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Induced YM theory with auxiliary bosons*

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We study pure SU(N_c) lattice gauge theory with a plaquette weight factor given by an inverse determinant which can be written as an integral over auxiliary bosonic fields (modifying a proposal of Budczies and Zirnbauer). We derive conditions for the existence of a continuum limit and its equivalence to Yang-Mills theory. Furthermore, we perturbatively compute the relation between the coupling constants of the 'induced' gauge action and the familiar Wilson gauge action using the background-field technique. The perturbative relation agrees well with numerical results for $N_c = 2$ in three dimensions.

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PoS(LATTICE 2015)276

1. Introduction

In general, persistent challenges for lattice QCD, e.g., the sign problem for real chemical potential, critical slowing down, or volume scaling, make it worthwhile to explore alternative discretizations of Yang-Mills theory. The idea we pursue here is to induce gauge dynamics by auxiliary fields coupled linearly to the gauge fields, rendering applicable analytic methods and simulation algorithms used in strong-coupling expansions. While earlier approaches based on that idea required an infinite number of infinitely heavy auxiliary fields (or did not exhibit the desired YM continuum limit), Budczies and Zirnbauer (BZ) succeeded in Ref. [1] to induce $\mathrm{U}(N_c)$ pure YM theory with $N_b \geq N_c$ auxiliary fields. A slightly modified version of the BZ approach (curing a trivial sign problem) adapted to $\mathrm{SU}(N_c)$ was introduced and investigated numerically in Ref. [2]. Numerical results for $N_c = 2$ in three dimensions showed good agreement with simulations using the usual Wilson action. In the following, we show analytically that the induced $\mathrm{SU}(N_c)$ gauge action indeed exhibits a continuum limit (for $N_b \geq N_c - \frac{5}{4}$) which is equivalent to YM theory for $N_b \geq N_c - \frac{3}{4}$ in two dimensions. Universality arguments [1] as well as numerical results [2] suggest that this equivalence persists in higher dimensions. Furthermore, we perturbatively compute the relation between the coupling constants of the induced gauge action and Wilson's gauge action using the background-field technique. For technical details, we refer to Ref. [3].

2. Induced lattice gauge action

Instead of the familiar Wilson weight factor $\omega_W(U_p) = \exp\left[\frac{1}{g_W^2}\operatorname{Tr}\left(U_p + U_p^\dagger - 2\right)\right]$ for the plaquette variables

$$U_p \equiv U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\nu})U_\nu^\dagger(x) \in \mathrm{SU}(N_c)\,, \qquad p \equiv (x;\mu<\nu)\,, \tag{2.1}$$

we consider the pure gauge plaquette weight factor [2]

$$\omega(U_p) = \det{}^{-N_b}\left(1 - \frac{\alpha}{2}(U_p + U_p^\dagger)\right) \tag{2.2}$$

with $0 < \alpha \leq 1$ and $N_b > 0$ not necessarily restricted to integer values.

For integer N_b, however, the inverse determinants can be written as integrals over N_b complex auxiliary boson fields (in the fundamental representation) with mass parameter $m \geq 2$ determined by the parameter α through $\frac{2}{\alpha} = m^4 - 4m^2 + 2$,

$$\prod_p \omega(U_p) = \int [d\phi]\, e^{-S_B[\bar{\phi},\phi,U]}\,, \tag{2.3}$$

$$\begin{aligned} S_B[\bar{\phi},\phi,U] = \sum_{b=1}^{N_b}\sum_p\sum_{j=1}^{4}\Big[& m\bar{\phi}_{b,p}(x_j^p)\phi_{b,p}(x_j^p) - \bar{\phi}_{b,p}(x_{j+1}^p)U(x_{j+1}^p,x_j^p)\phi_{b,p}(x_j^p) \\ & - \bar{\phi}_{b,p}(x_j^p)U(x_j^p,x_{j+1}^p)\phi_{b,p}(x_{j+1}^p)\Big]\,, \end{aligned} \tag{2.4}$$

where $U(x+\hat{\mu},x) \equiv U_\mu(x)$ etc. and x_j^p with $j = 1,\dots,4$ denote the four points of the plaquette p in the order of appearance following the plaquette orientation.

3. Continuum limit

In order to prove that the induced lattice gauge action indeed exhibits a continuum limit, we determine whether $\omega(U_p)$, as given in Eq. (2.2), reduces to a δ-function (located at $U_p = \mathbb{1}$) on the $\mathrm{SU}(N_c)$ group manifold in the limit $\alpha \to 1$. To this end, we make use of the Peter-Weyl theorem,

$$\delta(U) \propto \sum_{\text{all irreps } \lambda} d_\lambda \chi_\lambda(U)\,, \tag{3.1}$$

where the sum is over all irreducible representations λ with dimension d_λ and character χ_λ. Similarly expanding $\omega(U)$ in irreducible characters,

$$\omega(U) = \sum_\lambda c_\lambda(\alpha)\chi_\lambda(U)\,, \qquad\qquad c_\lambda(\alpha) = \int dU\, \omega(U)\chi_\lambda(U^{-1})\,, \tag{3.2}$$

we obtain the properly normalized weight function $\bar\omega(U)$ through ($\lambda = 0$ for the trivial rep.)

$$\mathscr{Z} \equiv \int dU\, \omega(U) = c_0(\alpha)\,, \qquad\qquad \bar\omega(U) \equiv \frac{1}{\mathscr{Z}}\omega(U) = \sum_\lambda \frac{c_\lambda(\alpha)}{c_0(\alpha)}\chi_\lambda(U)\,. \tag{3.3}$$

To compute $\lim_{\alpha\to 1} c_\lambda/c_0$, we parametrize the plaquette variable as $U = e^{i\sqrt{\gamma(\alpha)}H}$ with $\gamma(\alpha) = \frac{2}{\alpha}(1-\alpha)$ and H restricted to a domain $V(\alpha) \subset \mathrm{su}(N_c)$ such that $\mathrm{SU}(N_c)$ is covered exactly once. The corresponding integration measure is given by

$$dU = \gamma(\alpha)^{\frac{N_c^2-1}{2}} \sqrt{\det g(H)}\, dH\,, \qquad g(H) = \frac{1}{2} + \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k+2)!}\gamma(\alpha)^k H_{\text{adjoint}}^{2k} \tag{3.4}$$

with H_{adjoint} denoting the element of the adjoint representation of $\mathrm{su}(N_c)$ corresponding to H in the fundamental representation. Next, we expand the integrand of Eq. (3.2) in powers of $1-\alpha$, using

$$\det{}^{-N_b}\left(1 - \frac{\alpha}{2}\left(U + U^\dagger\right)\right) = \frac{\det^{-N_b}\left(1+H^2\right)}{(1-\alpha)^{N_b N_c}}\left(1 + N_b\frac{1-\alpha}{6\alpha}\operatorname{Tr}\frac{H^4}{1+H^2} + \dots\right), \tag{3.5}$$

$$\chi_\lambda\left(e^{-i\sqrt{\frac{2}{\alpha}(1-\alpha)}H}\right) = d_\lambda\left(1 - \frac{C_2^{\mathrm{SU}(N_c)}(\lambda)}{N_c^2-1}(1-\alpha)\operatorname{Tr}H^2 + \dots\right), \tag{3.6}$$

where C_2 denotes the quadratic Casimir invariant of $\mathrm{SU}(N_c)$. We then integrate over $V(\alpha)$ term by term and analyze the asymptotic behavior of these integrals in the limit $\alpha \to 1$.

In Ref. [3] we show in detail that the first-order term of the integrand of Eq. (3.2) dominates over all higher-order terms in the limit $\alpha \to 1$ as long as $\int_{V(\alpha)} dH \det^{-N_b}(1+H^2)$ is finite or at most logarithmically divergent as $\alpha \to 1$. In the eigenvalue parametrization

$$\int_{V(\alpha)} dH \det{}^{-N_b}(1+H^2) \propto \int_{-\pi/\sqrt{\gamma(\alpha)}}^{\pi/\sqrt{\gamma(\alpha)}} \left(\prod_{j=1}^{N_c} dz_j\right)\delta\left(\sum_{j=1}^{N_c} z_j\right)\left(\prod_{j<k}(z_j - z_k)^2\right)\prod_{j=1}^{N_c}\left(1+z_j^2\right)^{-N_b} \tag{3.7}$$

we see that this is the case for $N_b \geq N_c - \frac{5}{4}$. This immediately results in $c_\lambda(\alpha)/c_0(\alpha) = d_\lambda$ at leading order in $1-\alpha$ and consequently, see Eq. (3.1),

$$\lim_{\alpha\to 1} \bar\omega(U) = \sum_\lambda d_\lambda \chi_\lambda(U) \propto \delta(U) \qquad\qquad \text{for} \quad N_b \geq N_c - \frac{5}{4}\,. \tag{3.8}$$

PoS(LATTICE 2015)276

For $N_b < N_c - \frac{5}{4}$, all terms in the expansion of the integrand of Eq. (3.2) result in identical leading-order divergences as $\alpha \to 1$ (when integrated over $V(\alpha)$), which means that $\lim_{\alpha\to 1} c_\lambda/c_0$ is a non-trivial function of λ that generically differs from d_λ.

We therefore conclude that $N_b \geq N_c - \frac{5}{4}$ is a necessary condition for the existence of the continuum limit. The same method applied to the $\mathrm{U}(N_c)$ case leads to the condition $N_b \geq N_c - \frac{1}{2}$. These bounds have been verified through numerical simulations and explicit analytical calculations for $\mathrm{SU}(2)$, see Refs. [2] and [3].

To determine the nature of the continuum limit (provided that it exists), we need to expand the ratio $c_\lambda(\alpha)/c_0(\alpha)$ to linear order in $1-\alpha$. We use the same strategy as before, i.e., expand the integrand of Eq. (3.2) in powers of $1-\alpha$ and term-by-term integrate over $V(\alpha)$. For the next-order term, the relevant integral is $\int_{V(\alpha)} dH \det^{-N_b}(1+H^2)\,\mathrm{Tr}\,H^2$ which exists for $N_b > N_c - \frac{3}{4}$ and is logarithmically divergent for $N_b = N_c - \frac{3}{4}$. Concerning the dependence on λ, we hence obtain for $N_b > N_c - \frac{3}{4}$,

$$\frac{c_\lambda(\alpha)}{c_0(\alpha)} = d_\lambda \left(1 - (1-\alpha) \frac{C_2^{\mathrm{SU}(N_c)}(\lambda)}{N_c^2 - 1} \frac{\int_{\mathrm{su}(N_c)} dH \det^{-N_b}(1+H^2)\,\mathrm{Tr}\,H^2}{\int_{\mathrm{su}(N_c)} dH \det^{-N_b}(1+H^2)} + \ldots \right). \tag{3.9}$$

For $N_b = N_c - \frac{3}{4}$, $1-\alpha$ has to be replaced by $(1-\alpha)\log(1-\alpha)$ in the above expansion. For $N_b < N_c - \frac{3}{4}$, the NLO term generically has a more involved dependence on λ, not simply given by $C_2(\lambda)$.

Analogously to the Wilson weight factor for small g^2, the induced weight function $\bar{\omega}(U)$ therefore reduces to the heat-kernel weight factor for small $1-\alpha$, $\frac{c_\lambda(\alpha)}{c_0(\alpha)} = d_\lambda e^{-\tau C_2^{\mathrm{SU}(N_c)}(\lambda)}$ with diffusion parameter $\tau \equiv \tau(\alpha, N_b, N_c)$ as long as $N_b \geq N_c - \frac{3}{4}$. In two dimensions, the heat-kernel lattice action is exactly self-reproducing, i.e., the effective action for a doubled lattice cell, obtained by integrating over all internal link variables, has the same functional form as the original plaquette action (Migdal's recursion [4]). Hence, taking the continuum limit is trivial in 2d and we conclude that the continuum limit of the induced theory is in the universality class of $\mathrm{SU}(N_c)$ Yang-Mills theory for $N_b \geq N_c - \frac{3}{4}$. (For gauge group $\mathrm{U}(N_c)$, we obtain the condition $N_b \geq N_c + \frac{1}{2}$.)

Following Ref. [1], we conjecture that the equivalence with Yang-Mills theory persists also in three and four dimensions since the collective nature of the fields gets enhanced (compared to 2d) which should work in favor of universality. First numerical tests in 3d indeed confirm this to be the case [2].

4. Relation of coupling constants in perturbation theory

Since the continuum limit is essentially trivial in two dimensions, the relation of $1-\alpha$ and the Wilson coupling constant g_W^2 can be obtained simply by matching the character expansions of the plaquette weight functions. In three and four dimensions on the other hand, the continuum limit is more involved and we need perturbation theory to determine the relation of the coupling constants.

However, in a perturbative expansion in powers of $1-\alpha$ at fixed N_b, we encounter two problems: First, the expansion of the logarithm in (ignoring irrelevant constants)

$$S = N_b \sum_p \mathrm{Tr} \log \left(1 - \frac{\alpha}{2} \left(U_p + U_p^\dagger \right) \right) = N_b \sum_p \mathrm{Tr} \log \left(1 - \frac{\alpha}{2(1-\alpha)} \left(U_p + U_p^\dagger - 2 \right) \right) \tag{4.1}$$

is justified only if $\left|\frac{\alpha}{1-\alpha}(\cos\varphi-1)\right| \leq 1$ for all possible eigenvalues $e^{i\varphi}$ of U_p, i.e., $\alpha \leq \frac{1}{3}$. Second, after expanding the logarithm anyway, we see that a saddle-point analysis of the partition function is not possible since higher orders of $U+U^\dagger-2$ are not suppressed in

$$S = -N_b \sum_p \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{\alpha}{2(1-\alpha)}\right)^n \mathrm{Tr}\left(\left(U_p + U_p^\dagger - 2\right)^n\right) \tag{4.2}$$

and we end up with non-Gaussian integrals.

As a workaround, we will therefore first keep $\alpha \leq \frac{1}{3}$ fixed and take the limit $N_b \to \infty$, which allows for a systematic saddle-point analysis, and then analytically continue $g_W(\alpha, N_b)$ to small $1-\alpha$. Since the $(n=1)$-term in Eq. (4.2) corresponds to the Wilson gauge action, it is natural to define the coupling constant $\tilde{g}_I$ for the induced theory as

$$\frac{1}{\tilde{g}_I^2} = N_b \frac{\alpha}{2(1-\alpha)} \,. \tag{4.3}$$

In order to compute the relation of the coupling constants g_W and $\tilde{g}_I$ for fixed α,

$$\frac{1}{g_W^2} = \frac{1}{\tilde{g}_I^2}\left(1 + c_1(\alpha)\tilde{g}_I^2 + c_2(\alpha)\tilde{g}_I^4 + \ldots\right) , \tag{4.4}$$

it is convenient to use the background-field technique (based on Refs. [5, 6]). As usual, we parametrize the link variables in terms of quantum fields q_μ and background fields A_μ through

$$U_\mu(x) = e^{iagq_\mu(x)} U_\mu^{(0)}(x) \,, \qquad U_\mu^{(0)}(x) = e^{iaA_\mu(x)} \tag{4.5}$$

with $g = g_W$ and $g = \tilde{g}_I$, respectively, and compute the effective actions $\Gamma_{I/W}[A]$,

$$e^{-\Gamma_{I/W}[A]} \propto \int_{\text{1-PI}} [Dq] e^{-S_{I/W}[A,q]} \,, \tag{4.6}$$

to quadratic order in the background fields. The relation between the coupling constants $\tilde{g}_I$ and g_W is then obtained by requiring $\Gamma_I[A] = \Gamma_W[A]$ in the continuum limit $g \to 0$.

Since we need to expand the gauge action only to quadratic order in A, we write

$$S_I = S_W\big|_{g_W=\tilde{g}_I} + \sum_{n=2}^{\infty}\left(S_I^{(n,0)} + S_I^{(n,1)} + S_I^{(n,2)} + \mathscr{O}(A^3)\right) , \tag{4.7}$$

where $S_I^{(n,k)}$ includes all $\mathscr{O}(A^k)$ terms resulting from $\mathrm{Tr}\left(\left(U_p + U_p^\dagger - 2\right)^n\right)$ in the sum over n in Eq. (4.2). With $q_{\mu\nu}(x) \equiv q_\mu(x) + q_\nu(x+\mu) - q_\mu(x+\nu) - q_\nu(x)$ and $A_{\mu\nu}$ defined analogously, we obtain to leading order in the quantum field

$$S_I^{(n,0)} = (-1)^{n+1} \frac{a^{2n}\tilde{g}_I^{2n-2}}{(2/\alpha-2)^{n-1}} \sum_{x,\mu,\nu} \frac{1}{2n} \mathrm{Tr}\left[q_{\mu\nu}(x)^{2n} + \mathscr{O}\left(\tilde{g}_I q^{2n+1}\right)\right] , \tag{4.8}$$

$$S_I^{(n,1)} = (-1)^{n+1} \frac{a^{2n}\tilde{g}_I^{2n-3}}{(2/\alpha-2)^{n-1}} \sum_{x,\mu,\nu} \mathrm{Tr}\left[A_{\mu\nu}(x) q_{\mu\nu}(x)^{2n-1} + \mathscr{O}\left(\tilde{g}_I A q^{2n}\right)\right] , \tag{4.9}$$

$$S_I^{(n,2)} = (-1)^{n+1} \frac{a^{2n}\tilde{g}_I^{2n-4}}{(2/\alpha-2)^{n-1}} \sum_{x,\mu,\nu} \mathrm{Tr}\left[\sum_{m=0}^{n-2} A_{\mu\nu}(x) q_{\mu\nu}(x)^m A_{\mu\nu}(x) q_{\mu\nu}(x)^{2n-m-2} + \frac{1}{2}\left(A_{\mu\nu}(x) q_{\mu\nu}(x)^{n-1}\right)^2 + \mathscr{O}\left(A^2 \tilde{g}_I q^{2n-1}\right)\right] . \tag{4.10}$$

The expansion of the Wilson action (i.e., the $(n=1)$-term in the induced action) as well as the gauge-fixing procedure can be taken over one-to-one from the Wilson case.

Splitting the action in a 'free' action S_f for the quantum field q (i.e., terms of order $q^2 A^0$), a 'classical' piece $S_{\rm cl}[A]$ (terms independent of q), and 'interaction' terms $S_{\rm int}$ (all remaining terms), we get

$$e^{-\Gamma[A]} \propto e^{-S_{\rm cl}[A]} \int_{\text{1-PI}} [Dq] e^{-S_f[q]} \sum_k \frac{1}{k!} \left(-S_{\rm int}[A,q]\right)^k \propto e^{-S_{\rm cl}[A]} \sum_k \frac{1}{k!} \left\langle \left(-S_{\rm int}[A,q]\right)^k \right\rangle_{\text{1-PI}}, \tag{4.11}$$

where we omitted integrals over ghost fields since these do not contribute to LO and relevant NLO terms. Expectation values are taken w.r.t. the free action $S_f = \frac{a^4}{2} \sum_{x,\mu,b} q_\mu^b(x) \Box q_\mu^b(x)$ with lattice d'Alembert operator $\Box$,

$$\left\langle q_\mu^a(x) q_\nu^b(y) \right\rangle = \delta_{ab} \delta_{\mu\nu} D(x-y), \tag{4.12}$$

where $D(x)$ denotes the standard lattice propagator for a massless scalar field.

Since S_I includes S_W, the corresponding terms cancel in $\Gamma_I - \Gamma_W$ and the one-loop coefficient $c_1(\alpha)$ in Eq. (4.4) is exclusively determined from

$$\begin{aligned}\left\langle S_I^{(2,2)} \right\rangle &= -\frac{a^4 \alpha}{2(1-\alpha)} \sum_x \sum_{\mu,\nu} A_{\mu\nu}^a(x) A_{\mu\nu}^b(x) \left\langle q_{\mu\nu}^c(x) q_{\mu\nu}^d(x) \right\rangle \operatorname{Tr}\left[t_a t_b t_c t_d + \frac{1}{2} t_a t_c t_b t_d\right] \\ &\overset{a\to 0}{\to} -\frac{4}{d} \left(\frac{2N_c^2-3}{8N_c}\right) \frac{\alpha}{2(1-\alpha)} a^{4-d} \int d^d x \sum_{\mu,\nu} \operatorname{Tr} F_{\mu\nu}(x)^2 + \ldots \end{aligned} \tag{4.13}$$

with $\mathrm{SU}(N_c)$ generators t_a normalized to $\operatorname{Tr} t_a t_b = \frac{1}{2}\delta_{ab}$. In d dimensions, comparison with $S_{\rm cl}[A] = \frac{1}{2\tilde{g}_I^2} a^{4-d} \int d^d x \sum_{\mu,\nu} \operatorname{Tr} F_{\mu\nu}(x)^2 + \ldots$ leads to

$$c_1(\alpha) = c_{1,-1} \left(\frac{\alpha}{2(1-\alpha)}\right), \qquad c_{1,-1} = -\left(\frac{2N_c^2-3}{N_c d}\right). \tag{4.14}$$

At order $\tilde{g}_I^2$, Γ_I contains terms of order $(1-\alpha)^{-2}$ and $(1-\alpha)^{-1}$,

$$c_2(\alpha) = c_{2,-2} \left(\frac{\alpha}{2(1-\alpha)}\right)^2 + c_{2,-1} \left(\frac{\alpha}{2(1-\alpha)}\right). \tag{4.15}$$

However, we are only interested in the coefficient $c_{2,-2}$ (see Eq. (4.17) below), which is found to be given by

$$c_{2,-2} = \frac{N_c^4 - 3N_c^2 + 6}{d^2 N_c^2} - \frac{N_c^4 - 6N_c^2 + 18}{2N_c^2(d-1)} \left(\frac{3}{d^2} - 4(4-d) J_d\right) \tag{4.16}$$

with $J_2 = \frac{1}{32}$ and $J_3 \approx 0.0085535415$ from $\left\langle S_I^{(3,2)} - S_I^{(2,0)} S_I^{(2,2)} - \frac{1}{2} S_I^{(2,1)} S_I^{(2,1)} \right\rangle$.

If we now rewrite the RHS of Eq. (4.4) in terms of α and N_b, we obtain

$$\frac{1}{g_W^2} = \frac{\alpha}{2(1-\alpha)} \underbrace{\left[N_b + c_{1,-1} + c_{2,-2}/N_b + \mathscr{O}\left(N_b^{-2}\right)\right]}_{\equiv d_0(N_b)} + \mathscr{O}((1-\alpha)^0). \tag{4.17}$$

For the limit $\alpha \to 1$ at fixed N_b, a natural definition of a coupling constant is thus given by

$$\frac{1}{g_I^2} \equiv d_0(N_b)\frac{\alpha}{2(1-\alpha)}, \qquad \frac{1}{g_W^2} = \frac{1}{g_I^2}\left(1 + d_1(N_b)g_I^2 + \ldots\right). \tag{4.18}$$

Using the methods and results introduced in Ref. [2], we determined the coefficient d_0 numerically through simulations with both Wilson and induced gauge action for $N_c = 2$ in three dimensions. These numerical results are shown in Fig. 1 together with the results from perturbation theory. We observe surprisingly good agreement even for small values of N_b.

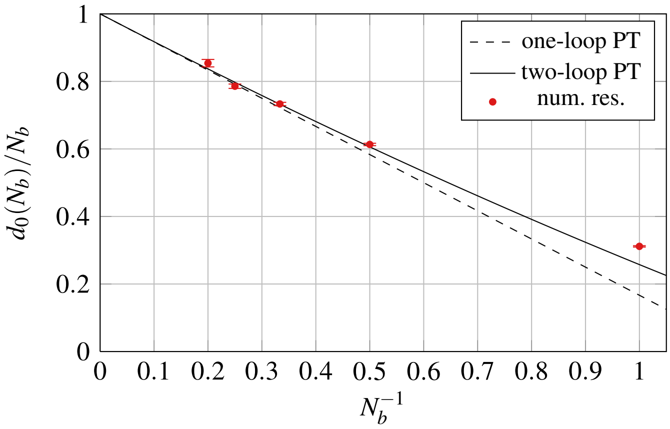

Figure 1: Perturbative and numerical results for d_0/N_b in $d = 3$ with $N_c = 2$.

5. Summary and perspectives

We have shown analytically that the induced $\mathrm{SU}(N_c)$ action exhibits a continuum limit in 2d as $\alpha \to 1$ for fixed $N_b \geq N_c - \frac{5}{4}$ and derived $N_b \geq N_c - \frac{3}{4}$ as a necessary condition for this continuum limit to be in the universality class of YM theory. The equivalence is expected to persist in higher dimensions. For gauge group $\mathrm{U}(N_c)$, the conditions are $N_b \geq N_c - \frac{1}{2}$ and $N_b \geq N_c + \frac{1}{2}$, respectively. Although perturbation theory for $\alpha \to 1$ is problematic, a relation between the coupling constants can be determined by first taking $N_b \to \infty$ at fixed $\alpha \leq \frac{1}{3}$ and analytic continuation to small $1-\alpha$. We observe good agreement with first numerical results for $\mathrm{SU}(2)$ in 3d. In the future, we will extend our numerical study to $\mathrm{SU}(3)$ in four dimensions and plan to use the bosonized version of the gauge action for full QCD.

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