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Neutral pion photoproduction on protons in fully covariant ChPT with $\Delta(1232)$ loop contributions

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We study the neutral pion photoproduction at near-threshold energies in fully covariant chiral perturbation theory up to $\mathscr{O}(p^3)$. When including only nucleonic virtual states in the model, the convergence is too slow. Therefore we test the model when introducing the $\Delta(1232)$ resonance as an additional degree of freedom. Some low-energy constants were fitted, converging to values in good agreement with those expected from literature.

The 8th International Workshop on Chiral Dynamics
29 June 2015 - 03 July 2015
Pisa, Italy

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PoS(CD15)078

1. Introduction

Pion photoproduction on nucleons has been the focus of many theoretical studies in the past years [1, 2, 3, 4, 5]. More specifically the neutral channels are particularly interesting, as their total cross sections are much smaller than for the charged channels, and there are big discrepancies between the models and data [6, 7]. It was pointed out that, when using chiral perturbation theory (ChPT) models it is therefore very important to take into account not only tree-level, but also loop-diagram contributions, as the lower-order contributions show very strong cancellations between amplitude pieces [8, 9, 10, 11, 12].

By obtaining data for higher energies [13], it became clear that also this approach is not sufficient to describe the right convergence, even at energies of about only 20 MeV above threshold. A possible solution that was performed in [14, 15] is studying higher-order contributions, i.e. going to $\mathscr{O}(p^4)$. Unfortunately this was not sufficient to reproduce the empirical behaviour, neither in the heavy-baryon quasi-static approach nor in the fully relativistical one. Another suggestion to solve the discrepancy, which we study here, is to consider the contributions coming from the $\Delta(1232)$ resonance [16, 17, 18]. Its effect is small close to pion production threshold, but when approaching the resonance's mass one would expect it to lead to an important modification of the cross-section's behaviour [19, 20, 21, 22, 23, 24]. A first such study has already been performed in [25], which is a calculation up to chiral order p^3. There it has been shown that, although no new fitting constants are introduced, the convergence between model and data is very good, even at energies higher than 200 MeV.

Here we study the low-energy constants. They are taken as fitting parameters and their convergence to the literature values is tested. Furthermore, we propose the inclusion of the next possible set of loop diagrams. It corresponds to an order $p^{7/2}$ calculation when following the counting of [26], which is reasonable for photon energies sufficiently below the $\Delta(1232)$ resonance mass. The goal is to study the effect of this higher order on the modelling of polarization observables, which is expected to be small if the theory is convergent and the power counting consistent.

2. Specifics of the chiral Lagrangian to calculate the $\gamma p \to p\pi^0$ channel

We study the process in a fully covariant ChPT calculation, including nucleons, pions, photons and the $\Delta(1232)$ resonance. The power-counting scheme we use is the δ expansion introduced in [26], where the small parameter $\delta = M_\Delta - m$ is treated as being of order $p^{1/2}$. So being, the diagrams are ordered by the rule

$$D = 4L + \sum k V_k - 2N_\pi - N_N - \frac{1}{2}N_\Delta, \tag{2.1}$$

where D is the order of the diagram, L the number of loops, V_k the number of vertices of a specific order k, and the N_i are the three propagators of the respective hadrons. Following this counting scheme, for a calculation up to $\mathscr{O}(p^{7/2})$ we need the following pieces of the nucleonic Lagrangian:

$$\begin{aligned}\mathscr{L}_{\pi N} = \bar{\Psi}\Big\{ & \mathrm{i}\not{D} - m + \frac{g_A}{2}\not{u}\gamma_5 + \frac{1}{8m}\left(c_6 f^+_{\mu\nu} + c_7 \mathrm{Tr}\left[f^+_{\mu\nu}\right]\right)\sigma^{\mu\nu} \\ & + \left(\frac{\mathrm{i}}{2m}\varepsilon^{\mu\nu\alpha\beta}\left(d_8 \mathrm{Tr}\left[\tilde{f}^+_{\mu\nu}u_\alpha\right] + d_9 \mathrm{Tr}\left[f^+_{\mu\nu}\right]u_\alpha\right) + \mathrm{h.c.}\right)\mathrm{D}_\beta \\ & + \frac{\gamma^\mu\gamma_5}{2}\left(d_{16}\mathrm{Tr}\left[\chi_+\right]u_\mu + \mathrm{i}d_{18}\gamma^\mu\gamma_5[\mathrm{D}_\mu,\chi_-]\right)\Big\}\Psi + \ldots,\end{aligned} \tag{2.2}$$

with the definitions of [27]. While the low-energy constants g_A, c_6 and c_7 are already well determined at the order at which they appear, corrections to their values are expected when moving to higher orders and including the $\Delta(1232)$ resonance. As for the d_8 and d_9, they are mainly sensitive to the pion production processes, as they represent the third-order contact term. Fitting them to our model is therefore an important study of their values. Finally, the d_{16} and d_{18} have already been thoroughly studied in [24] in the same ChPT approach, degrees of freedom and renormalization as in the present work. Therefore, it would be extremely interesting to compare the fitting results of these two works. The inclusion of the degrees of freedom of the isospin-3/2 Δ quadruplet requires the additional Lagrangian terms

$$\mathscr{L}_{\Delta\pi N} = \mathrm{i}\bar{\Psi}\left(\frac{h_A}{2FM_\Delta}T^a\gamma^{\mu\nu\lambda}(\mathrm{D}^{ab}_\lambda\pi^b) + \frac{3e}{2m(m+M_\Delta)}T^3\left(\mathrm{i}g_M\tilde{F}^{\mu\nu} - g_E\gamma_5 F^{\mu\nu}\right)\right)\partial_\mu\Delta_\nu + \mathrm{H.c.}, \tag{2.3}$$

with the definitions in [28]. The constants F, h_A and g_M are very well studied and we don't expect them to change in our fits. As for g_E, it is still not too well known, so we can leave it as a free fitting parameter.

With the help of the Feynman rules extracted from these Lagrangians, one can now calculate all the amplitudes of the diagrams entering this channel at the considered order, shown in Figs. 1 and 2. We parameterize the amplitude $\mathscr{M}$ as

$$\varepsilon_\mu\mathscr{M}^\mu = \bar{u}(p')\left(V_N q\cdot\varepsilon\gamma_5 + V_K q\cdot\varepsilon\not{k}\gamma_5 + V_E\not{\varepsilon}\gamma_5 + V_{EK}\not{\varepsilon}\not{k}\gamma_5\right)u(p), \tag{2.4}$$

where V_N, V_K, V_E and V_{EK} are structure functions of the photon energy and the scattering angle, and where ε_μ is the photon-polarization 4-vector, k_μ its 4-momentum and q_μ the momentum of the outgoing π^0. The Dirac spinors $u(p)$ and $\bar{u}(p')$ are those of the nucleon in the initial and final states, respectively. Another common representation is current conserving by definition and has the form [3]

$$\varepsilon_\mu\mathscr{M}^\mu = \varepsilon_\mu\bar{u}(p')\left(\sum_{i=1}^{4}A_i M_i^\mu\right)u(p),$$

with

$$\begin{aligned}\varepsilon\cdot M_1 &= \mathrm{i}\not{k}\not{\varepsilon}\gamma_5, \\ \varepsilon\cdot M_2 &= \mathrm{i}(p'\cdot\varepsilon k\cdot q - q\cdot\varepsilon k\cdot(p+p'))\gamma_5, \\ \varepsilon\cdot M_3 &= \mathrm{i}(\not{\varepsilon}k\cdot q - \not{k}q\cdot\varepsilon)\gamma_5, \\ \varepsilon\cdot M_4 &= \mathrm{i}(\not{\varepsilon}k\cdot(p+p') - \not{k}p'\cdot\varepsilon - 2m\not{k}\not{\varepsilon})\gamma_5.\end{aligned}$$

The conversion between parameterizations is straightforward:

$$\begin{aligned}
A_1 =& \mathrm{i}\left(V_{EK} - \frac{m}{k\cdot p}\left(V_E + k\cdot q V_K\right)\right),\\
A_2 =& \mathrm{i}\frac{V_N}{2k\cdot p},\\
A_3 =& \mathrm{i}\left(V_K\left(1-\frac{k\cdot q}{2k\cdot p}\right) - \frac{V_E}{2k\cdot p}\right),\\
A_4 =& -\frac{\mathrm{i}}{2k\cdot p}\left(V_E + k\cdot q V_K\right).
\end{aligned}$$

Finally, for the calculation of multipoles it is useful to write the expressions in terms of the Chew–Goldberger–Low–Nambu (CGLM) amplitudes [19],

$$\varepsilon_\mu \mathscr{M}^\mu = \frac{4\pi W}{m}\chi_f^\dagger \mathscr{F}\chi_i,$$

where χ_i and χ_f are the initial and final state Pauli spinors, respectively, and W the center of mass energy. For real photons, the amplitude $\mathscr{F}$ may be written as

$$\mathscr{F} = \mathrm{i}\vec{\tau}\cdot\vec{\varepsilon}\mathscr{F}_1 + \vec{\tau}\cdot\hat{q}\vec{\tau}\cdot\hat{k}\times\vec{\varepsilon}\mathscr{F}_2 + \mathrm{i}\vec{\tau}\cdot\hat{k}\hat{q}\cdot\varepsilon\mathscr{F}_3 + \mathrm{i}\vec{\tau}\cdot\hat{q}\hat{q}\cdot\varepsilon\mathscr{F}_4.$$

The conversion between parameterizations is given by

$$\begin{aligned}
\mathscr{F}_1 =& \frac{\sqrt{(E_i+m)(E_f+m)}}{8\pi W}\left[-\left(k_0+\frac{k_0^2}{E_i+m}\right)A_1 - k\cdot q A_3\right.\\
&\left. + \left(-k_0^2 + 2k_0 m + \frac{2k_0^2 m}{E_i+m} - k_0(E_i+E_f) - k_0|\vec{q}|\cos\theta\right)A_4\right],
\end{aligned}$$

$$\begin{aligned}
\mathscr{F}_2 =& \frac{\sqrt{(E_i+m)(E_f+m)}}{8\pi W}|\vec{q}|\left[\left(\frac{k_0}{E_f+m} + \frac{k_0^2}{(E_i+m)(E_f+m)}\right)A_1\right.\\
& - \frac{k_0 k\cdot q}{(E_i+m)(E_f+m)}A_3\\
&\left. - \left(k_0\frac{k_0^2 + 2k_0 m + k_0(E_i+E_f) + k_0|\vec{q}|\cos\theta}{(E_i+m)(E_f+m)} + \frac{2k_0 m}{E_f+m}\right)A_4\right],
\end{aligned}$$

$$\begin{aligned}
\mathscr{F}_3 =& \frac{\sqrt{(E_i+m)(E_f+m)}}{8\pi W}|\vec{q}|\left[-k_0^2\frac{E_i+E_f+k_0+q_0}{E_i+m}A_2\right.\\
&\left. + \left(k_0 + \frac{k_0^2}{E_i+m}\right)(A_4 - A_3)\right],
\end{aligned}$$

$$\begin{aligned}
\mathscr{F}_4 =& \frac{\sqrt{(E_i+m)(E_f+m)}}{8\pi W}|\vec{q}|^2\left[\left(k_0\frac{k_0+E_i+E_f+q_0}{E_f+m}\right)A_2\right.\\
&\left. + \left(\frac{k_0}{E_f+m} + \frac{k_0^2}{(E_i+m)(E_f+m)}\right)(A_4 - A_3)\right].
\end{aligned}$$

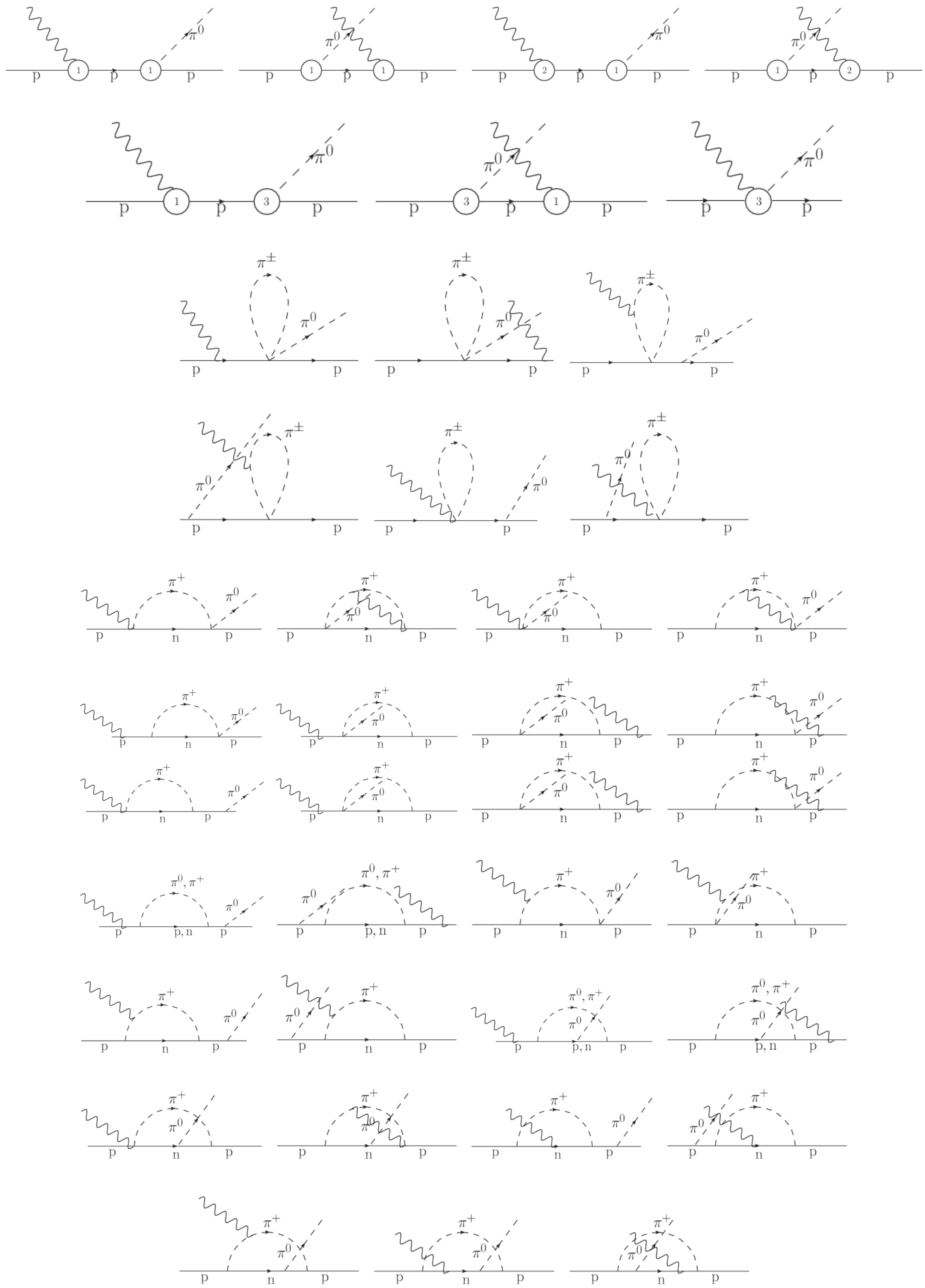

Figure 1: Diagrams with nucleonic virtual states only contributing to the neutral pion photoproduction up to $\mathscr{O}(p^3)$.

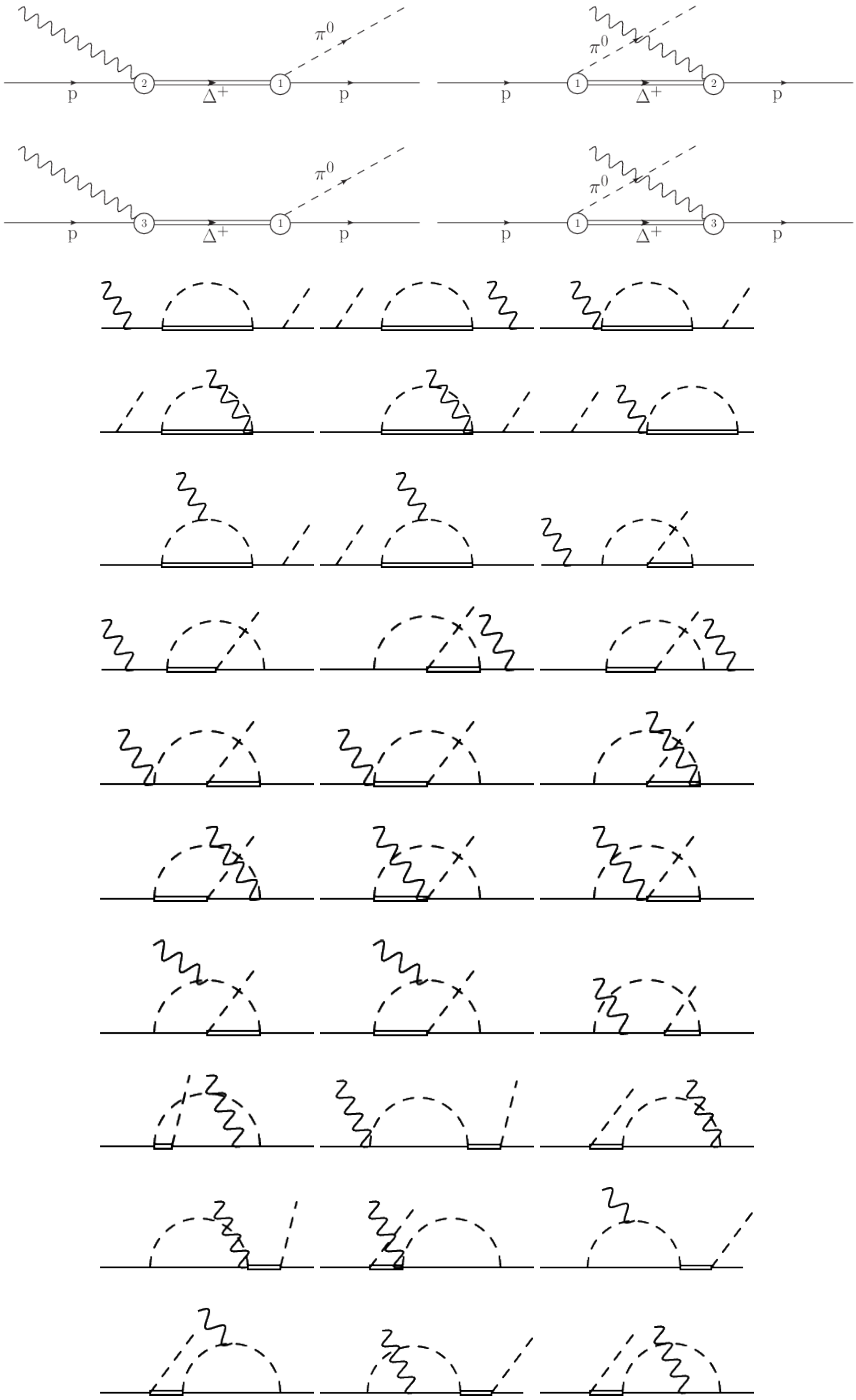

Figure 2: Diagrams with isospin-$3/2$ virtual states contributing to the neutral pion photoproduction up to $\mathscr{O}(p^{7/2})$.

The clear ordering of the mesonic ChPT is spoiled by the inclusion of baryons: power-counting breaking terms appear in those diagrams that have baryons inside of a loop. When integrating over the loop momenta, in addition to the usual divergences of dimensional regularization, there are now also terms that belong to a lower chiral order than the nominal order of the diagram. A scheme which has proven to be straightforward and effectively renormalizes both these issues is the EOMS regularization scheme [29, 30]. The expressions of the divergences and power-counting breaking terms are fully analytical. Therefore they are absorbed into the low-energy constants of the corresponding order. This scheme has been broadly studied in [31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43]. The specific subtraction we perform is the modified minimal subtraction $\widetilde{MS}$ [41]. For that purpose, all terms proportional to

$$L = \frac{2}{\varepsilon} + \log(4\pi) - \gamma_E + 1$$

are subtracted, with $\varepsilon = 4 - \text{dim}$. Then, after making an expansion of the amplitudes[1], we have removed the power-counting breaking terms. The analytical expression obtained for the power-counting breaking terms in the isospin-$1/2$ sector reads

$$\frac{ieg_A^3 m}{32F^3\pi^2}\left[\left(4\nu - 3\frac{m_\pi^2}{\nu}\right)\not{\varepsilon}\gamma_5 + \left(3 - 3\frac{m_\pi^2}{\nu^2}\right)\not{\varepsilon}\not{k}\gamma_5 + \frac{1}{\nu} q\cdot\varepsilon\not{k}\gamma_5 - \frac{2m}{\nu} q\cdot\varepsilon\gamma_5\right].$$

The expressions of the additional power-counting breaking terms coming from the introduction of the Δ loops are larger and thus not shown here.

At the considered order, the wave-function renormalization has to be taken into account for the external proton legs of the $\mathcal{O}(p^1)$ tree diagrams, as the correction amounts to multiplying this tree-level amplitude by the residuum Z_p, a term of $\mathcal{O}(p^2)$. All the corrections to higher-order amplitudes or to the other external legs would be at least of $\mathcal{O}(p^4)$. The analytical expression for this correction factor when including only isospin-$1/2$ intermediate states reads

$$Z_p = \left.\frac{1}{1-\Sigma_p'}\right|_{\not{p}=m} = 1 + \Sigma_p' + \mathcal{O}(p^3)|_{\not{p}=m} = 1 + \frac{3g_A^2 m_\pi^2}{32\pi^2 F^2}\left(3\log\left(\frac{m}{m_\pi}\right) - 2\right), \tag{2.5}$$

where Σ_p is the self energy of the proton. Since we are considering the $\Delta(1232)$ as an intermediate state, we also have to take into account the additional self-energy loop that enters the wave-function renormalization. Also in this case, we took the $\mathcal{O}(p^2)$ term (there is a power-counting breaking term at $\mathcal{O}(p^0)$ which is subtracted) and added it to Eq. 2.5.

We compare our model with the full set of data of Refs. [13] on the angular cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{|\vec{q}|m^2}{2\pi W(s-m^2)}\sum_{\varepsilon}\frac{\mathrm{Tr}\left[\mathscr{M}^*\cdot(\not{p}'+m)\cdot\mathscr{M}\cdot(\not{p}+m)\right]}{2}, \tag{2.6}$$

where the sum over polarizabilities is taken because we are working with unpolarized photons, and with the linearly polarized photon asymmetry

$$\Sigma = \frac{\mathrm{d}\sigma_\perp - \mathrm{d}\sigma_\parallel}{\mathrm{d}\sigma_\perp + \mathrm{d}\sigma_\parallel}, \tag{2.7}$$

[1] The chosen small parameters were m_π, $\nu = (s-u)/(4m)$ with s and u the Mandelstam variables of $\mathcal{O}(p^1)$, and the Mandelstam variable t of order $\mathcal{O}(p^2)$ as in [24].

with $d\sigma_\perp$ and $d\sigma_\parallel$ the angular cross sections for photon polarization perpendicular and parallel to the reaction plane with the pion and the outgoing proton. In the CGLM representation, the differential cross section and photon asymmetry are written with the help of the response functions

$$\begin{aligned}
R_T =& |\mathscr{F}_1|^2+|\mathscr{F}_2|^2+\frac{1}{2}\sin^2\theta\left(|\mathscr{F}_3|^2+|\mathscr{F}_4|^2\right)\\
&-\mathrm{Re}\left[2\cos\theta\mathscr{F}_1^*\mathscr{F}2-\sin^2\theta\left(\mathscr{F}_1^*\mathscr{F}_4+\mathscr{F}_2^*\mathscr{F}_3+\cos\theta\mathscr{F}_3^*\mathscr{F}_4\right)\right],\\
R_{TT} =& \frac{1}{2}\sin^2\theta\left(|\mathscr{F}_3|^2+|\mathscr{F}_4|^2\right)\\
&+\mathrm{Re}\left[\sin^2\theta\left(\mathscr{F}_1^*\mathscr{F}_4+\mathscr{F}_2^*\mathscr{F}_3+\cos\theta\mathscr{F}_3^*\mathscr{F}_4\right)\right],
\end{aligned}$$

with which one obtains

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_\pi}=\frac{|\vec{q}|}{k_\gamma}R_T \quad \text{and} \quad \Sigma=-\frac{R_{TT}}{R_T}.$$

As for the lowest multipoles E_{0+}, M_{1+}, M_{1-} and E_{1+}, they read [9]:

$$\begin{pmatrix} E_{0+}\\ M_{1+}\\ M_{1-}\\ E_{1+}\end{pmatrix}=\int_{-1}^{1}\mathrm{d}x\begin{pmatrix}
\frac{1}{2}P_0(x) & -\frac{1}{2}P_1(x) & 0 & \frac{1}{6}\left[P_0(x)-P_2(x)\right]\\
\frac{1}{4}P_1(x) & -\frac{1}{4}P_2(x) & \frac{1}{12}\left[P_2(x)-P_0(x)\right] & 0\\
-\frac{1}{2}P_1(x) & \frac{1}{2}P_0(x) & \frac{1}{6}\left[P_0(x)-P_2(x)\right] & 0\\
\frac{1}{4}P_1(x) & -\frac{1}{4}P_2(x) & \frac{1}{12}\left[P_0(x)-P_2(x)\right] & \frac{1}{10}\left[P_1(x)-P_3(x)\right]
\end{pmatrix}\begin{pmatrix}\mathscr{F}_1(x)\\ \mathscr{F}_2(x)\\ \mathscr{F}_3(x)\\ \mathscr{F}_4(x)\end{pmatrix},$$

where $x=\cos(\theta)$.

3. Results and discussion

Following the calculations performed in [25], we studied the neutral pion photoproduction on the proton in a fully covariant ChPT calculation and with the inclusion of the isospin-3/2 virtual states. There the obtained fits described the steep increase of the cross section with the photon energy very well, even at energies higher than 200 MeV. The low-energy constants obtained in the fits are shown in Table 1. One can see that the low-energy constant g_A perfectly agrees with the expected result obtained from β-decay data. As for $\tilde{c}_{67}=c_6+c_7$, it corresponds to a combination of the nuclear magnetic moments, which is calculated in the $\overline{MS}$ scheme and with the inclusion of the $\Delta(1232)$ in [39]. An analogous calculation in the $\widetilde{MS}$ scheme used in the present work yields an expected value of $\tilde{c}_{67}=2.5$, which is also very close to the free fit performed here. There are not yet any studies of $\tilde{d}_{89}=d_8+d_9$ in the EOMS scheme with $\Delta(1232)$ degrees of freedom. Therefore we let it completely free an obtain a very reasonable and natural value for its result. The low-energy constants d_{16} and d_{18} have been studied in the $\widetilde{MS}$ scheme with inclusion of $\Delta(1232)$ in [24]. The couplings are the same as in the present work and the study is performed up to the same order. The constant d_{16} is absorbed into the renormalization of g_A. Therefore our combined value $\tilde{d}_{168}=2d_{16}-d_{18}$ corresponds to a d_{18} of around 7.6 MeV. This is interestingly quite different from the expected value in the above-mentioned work. Another interesting fact is that, when letting g_M be a free fitting constant, it assumes the value of 3.1. A study performed in [44] showed that the

	g_A	$\tilde{c}_{67}$	$\tilde{d}_{89}$ [GeV^{-2}]	$\tilde{d}_{168}$ [GeV^{-2}]	χ^2/d.o.f.
Full Model	**1.27**	2.33	1.46	-12.1	0.69
Full Model	1.24	2.36	1.46	-11.1	0.68

Table 1: LEC values in different versions of the model. Fixed values appear in boldface.

expected value of g_M from electromagnetic decay data of the $\Delta(1232)$ is 3.16, which is in perfect agreement with the result from our fit. Finally, for the shown results g_E was fixed to -1, which is the value presented in [22].

As an extension of this work, we also test the convergence of the model, by proposing the calculation up to the next chiral order, which in the presented counting scheme corresponds to a calculation at $\mathscr{O}(p^{7/2})$. The additional contribution is expected to be small. With this model, it will then be possible to study observables like the differential cross sections, photon asymmetries and multipoles, among others. By fits to experimental data, one obtains further insight into the low-energy constants' values at the considered order, and therefore it is then possible to make more reliable statements about the consistency between these results and those from previous works.

Acknowledgements

This research was supported by the Spanish Ministerio de Economía y Competitividad and European FEDER funds under Contracts No. FIS2011-28853-C02-01 and FIS2014-51948-C2-2-P, by Generalitat Valenciana under Contract No. PROMETEO/20090090 and by the EU HadronPhysics3 project, Grant Agreement No. 283286. A.N. Hiller Blin acknowledges support from the Santiago Grisolía program of the Generalitat Valenciana. We thank D. Hornidge for providing us with the full set of data from Ref. [13].

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