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$B_{\ell 4}$ decay and extraction of $|V_{ub}|$

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To reduce the model dependence inherent to the extraction of $|V_{ub}|$ from $B \to \rho \ell \bar{\nu}_\ell$, we propose to investigate the full four-body semileptonic decay $B \to \pi\pi\ell\bar{\nu}_\ell$, where the hadronic $B \to \pi\pi$ transition form factors are analyzed in dispersion theory, a model-independent approach that rigorously takes into account the $\pi\pi$ final-state interaction. In this way, both the finite-width effect of the ρ meson as well as the scalar $\pi\pi$ contribution can be included in a systematic way.

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1. Introduction

In the Standard Model, *CP* violation in the quark sector is described by the Cabibbo–Kobayashi–Maskawa (CKM) matrix [1, 2]

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \tag{1.1}$$

Up to now, the test of *CP* violation for s and b flavors has been comprehensively performed in K- and B-meson factories. Determining the CKM matrix elements in a precise manner becomes a necessity, especially for testing its unitarity: any deviation therefrom would be a signal for physics beyond the Standard Model. For the details of determining these elements, see the review "The CKM quark-mixing matrix" in Ref. [3]. Among them, $|V_{ub}|$ has still not been well determined. It can be extracted from the inclusive decays $B \to X_u \ell \bar{\nu}_\ell$ as well as exclusive decays like $B \to \pi \ell \bar{\nu}_\ell$ or $B \to \rho \ell \bar{\nu}_\ell$. However, there currently is some tension between the results from these two methods (see the review "Determination of V_{cb} and V_{ub}" in Ref. [3]):

$$\begin{aligned} |V_{ub}| &= (4.41 \pm 0.15^{+0.15}_{-0.17}) \times 10^{-3} \quad \text{(inclusive)}, \\ |V_{ub}| &= (3.28 \pm 0.29) \times 10^{-3} \quad \text{(exclusive)}. \end{aligned} \tag{1.2}$$

The sizable width of the ρ as well as the low statistics of the measurement, which hinders a partial-wave decomposition, makes the extraction of $|V_{ub}|$ from $B \to \rho \ell \bar{\nu}_\ell$ with controlled uncertainties difficult. We thus propose to study the full process $B \to \pi\pi \ell \bar{\nu}_\ell$. The hadronic or strong part of the decay can be described in terms of form factors, which we analyze in terms of dispersion theory. Free parameters of the formalism (in the terminology of dispersion theory: subtraction constants), in particular for the normalization condition, are obtained by a matching to heavy-meson chiral perturbation theory (HMChPT) [4]. We will briefly explain this procedure and present our description of the form factors below. For more details we refer to the original publication [5].

2. Form factors

2.1 Kinematics

The four-body decay kinematics is discussed, e.g., in Ref. [6]. For completeness, we here present the part of the notation relevant below. The decay can be described in terms of five kinematic variables:

$$s = M_{\pi\pi}^2 = (p_+ + p_-)^2, \quad s_\ell = (p_\ell + p_\nu)^2, \quad \theta_\pi, \quad \theta_\ell, \quad \phi, \tag{2.1}$$

where p_+ (p_-) denotes the four-momentum of the π^+ (π^-), and similarly for the lepton pair. The angle θ_π (θ_ℓ) is formed by the outgoing direction of the π^+ (ℓ) in the dipion (dilepton) center-of-mass system (CMS) and the dipion (dilepton) line-of-flight in the B-meson rest frame, while ϕ is the angle between the decay planes of the $\pi\pi$ and the $\ell\nu$ pair. We denote the $\pi\pi$ phase-space factor by $\sigma_\pi = \sqrt{1 - 4M_\pi^2/s}$ and define

$$X = \frac{1}{2}\sqrt{\left[m_B^2 - (\sqrt{s} + \sqrt{s_\ell})^2\right]\left[m_B^2 - (\sqrt{s} - \sqrt{s_\ell})^2\right]} \tag{2.2}$$

related to the modulus of the three-momentum formed by two particles with mass $\sqrt{s}$ and $\sqrt{s_\ell}$ in B-meson rest frame. The $B \to \pi\pi$ transition form factors are defined according to

$$\langle \pi^+(p_+)\pi^-(p_-)|\bar{u}\gamma_\mu(1-\gamma_5)b|B^-(p_B)\rangle = -\frac{i}{m_B}(P_\mu F + Q_\mu G + L_\mu R) - \frac{H}{m_B^3}\varepsilon_{\mu\nu\rho\sigma}L^\nu P^\rho Q^\sigma\,, \tag{2.3}$$

with $P = p_+ + p_-$, $Q = p_+ - p_-$, and $L = p_\ell + p_\nu$. In the data analysis, the following set of form factors is often more convenient

$$F_1 = X\cdot F + \sigma_\pi (PL)\cos\theta_\pi G\,, \quad F_2 = G\,, \quad F_3 = H\,, \quad F_4 = -(PL)F - s_\ell R - \sigma_\pi X \cos\theta_\pi G\,, \tag{2.4}$$

whose partial-wave expansions are given by

$$\begin{aligned} F_1 &= X\sum_{l\geq 0} P_l(\cos\theta_\pi)\, f_l\,, & F_2 &= \sum_{l\geq 1} P_l'(\cos\theta_\pi) g_l\,, \\ F_3 &= \sum_{l\geq 1} P_l'(\cos\theta_\pi) h_l\,, & F_4 &= \sum_{l\geq 0} P_l(\cos\theta_\pi)\tilde{r}_l\,, \end{aligned} \tag{2.5}$$

where the $P_l(z)$ denote the standard Legendre polynomials and $P_l'(z)$ their derivatives with respect to z. The decay rate, after integration over the angles ϕ and θ_ℓ, reads

$$\begin{aligned} d\Gamma &= G_F^2|V_{ub}|^2 N(s,s_\ell) J_3(s,s_\ell,\theta_\pi)\, ds\, ds_\ell\, d\cos\theta_\pi\,, \\ J_3(s,s_\ell,\theta_\pi) &= \frac{2+z_\ell}{3}|F_1|^2 + \frac{(2+z_\ell)\sigma_\pi^2 s s_\ell}{3}\left(|F_2|^2 + \frac{X^2}{m_B^4}|F_3|^2\right)\sin^2\theta_\pi\,, \\ N(s,s_\ell) &= \frac{(1-z_\ell)^2\sigma_\pi X}{2(4\pi)^5 m_B^5}\,, \end{aligned} \tag{2.6}$$

where the F_4 term has been neglected since it is suppressed by a factor of $z_\ell = m_\ell^2/s_\ell$, a very small quantity for the kinematics of relevance here.

2.2 Form factors in dispersion theory

Using the Lagrangian of HMChPT [4], one can calculate the form factors perturbatively. At leading order the contributing diagrams are shown in Fig. 1. The resulting expressions are given explicitly in Ref. [5]. Performing a strict chiral expansion in the heavy-meson limit, the B^* pole terms of diagrams (B) and (C) are actually the dominant contributions.

In order to take into account the final-state interaction, we employ dispersion theory based on Omnès functions. The standard Omnès problem is to solve the relation

$$\mathrm{Im}\, f_l(s) = f_\ell(s)e^{-i\delta_l^I(s)}\sin\delta_l^I(s),$$

where the subscript and superscript denote partial wave l and isospin I, respectively, and the $\pi\pi$ phase shifts δ_l^I are known. The solution is [7]

$$f_l(s) = P_n(s)\Omega_l^I(s)\,, \qquad \Omega_l^I(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^\infty \frac{\delta_l^I(s')}{s'(s'-s-i\varepsilon)}ds'\right), \tag{2.7}$$

where P_n is a polynomial of order n, and $\Omega(s)$ is the Omnès function. Watson's theorem [8] states that the phases of the partial-wave amplitudes (f_l, g_l, h_l) coincide with the corresponding $\pi\pi$

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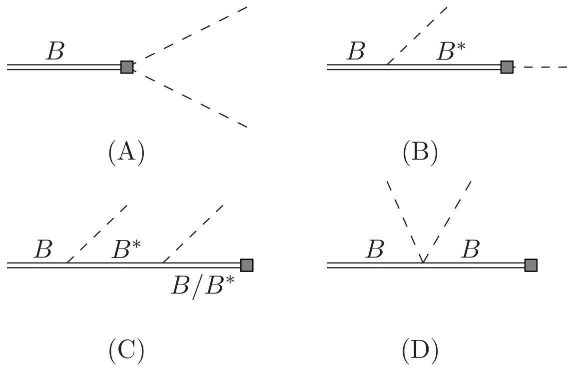

Figure 1: Leading-order diagrams for $B \to \pi\pi$ matrix elements of the hadronic current. Diagrams (B) and (C) contain B^* pole terms. Solid double lines and dashed lines represent heavy mesons and pseudo-Goldstone bosons, respectively. The shaded square denotes an insertion of the left-handed leptonic current. Diagram (C) involves both $BB^*\pi$ and $B^*B^*\pi$ vertices. Diagrams (A) and (D) are suppressed in the chiral expansion as long as the lepton mass is neglected.

scattering phase shifts between $\pi\pi$ threshold and the energy where the first significant inelastic channels start. The inelasticity starts from the 4π threshold, however, in practice, the range of validity of Watson's theorem can be extended approximately up to around 1 GeV ($K\bar{K}$ threshold for S-waves and even further for P-waves), since multipion production is strongly suppressed by phase space and chiral symmetry [9]. The $\pi\pi$ scattering phase shift is well known up to 1.4 GeV [10, 11].

The Omnès problem needs to be generalized in the presence of left-hand singularities. Equation (2.7) is replaced by the modified Omnès problem

$$\mathrm{Im}\, M_l(s) = \Big(M_l(s) + \hat{M}_l(s)\Big) e^{-i\delta_l^I(s)} \sin\delta_l^I(s)\,, \tag{2.8}$$

where the left-hand cuts are included in the real function $\hat{M}_l$. The solution for M_l is given by [12]

$$M_l(s) = \Omega_l^I(s)\left\{P_{n-1}(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\hat{M}_l(s')\sin\delta_l^I(s')ds'}{|\Omega_l^I(s')|(s'-s-i\varepsilon)s'^n}\right\}. \tag{2.9}$$

Again, P_{n-1} denotes a polynomial of order $n-1$. The number of subtractions n is chosen such that the dispersive integration converges. In our case, $\hat{M}_l$ is obtained from the partial-wave projection of the B^* pole terms, which are calculated from leading-order Feynman diagrams. This introduces an implicit s_ℓ dependence. For a fixed value of s_ℓ (in the expressions below, the s_ℓ dependence will be omitted to simplify notation), we can decompose the form factors according to, e.g.,

$$F_1 \sim F_1^{\mathrm{pole}} + M_0(s) + \cos\theta_\pi M_1(s)\,, \tag{2.10}$$

i.e., perform a partial-wave expansion for the pole-term-subtracted form factors. By construction one finds for the imaginary parts of the partial waves

$$\mathrm{Im}\, f_0(s) = \mathrm{Im}\, M_0(s)\,, \tag{2.11}$$

so $f_0(s)$ can be written as

$$f_0(s) = M_0(s) + \hat{M}_0(s) \tag{2.12}$$

with a real function $\hat{M}_0$ that is obtained from projecting $F_1^{\rm pole}$ onto the S-wave. As argued in Ref. [5], the number of subtractions for M_0 is chosen as $n=2$. A similar procedure can be applied to the other form factors. Note that a dispersion-theoretical treatment of the related semileptonic kaon decay $K_{\ell 4}$ has been performed in Ref. [13], albeit with a more elaborate, self-consistent calculation of left- and right-hand-cut structures. Procedures similar to the above, with left-hand cuts approximated by fixed resonance-pole contributions, have been applied e.g. to $\gamma\gamma\to\pi\pi$ [14] or $\eta\to\pi\pi\gamma$ [15].

3. Results and discussion

We next discuss the necessary input to solve the dispersion relations above: $\pi\pi$ phase shifts and the subtraction constants. As we describe the S-wave in a simplified single-channel approximation, i.e. without taking inelasticities due to $K\bar{K}$ intermediate states into account explicitly, we employ the phase of the non-strange pion scalar form factor (as determined in Ref. [16]) instead of δ_0^0. For the P-wave, we use the parametrizations for δ_1^1 given by the Madrid group [10]. Both phases are smoothly continued to approach π for $s\to\infty$. We have checked that the results at low energies (the decay rate below 1 GeV considered here) are insensitive to the precise details of the high-energy continuation.

As for fixing the subtraction constants, we argue in Ref. [5] that the coefficient of the highest power in the subtraction polynomial should be adjusted to provide a reasonable high-energy behavior of the corresponding partial-wave amplitude, while the lowest terms receive contributions from matching to the non-pole, polynomial part of leading-order HMChPT. More explicitly, for the S-wave subtraction polynomial $a_0+a_1 s$, we find

$$a_0=-\frac{(1-g)^2 f_B m_B}{4f_\pi^2}\,,\quad a_1=\hat{M}_0(m_\rho^2)\times\frac{d\Omega_l^I(s)}{ds}\bigg|_{s=0}\,, \tag{3.1}$$

where f_B and f_π are the decay constants of B meson and pion, respectively.

In Fig. 2, we show our result for the decay spectrum with respect to s for fixed $s_\ell=(m_B-1\,\text{GeV})^2$. We find that at low $\pi\pi$ invariant masses, the S-wave contribution is very significant, and leads to noticeable corrections to simple ρ dominance for this decay; compare also Ref. [17]. For the similar observations in the $K\pi$ sector, see, e.g., Ref. [18]. The band in Fig. 2 is obtained by varying partial higher-order contributions. We stress that in our approach, the left-hand structure in the πB interaction is approximated by the B^* pole. Ultimately, the subtraction constants should be fitted to experimental data, combined with HMChPT to fix the normalization. We point out that while the dispersive approach to determine the $\pi\pi$ invariant mass distribution up to about 1 GeV from $\pi\pi$ phase shifts is universal, our current matching procedure to fix the subtraction constants relies on HMChPT, and is therefore limited to the kinematic region of large s_ℓ. Moreover, even in the realm of applicability of HMChPT, higher-order corrections may be significant, and it deserves further study to better quantify the theoretical uncertainty.

4. Summary

In summary, we have analyzed the $B\to\pi\pi$ hadronic transition form factors using dispersion theory, which takes into account the $\pi\pi$ final-state interaction model-independently. The approach

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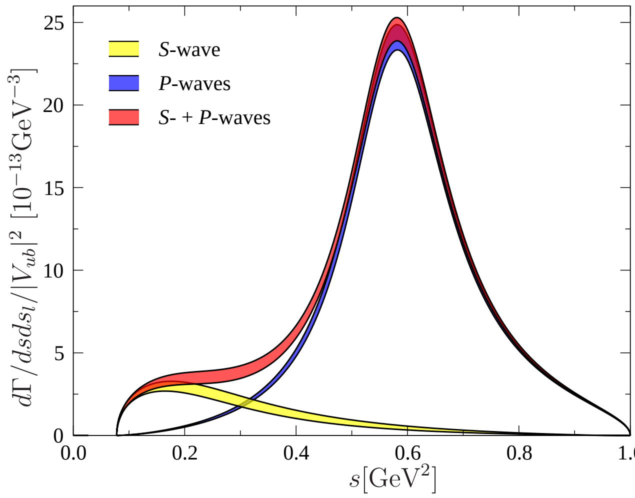

Figure 2: Differential decay width $d\Gamma/dsds_\ell$ divided by $|V_{ub}|^2$ for the example value of $s_\ell = (m_B - 1\,\mathrm{GeV})^2$, decomposed into S- and P-wave contributions.

allows us to quantify the S-wave contribution as a background to the ρ-dominated P-wave. For fixed s_ℓ, we have predicted the decay spectrum with respect to the $\pi\pi$ invariant mass. Using this formalism, the CKM matrix element $|V_{ub}|$ can be fixed by fitting a small number of subtraction constants to experimental data, together with the overall normalization constrained either by lattice calculations or heavy-meson chiral perturbation theory.

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References

[1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).

[2] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[3] K. A. Olive *et al.* [Particle Data Group Collaboration], Chin. Phys. C **38**, 090001 (2014).

[4] M. B. Wise, Phys. Rev. D **45**, R2188 (1992).

[5] X.-W. Kang, B. Kubis, C. Hanhart and U.-G. Meißner, Phys. Rev. D **89**, 053015 (2014) [arXiv:1312.1193 [hep-ph]].

[6] J. Bijnens, G. Colangelo and J. Gasser, Nucl. Phys. B **427**, 427 (1994) [hep-ph/9403390].

[7] R. Omnès, Nuovo Cim. **8**, 316 (1958).

[8] K. M. Watson, Phys. Rev. **95**, 228 (1954).

[9] J. F. Donoghue, J. Gasser and H. Leutwyler, Nucl. Phys. B **343**, 341 (1990).

[10] R. García-Martín, R. Kamiński, J. R. Peláez, J. Ruiz de Elvira and F. J. Yndurain, Phys. Rev. D **83**, 074004 (2011) [arXiv:1102.2183 [hep-ph]].

[11] B. Ananthanarayan, G. Colangelo, J. Gasser and H. Leutwyler, Phys. Rept. **353**, 207 (2001) [hep-ph/0005297].

[12] A. V. Anisovich and H. Leutwyler, Phys. Lett. B **375**, 335 (1996) [hep-ph/9601237].

[13] G. Colangelo, E. Passemar and P. Stoffer, Eur. Phys. J. C **75**, 172 (2015) [arXiv:1501.05627 [hep-ph]].

[14] R. García-Martín and B. Moussallam, Eur. Phys. J. C **70**, 155 (2010) [arXiv:1006.5373 [hep-ph]].

[15] B. Kubis and J. Plenter, Eur. Phys. J. C **75** 6, 283 (2015) [arXiv:1504.02588 [hep-ph]].

[16] M. Hoferichter, C. Ditsche, B. Kubis and U.-G. Meißner, JHEP **1206**, 063 (2012) [arXiv:1204.6251 [hep-ph]].

[17] G. Burdman and J. F. Donoghue, Phys. Lett. B **280**, 287 (1992).

[18] U.-G. Meißner and W. Wang, JHEP **1401**, 107 (2014) [arXiv:1311.5420 [hep-ph]].