

## Determination of $\varepsilon_K$ using lattice QCD inputs

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We present results for the indirect CP violation parameter  $\varepsilon_K$  determined directly from the standard model using lattice QCD to fix the inputs  $\hat{B}_K$ ,  $\xi_0$ ,  $|V_{us}|$ , and  $|V_{cb}|$ . We use the FLAG and SWME results for  $\hat{B}_K$ . We use the RBC-UKQCD result for  $\xi_0$  determined using the experimental value of  $\varepsilon'/\varepsilon$  and the lattice result of  $\text{Im}A_2$ . To set the Wolfenstein parameter  $\lambda$ , we use  $|V_{us}|$ , which is determined from  $K_{\ell 3}$  and  $K_{\mu 2}$  decays combined with lattice evaluations of the  $K \rightarrow \pi \ell \nu$  vector form factor and  $f_K$ . To set the Wolfenstein parameter  $A$ , we use the FNAL/MILC results for  $|V_{cb}|$ , which are determined from the exclusive decay  $\bar{B} \rightarrow D^* \ell \bar{\nu}$  and the axial form factor at zero recoil. We also use the inclusive  $|V_{cb}|$  obtained using the heavy quark expansion based on QCD sum rules and the OPE. We compare the results with those for exclusive  $|V_{cb}|$ . We find that the standard model prediction of  $\varepsilon_K$  with exclusive  $|V_{cb}|$  (lattice QCD results) is lower than the experimental value by  $3.4\sigma$ . However, we observe no tension in  $\varepsilon_K$  determined from inclusive  $|V_{cb}|$ .

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## 1. Introduction

Indirect CP violation in neutral kaons is parametrized by  $\varepsilon_K$

$$\varepsilon_K \equiv \frac{\mathcal{A}(K_L \rightarrow \pi\pi(I=0))}{\mathcal{A}(K_S \rightarrow \pi\pi(I=0))}. \quad (1.1)$$

Here,  $K_L$  and  $K_S$  are the neutral kaon states in nature. We can also calculate  $\varepsilon_K$  directly from the standard model (SM) using tools in lattice QCD. Hence, we can test the SM through CP violation by comparing the experimental and theoretical value of  $\varepsilon_K$ .

In order to calculate  $\varepsilon_K$  directly from the SM, we use input parameters obtained from lattice QCD and experiments. In particular, we use lattice QCD inputs for  $\hat{B}_K$ ,  $|V_{cb}|$ ,  $|V_{us}|$ , and  $\xi_0$  in this paper. In addition, in order to avoid unwanted correlation through  $\varepsilon_K$  between the Wolfenstein parameters of the CKM matrix and the inputs, we adopt the angle-only fit (AOF) from the UTfit collaboration [1] to determine the apex  $(\bar{\rho}, \bar{\eta})$  of the unitarity triangle.

## 2. Master formula for $\varepsilon_K$

In the SM, the master formula for  $\varepsilon_K$  is

$$\varepsilon_K = e^{i\theta} \sqrt{2} \sin \theta \left( C_\varepsilon X_{\text{SD}} \hat{B}_K + \frac{\xi_0}{\sqrt{2}} + \xi_{\text{LD}} \right) + \mathcal{O}(\omega\varepsilon') + \mathcal{O}(\xi_0 \Gamma_2 / \Gamma_1), \quad (2.1)$$

where

$$C_\varepsilon = \frac{G_F^2 F_K^2 m_{K^0} M_W^2}{6\sqrt{2} \pi^2 \Delta M_K}, \quad \xi_{\text{LD}} = \frac{m'_{\text{LD}}}{\sqrt{2} \Delta M_K}, \quad m'_{\text{LD}} = -\text{Im} \left[ \mathcal{P} \sum_C \frac{\langle \bar{K}^0 | H_w | C \rangle \langle C | H_w | K^0 \rangle}{m_{K^0} - E_C} \right] \quad (2.2)$$

Here,  $X_{\text{SD}}$  is the short distance contribution from the box diagrams:

$$X_{\text{SD}} = \text{Im} \lambda_t \left[ \text{Re} \lambda_c \eta_{cc} S_0(x_c) - \text{Re} \lambda_t \eta_{tt} S_0(x_t) - (\text{Re} \lambda_c - \text{Re} \lambda_t) \eta_{ct} S_0(x_c, x_t) \right], \quad (2.3)$$

where  $S_0$  are the Inami-Lim functions given in Ref. [2],  $\lambda_i \equiv V_{is}^* V_{id}$ , and  $x_i = m_i^2 / M_W^2$  with  $m_i = m_i(m_i)$  defined as the scale invariant  $\overline{\text{MS}}$  quark mass [3]. The  $\xi_0$  term represents the long distance effect from the absorptive part of the effective Hamiltonian:  $\xi_0 = \text{Im} A_0 / \text{Re} A_0$ . The  $\xi_{\text{LD}}$  term represents the long distance effect from the dispersive part of the effective Hamiltonian. Details of how to derive the master formula in Eq. (2.1) directly from the standard model using Wigner-Weisskopf perturbation theory are given in Ref. [4].

## 3. Input parameters

The CKMfitter and UTfit groups provide the Wolfenstein parameters  $\lambda$ ,  $\bar{\rho}$ ,  $\bar{\eta}$  and  $A$  from the global unitarity triangle (UT) fit, which are summarized in Table 1 (a). Here, the parameters  $\varepsilon_K$ ,  $\hat{B}_K$ , and  $|V_{cb}|$  are inputs to the global UT fit. Hence, the Wolfenstein parameters extracted from the global UT fit contain unwanted dependence on  $\varepsilon_K$ . In order to avoid this unwanted correlation and to determine  $\varepsilon_K$  self-consistently, we take another input set from the angle-only fit (AOF) in Ref. [1]. The AOF does not use  $\varepsilon_K$ ,  $\hat{B}_K$ , or  $|V_{cb}|$  as input to determine the UT apex  $(\bar{\rho}, \bar{\eta})$ . We

CKMfitter			UTfit			AOF [1]			Input			Value			Ref.		
$\lambda$	0.22537(61) / [5]		0.2255(6) / [5]			0.2253(8) / [5]			$\eta_{cc}$	1.72(27)						[4]	
$\bar{\rho}$	0.117(21) / [5]		0.124(24) / [5]			0.139(29) / [6]			$\eta_{tt}$	0.5765(65)						[7]	
$\bar{\eta}$	0.353(13) / [5]		0.354(15) / [5]			0.337(16) / [6]			$\eta_{ct}$	0.496(47)						[8]	

(a) Wolfenstein parameters

Inclusive (Kin.)			Inclusive (1S)			Exclusive			FLAG			SWME			
42.21(78) / [9]			41.96(45)(07) / [10]			39.04(49)(53)(19) / [11]			0.7661(99) / [12]			0.7379(47)(365) / [13]			

(c)  $|V_{cb}|$  in units of  $10^{-3}$

Input			Value			Ref.		
$G_F$			$1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$			[5]		
$M_W$			80.385(15) GeV			[5]		
$m_c(m_c)$			1.275(25) GeV			[5]		
$m_t(m_t)$			163.3(2.7) GeV			[16]		
$\theta$			43.52(5) $^\circ$			[5]		
$m_{K^0}$			497.614(24) MeV			[5]		
$\Delta M_K$			$3.484(6) \times 10^{-12} \text{ MeV}$			[5]		
$F_K$			156.2(7) MeV			[5]		

(d)  $\hat{B}_K$

Input			Value			Ref.		
$\xi_0$			$-1.63(19)(20) \times 10^{-4}$			[14]		
$\xi_{LD}$			$(0 \pm 1.6) \%$			[15]		

(e) Long distance effects

(f) Other input parameters

**Table 1:** Input parameters

take  $\lambda$  independently from  $|V_{us}|$ , which has been extracted from the  $K_{\ell 3}$  and  $K_{\mu 2}$  decays with lattice QCD inputs [5].

The input values for  $|V_{cb}|$  are summarized in Table 1 (c). The inclusive determination takes into account the inclusive decay modes:  $B \rightarrow X_c \ell \nu$  (essential) and  $B \rightarrow X_s \gamma$  (optional). Moments of lepton energy, hadron masses, and photon energy (optional) are measured from the relevant decays. These moments are fit to the theoretical formula which is a heavy quark expansion obtained with the aid of the operator product expansion (OPE) [5, 9]. Here, we use the most updated value, given in Ref. [9].

For the exclusive  $|V_{cb}|$ , we use the most precise value from the FNAL/MILC lattice calculation of the form factor  $\mathcal{F}(w)$  of the semileptonic decay  $\bar{B} \rightarrow D^* \ell \bar{\nu}$  at zero recoil ( $w = 1$ ) [11]. They combined their lattice result with the HFAG average [17] of  $\mathcal{F}(1) |\bar{\eta}_{EM}| |V_{cb}|$  to extract  $|V_{cb}|$ .

There have been a number of lattice QCD calculations of  $\hat{B}_K$  with  $N_f = 2 + 1$  [18, 19, 20, 21, 22]. Here, we use the FLAG average in Ref. [12] and the SWME result in Ref. [13], which deviates most from the FLAG average. They are summarized in Table 1 (d).

The RBC/UKQCD collaboration provides lattice results for  $\text{Im}A_2$  and  $\xi_0$  in Ref. [14, 23]. The long distance effect  $\xi_0$  is given in Table 1 (e). In the master formula in Eq. (2.1),  $\xi_{LD}$  represents the long distance effect of  $\approx 2\%$  which comes from the dispersive part of the effective Hamiltonian. The precise evaluation of  $\xi_{LD}$  from lattice QCD is not available yet. Hence, we do not include this effect in the central value of  $\varepsilon_K$ , but we take it as a systematic error with the value given in Table

Input Method	Inclusive $V_{cb}$	Exclusive $V_{cb}$	Input Method	Inclusive $V_{cb}$	Exclusive $V_{cb}$
CKMfitter	2.31(23)	1.73(18)	CKMfitter	$-0.34\sigma$	$2.7\sigma$
UTfit	2.30(24)	1.73(19)	UTfit	$-0.31\sigma$	$2.7\sigma$
AOF	2.15(23)	1.61(18)	AOF	$0.33\sigma$	$3.4\sigma$

(a)  $\varepsilon_K^{\text{SM}}$  (b)  $\Delta\varepsilon_K$

**Table 2:** (a)  $\varepsilon_K^{\text{SM}}$  in units of  $10^{-3}$ , and (b)  $\Delta\varepsilon_K$  in units of  $\sigma$ . The  $\sigma$  is obtained by combining errors of  $\varepsilon_K^{\text{SM}}$  and  $\varepsilon_K^{\text{Exp}}$  in quadrature.

1(e). The correction terms  $\mathcal{O}(\omega\varepsilon')$  and  $\mathcal{O}(\xi_0\Gamma_2/\Gamma_1)$  are of order  $10^{-7}$ , and we neglect them in this analysis. A rough estimate of  $\xi_{\text{LD}}$  is available from Ref. [15].

The  $\eta_{ij}$  parameters in Table 1 (b) represent the QCD corrections to the coefficients of Inami-Lim functions. The factor  $\eta_{tt}$  is given at NLO, whereas  $\eta_{cc}$  and  $\eta_{ct}$  are known up to NNLO. Refer to Ref. [4] for more details. The rest of the input parameters are given in Table 1 (f).

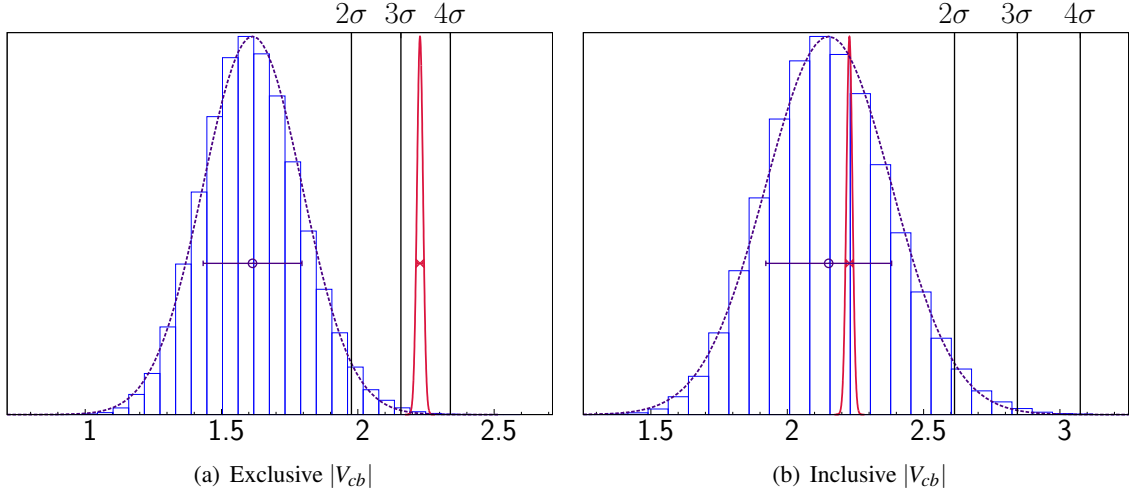
## 4. Results

Let us define  $\varepsilon_K^{\text{SM}}$  as the theoretical evaluation of  $|\varepsilon_K|$  using the master formula of Eq. (2.1). We define  $\varepsilon_K^{\text{Exp}}$  as the experimental value of  $|\varepsilon_K|$ :  $\varepsilon_K^{\text{Exp}} = (2.228 \pm 0.011) \times 10^{-3}$  [5]. Let us define  $\Delta\varepsilon_K$  as the difference between  $\varepsilon_K^{\text{Exp}}$  and  $\varepsilon_K^{\text{SM}}$ :  $\Delta\varepsilon_K \equiv \varepsilon_K^{\text{Exp}} - \varepsilon_K^{\text{SM}}$ . Here, we assume that the theoretical phase  $\theta$  is equal to the experimental phase  $\phi_\varepsilon$ , although it is not fully confirmed in lattice QCD yet.

In Table 2 (a), we present results for  $\varepsilon_K^{\text{SM}}$ . They are obtained using the FLAG average for  $\hat{B}_K$  [12], inclusive  $|V_{cb}|$  from Ref. [9], and exclusive  $|V_{cb}|$  from Ref. [11]. The corresponding probability distributions for  $\varepsilon_K^{\text{SM}}$  are presented in Fig. 1.

In Table 2 (b), we present results for  $\Delta\varepsilon_K$  for both inclusive and exclusive  $|V_{cb}|$ . From Table 2, we observe no tension in  $\Delta\varepsilon_K$  in the inclusive decay channels for  $|V_{cb}|$ , which are obtained using QCD sum rules and the heavy quark expansion. However, from Table 2, we find that there exists a  $3.4\sigma$  tension between  $\varepsilon_K^{\text{Exp}}$  and  $\varepsilon_K^{\text{SM}}$  obtained using the exclusive  $|V_{cb}|$ , which is determined using lattice QCD tools. In other words,  $\varepsilon_K^{\text{SM}}$  with exclusive  $|V_{cb}|$  and the most reliable input method (AOF) is only 72% of  $\varepsilon_K^{\text{Exp}}$ . The largest contribution that we neglect in our estimate of  $\varepsilon_K^{\text{SM}}$  is much less than 2%. Hence, the neglected contributions cannot explain the gap  $\Delta\varepsilon_K$  of 28% with exclusive  $|V_{cb}|$ .

In Fig. 2, we present the chronological evolution of  $\Delta\varepsilon_K/\sigma$  as the progress in lattice and perturbative QCD goes on. In 2012, RBC/UKQCD reported  $\xi_0$  in Ref. [14], and the lattice average for  $\hat{B}_K$  by LLV became available in Ref. [24]. Based on these works, SWME reported  $\Delta\varepsilon_K = 2.5\sigma$  in Ref. [25] in 2012. The FLAG average for  $\hat{B}_K$  became available in Ref. [12] in 2013. In 2014, FNAL/MILC reported an updated  $|V_{cb}|$  in the exclusive decay channel, and the NNLO value of  $\eta_{ct}$  in Ref. [8] became known to us. In 2014, SWME reported the updated  $\Delta\varepsilon_K = 3.0\sigma$  in Ref. [26]. In 2015, a remaining issue on the NNLO calculation of  $\eta_{cc}$  was addressed in Refs. [4, 27, 28]. In 2015, SWME reported the updated  $\Delta\varepsilon_K = 3.4\sigma$  in Ref. [4].

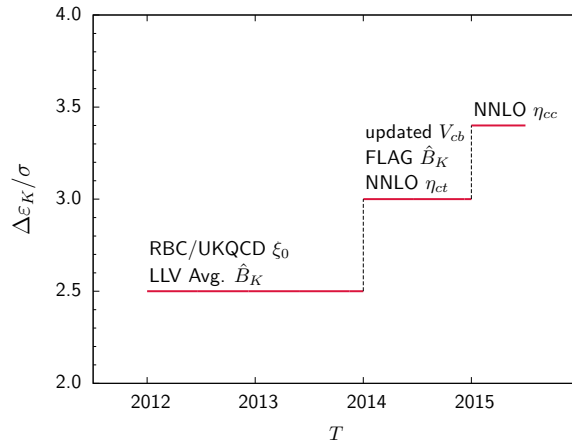


**Figure 1:** Gaussian probability distributions for  $\epsilon_K^{\text{SM}}$  (blue dotted line) and  $\epsilon_K^{\text{Exp}}$  (red solid line). Here, the results are obtained using the FLAG  $\hat{B}_K$  and the AOF parameters for the CKM matrix elements.

## 5. Conclusion

Here, we find that there is a substantial  $3.4\sigma$  tension in  $\epsilon_K$  between experiment and the SM with lattice QCD inputs. For the SM estimate of  $\epsilon_K$ , we use the AOF parameters and lattice QCD inputs for exclusive  $|V_{cb}|$ ,  $\hat{B}_K$ ,  $|V_{us}|$  and  $\xi_0$ . Since the AOF Wolfenstein parameters do not have unwanted correlation with the lattice inputs via  $\epsilon_K$ , the AOF method is relevant to the data analysis in this paper. We also find that the tension disappears for the inclusive  $|V_{cb}|$ , which is determined using QCD sum rules and the heavy quark expansion.

In Table 3, we present the error budget for  $\epsilon_K^{\text{SM}}$ . In the second column of the tables, we show the fractional contribution of each input parameter to the total error of  $\epsilon_K^{\text{SM}}$ . From this error budget, we find that  $|V_{cb}|$  dominates the error in  $\epsilon_K^{\text{SM}}$ . Therefore, it is essential to reduce the error of  $|V_{cb}|$  down to the sub-percent level. For this purpose, we plan to extract  $|V_{cb}|$  from the exclusive



**Figure 2:** Recent history of  $\Delta\epsilon_K$  along with the theoretical progress.

source	error (%)	memo	source	error (%)	memo
$V_{cb}$	39.3	FNAL/MILC	$\xi_0$	2.2	RBC/UKQCD
$\bar{\eta}$	20.4	AOF	$\xi_{LD}$	2.0	RBC/UKQCD
$\eta_{ct}$	16.9	$c-t$ Box	$\hat{B}_K$	1.5	FLAG
$\eta_{cc}$	7.1	$c-c$ Box	$m_c$	1.0	
$\bar{\rho}$	5.4	AOF	$\vdots$	$\vdots$	
$m_t$	2.4		$\vdots$	$\vdots$	

(a) First

(b) Second

**Table 3:** Error budget for  $\epsilon_K^{\text{SM}}$  obtained using the AOF method, the exclusive  $V_{cb}$ , and the FLAG  $\hat{B}_K$ . Here, the values are fractional contributions to the total error obtained using the formula given in Ref. [4].

channel using the Oktay-Kronfeld (OK) action [29] for heavy quarks to calculate the form factors for  $\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}$  decays. The first stage ground work for this goal is underway and preliminary results are reported in Ref. [30, 31].

Several lattice QCD inputs are obtained in the isospin limit,  $m_u = m_d$ . In particular, the isospin breaking effect from  $\epsilon'/\epsilon$  in  $\xi_0$  could be substantial [32, 33, 34]. The isospin breaking effects on  $\xi_0$  and other input parameters are of order 1% in  $\epsilon_K$ . Here we neglect them, but will incorporate them into the evaluation of  $\epsilon_K$  in the future.

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## References

- [1] A. Bevan, M. Bona, M. Ciuchini, D. Derkach, E. Franco, *et al.* *Nucl.Phys.Proc.Suppl.* **241-242** (2013) 89–94.
- [2] T. Inami and C. Lim *Prog.Theor.Phys.* **65** (1981) 297.
- [3] K. Chetyrkin, J. H. Kuhn, and M. Steinhauser *Comput.Phys.Commun.* **133** (2000) 43–65, [[hep-ph/0004189](https://arxiv.org/abs/hep-ph/0004189)].
- [4] J. A. Bailey, Y.-C. Jang, W. Lee, and S. Park [1503.05388](https://arxiv.org/abs/1503.05388).
- [5] K. Olive *et al.* *Chin.Phys.* **C38** (2014) 090001.
- [6] <http://www.utfit.org/UTfit/ResultsSummer2014PostMoriondSM>.

- [7] A. J. Buras and D. Guadagnoli *Phys.Rev.* **D78** (2008) 033005, [[0805.3887](#)].
- [8] J. Brod and M. Gorbahn *Phys.Rev.* **D82** (2010) 094026, [[1007.0684](#)].
- [9] A. Alberti, P. Gambino, K. J. Healey, and S. Nandi *Phys.Rev.Lett.* **114** (2015), no. 6 061802, [[1411.6560](#)].
- [10] C. W. Bauer, Z. Ligeti, M. Luke, A. V. Manohar, and M. Trott *Phys.Rev.* **D70** (2004) 094017, [[hep-ph/0408002](#)].
- [11] J. A. Bailey, A. Bazavov, C. Bernard, *et al.* *Phys.Rev.* **D89** (2014) 114504, [[1403.0635](#)].
- [12] S. Aoki, Y. Aoki, C. Bernard, T. Blum, G. Colangelo, *et al.* [1310.8555](#).
- [13] T. Bae *et al.* *Phys.Rev.* **D89** (2014) 074504, [[1402.0048](#)].
- [14] T. Blum, P. Boyle, N. Christ, N. Garron, E. Goode, *et al.* *Phys.Rev.Lett.* **108** (2012) 141601, [[1111.1699](#)].
- [15] N. Christ, T. Izubuchi, C. Sachrajda, A. Soni, and J. Yu *Phys.Rev.* **D88** (2013), no. 1 014508, [[1212.5931](#)].
- [16] S. Alekhin, A. Djouadi, and S. Moch *Phys.Lett.* **B716** (2012) 214–219, [[1207.0980](#)].
- [17] Y. Amhis *et al.* [1207.1158](#).
- [18] T. Bae *et al.* *Phys.Rev.Lett.* **109** (2012) 041601, [[1111.5698](#)].
- [19] S. Durr, Z. Fodor, C. Hoelbling, *et al.* *Phys.Lett.* **B705** (2011) 477–481, [[1106.3230](#)].
- [20] C. Aubin, J. Laiho, and R. S. Van de Water *Phys.Rev.* **D81** (2010) 014507, [[0905.3947](#)].
- [21] R. Arthur *et al.* *Phys. Rev.* **D87** (2013) 094514, [[1208.4412](#)].
- [22] T. Blum *et al.* [1411.7017](#).
- [23] T. Blum *et al.* *Phys. Rev.* **D91** (2015), no. 7 074502, [[1502.00263](#)].
- [24] J. Laiho, E. Lunghi, and R. S. Van de Water *Phys.Rev.* **D81** (2010) 034503, [[0910.2928](#)].  
<http://latticeaverages.org/>.
- [25] Y.-C. Jang and W. Lee *PoS LATTICE2012* (2012) 269, [[1211.0792](#)].
- [26] J. A. Bailey, Y.-C. Jang, and W. Lee *PoS LATTICE2014* (2014) 371, [[1410.6995](#)].
- [27] J. Brod and M. Gorbahn *Phys.Rev.Lett.* **108** (2012) 121801, [[1108.2036](#)].
- [28] A. J. Buras and J. Girrbach *Eur.Phys.J.* **C73** (2013) 2560, [[1304.6835](#)].
- [29] M. B. Oktay and A. S. Kronfeld *Phys.Rev.* **D78** (2008) 014504, [[0803.0523](#)].
- [30] J. A. Bailey, Y.-C. Jang, W. Lee, C. DeTar, A. S. Kronfeld, *et al.* *PoS LATTICE2014* (2014) 097, [[1411.1823](#)].
- [31] J. A. Bailey, Y.-C. Jang, W. Lee, C. DeTar, A. S. Kronfeld, *et al.* *PoS LATTICE2015* (2015) 099.
- [32] V. Cirigliano, A. Pich, G. Ecker, and H. Neufeld *Phys. Rev. Lett.* **91** (2003) 162001, [[hep-ph/0307030](#)].
- [33] V. Cirigliano, G. Ecker, H. Neufeld, and A. Pich *Eur. Phys. J.* **C33** (2004) 369–396, [[hep-ph/0310351](#)].
- [34] S. Gardner and G. Valencia *Phys. Lett.* **B466** (1999) 355–362, [[hep-ph/9909202](#)].