and $d\sigma/dt$ are presented for the two channels $\pi^- p \to \pi^+\pi^- n$ and $\pi^+ p \to \pi^+\pi^+ n$. Recalling that the total cross section in the $\pi^+\pi^-$ channel is very accurately predicted at order ε^2, both single-differential cross sections are also well described in the Δ-full approach. The Δ-less results at order Q^2 strongly underestimate the single-differential cross sections, which is fully in line with the observed underprediction of the total cross section. In contrast, the single-differential cross sections in the $\pi^+\pi^+$-channel are poorly predicted at both orders, Q^2 and ε^2. Recalling that the total cross section is significantly overpredicted in both approaches, the observerd large deviations for the single-differential cross sections should not be suprising.

Finally, a comparison with the previous calculations within the covariant [15] and HB [14] frameworks shows consistent predictions of total and differential cross sections for both Δ-less approaches. A comparison with the calculation of Ref. [13] points out that their leading-order results with explicit Δ and Roper give a similar description of the total cross sections as our next-to-leading-order Δ-full results. Note that they additionally included the Roper resonance explicitly and that there is no strict power counting underlying their calculations.

6. Summary and outlook

In this paper an analysis of the reaction $\pi N \to \pi\pi N$ at tree level up to next-to-leading order using the Δ-full and Δ-less HB and manifestly covariant formulations of χPT was performed. The main results are:

- Total cross section data of all five physically accessible channels with $T_\pi < 250$ MeV was fitted in order to determine the low-energy constants b_i. Due to strong anticorrelations, only the linear combinations $b_4 + b_5$ and $b_3 + b_6$ were reliably determined. They are found to be of natural size in the covariant approach.

- The description of the experimental data is clearly superior in the covariant subleading calculation compared to the HB one. Furthermore, the explicit treatment of the Δ leads to an improvement in the description of the data.

This work is going to be extended in the near future by calculations at the next-higher orders in the chiral expansion. The aim is to test the convergence of the chiral expansion and also to constrain futher higher order LECs by a combined analysis of the reactions $\pi N \to \pi N$ and $\pi N \to \pi\pi N$.

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PoS(CD15)086

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PoS(CD15)086

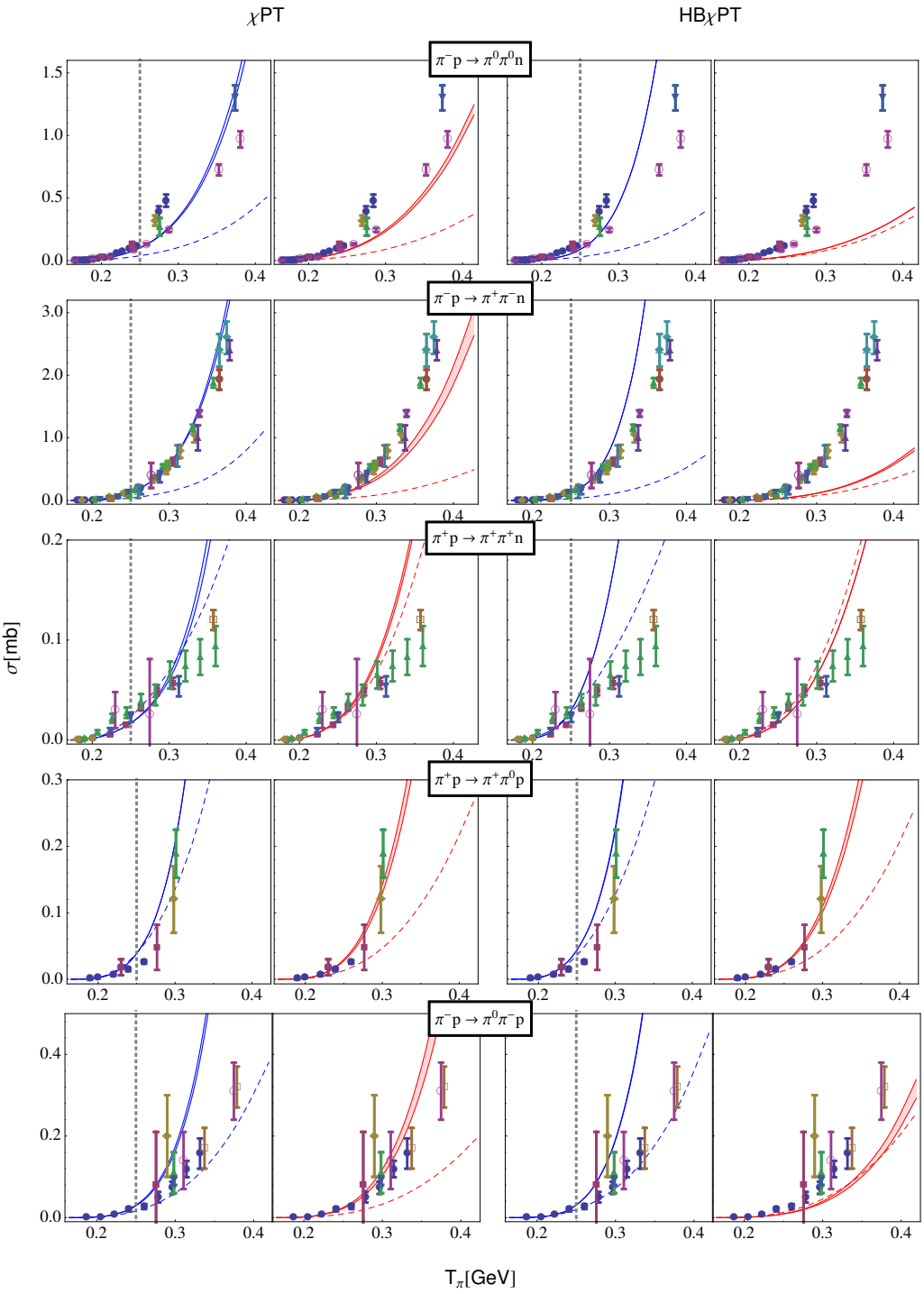

Figure 2: Predictions for the total cross section up to $T_\pi \simeq 400$ MeV. The columns from left to right correspond to the Δ-full covariant, Δ-less covariant, Δ-full HB and Δ-less HB χPT predictions, respectively. The dashed lines and solid bands refer to leading-order and next-to-leading-order results. The bands correspond to using the KH and GW sets of LECs c_i. The energies used in the fit are below the vertical dotted line.

PoS(CD15)086

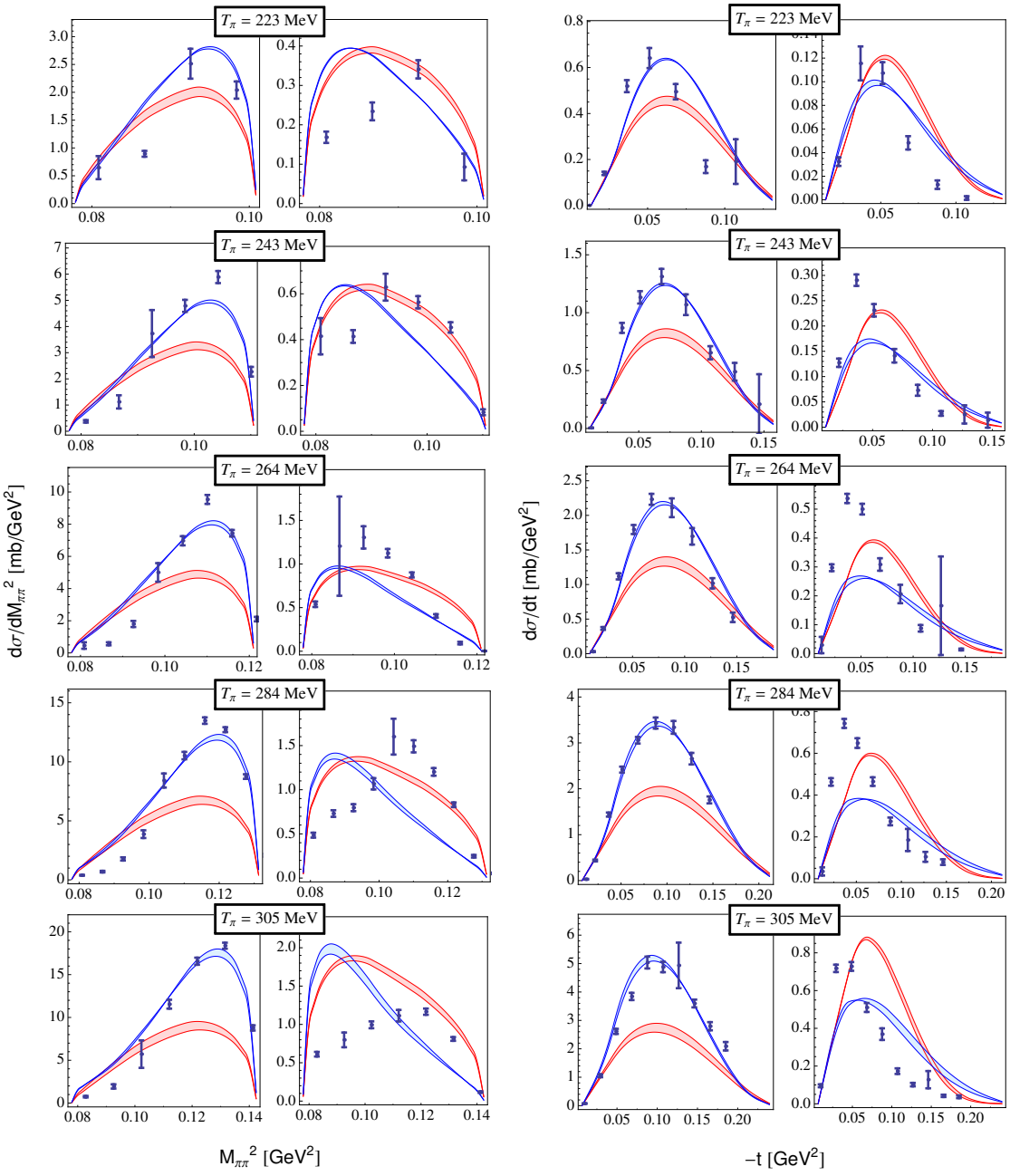

Figure 3: Comparison of next-to-leading-order covariant Δ-full and Δ-less χPT predictions for the single-differential cross sections with respect to $M_{\pi\pi}^2$ and t, respectively, between the two channels $\pi^- p \to \pi^+\pi^- n$ (left) and $\pi^+ p \to \pi^+\pi^+ n$ (right). The red and blue bands refer to Δ-less and Δ-full calculations, respectively.

PoS(CD15)086