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Exploring possibly existing $qq\bar{b}\bar{b}$ tetraquark states with $qq = ud, ss, cc$

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We compute potentials of two static antiquarks in the presence of two quarks qq of finite mass using lattice QCD. In a second step we solve the Schrödinger equation, to determine, whether the resulting potentials are sufficiently attractive to host a bound state, which would indicate the existence of a stable $qq\bar{b}\bar{b}$ tetraquark. We find a bound state for $qq = (ud - du)/\sqrt{2}$ with corresponding quantum numbers $I(J^P) = 0(1^+)$ and evidence against the existence of bound states with isospin $I = 1$ or $qq \in \{cc, ss\}$.

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1. Motivation

A number of mesons observed in experiments like LHCb or Belle are not well understood. Those mesons have masses and quantum numbers, which are not typical for standard quark-antiquark states, but indicate an exotic four-quark structure. Prominent examples are the charged charmonium-like and bottomonium-like states $Z_c^{\pm}$ and $Z_b^{\pm}$ (cf. e.g. [1]). Their masses and decay products suggest the presence of a $c\bar{c}$ or $b\bar{b}$ pair, respectively. On the other hand their electric charge indicates additionally a light quark-antiquark pair $u\bar{d}$ or $d\bar{u}$. Those four-quark systems, in the following also referred to as tetraquarks, are expected to be studied in more detail in the near future by experimental collaborations. Therefore, a sound theoretical understanding of those systems is crucial and of great interest.

Here we summarize the main results of our recently published work [2], where we have studied four-quark systems with two heavy antiquarks $\bar{b}\bar{b}$ and two lighter quarks qq using lattice QCD and the Born-Oppenheimer approximation. First $\bar{b}\bar{b}$ potentials in the presence of lighter quarks qq are computed. Then the Schrödinger equation is solved using these potentials, where possibly existing bound states indicate stable tetraquarks. Other papers studying the same systems with similar methods include [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

2. Qualitative discussion of $qq\bar{b}\bar{b}$ systems

At small $\bar{b}\bar{b}$ separations the $\bar{b}\bar{b}$ interaction is dominated by 1-gluon exchange. For a bound state the $\bar{b}\bar{b}$ pair must, therefore, be in an attractive color triplet. Due to the Pauli principle and because we assume a spatially symmetric s-wave, $\bar{b}\bar{b}$ has to form an antisymmetric color-spin-flavor combination and, hence, a symmetric spin combination, i.e. $\bar{b}\bar{b}$ spin $j_b = 1$. Since the complete four-quark system is color neutral, the light quarks qq must be in an antisymmetric color antitriplet. Again due to the Pauli principle qq has to form an antisymmetric color-spin-flavor combination and, hence, a symmetric spin-flavor combination. Candidates for tetraquarks are, therefore, the (spin) *scalar isosinglet* (i.e. a qq spin singlet $j = 0$ with antisymmetric flavor, e.g. $qq \in \{(ud - du)/\sqrt{2}\,, (s^{(1)}s^{(2)} - s^{(2)}s^{(1)})/\sqrt{2}\,, (c^{(1)}c^{(2)} - c^{(2)}c^{(1)})/\sqrt{2}\}$[1]) and the (spin) *vector isotriplet* (i.e. a qq spin triplet $j = 1$ with symmetric flavor, e.g. $qq \in \{uu\,, (ud + du)/\sqrt{2}\,, dd\,, ss\,, cc\}$). The overall quantum numbers of a bound $qq\bar{b}\bar{b}$ system are $I(J^P) = 0(1^+)$ for the scalar isosinglet channel and $I(J^P) \in \{1(0^+)\,, 1(1^+)\,, 1(2^+)\}$ for the vector isotriplet channel.

At large $\bar{b}\bar{b}$ separations the $\bar{b}\bar{b}$ interaction is screened by the light quarks qq, i.e. the four quarks form a system of two heavy-light mesons. One expects stronger screening for increasing quark mass m_q, because the wave functions of the corresponding mesons $q\bar{b}$ are then more compact.

3. Lattice QCD computation of static antiquark-antiquark potentials

We extract potentials of two static antiquarks $\bar{Q}\bar{Q}$ (approximating the two $\bar{b}$ quarks of the $qq\bar{b}\bar{b}$ system) in the presence of two light quarks qq from correlation functions

$$C(t,r) \;=\; \langle\Omega|\, \mathscr{O}^\dagger(t)\mathscr{O}(0)\,|\Omega\rangle \underset{t\to\infty}{\propto} \exp(-V(r)t). \tag{3.1}$$

[1] To be able to study flavor antisymmetric qq combinations with $q = s$, we consider two hypothetical degenerate flavors with the mass of the s quark, $s^{(1)}$ and $s^{(2)}$, and similarly for $q = c$, $c^{(1)}$ and $c^{(2)}$.

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$\mathcal{O}$ denotes a four-quark creation operator,

$$\mathcal{O} \;=\; (\mathcal{C}\Gamma)_{AB}(\mathcal{C}\tilde{\Gamma})_{CD}\Big(\bar{Q}_C(\mathbf{r}_1)q_A^{(1)}(\mathbf{r}_1)\Big)\Big(\bar{Q}_D(\mathbf{r}_2)q_B^{(2)}(\mathbf{r}_2)\Big) \quad , \quad r \;=\; |\mathbf{r}_1-\mathbf{r}_2|, \tag{3.2}$$

where Γ is an appropriate combination of γ matrices accounting for defined quantum numbers light quark spin $|j_z|$, parity P and P_x (cf. [8] for details). $\tilde{\Gamma} \in \{(1-\gamma_0)\gamma_5\,,\,(1-\gamma_0)\gamma_j\}$ does not affect the resulting potential $V(r)$, since the static quark spin is irrelevant. $\mathcal{C} = \gamma_0\gamma_2$ denotes the charge conjugation matrix. Note that operators like (3.2) generate overlap not only to mesonic molecule structures, but also to diquark-antidiquark structures [15, 16].

The asymptotic value of a potential and whether it is attractive or repulsive depends on the quantum numbers $(|j_z|, P, P_x)$ and, hence, on Γ. In the following we are exclusively interested in attractive potentials between two ground state static-light mesons: the scalar isosinglet corresponding to $\Gamma = (1+\gamma_0)\gamma_5$ and the vector isotriplet corresponding to $\Gamma = (1+\gamma_0)\gamma_j$.

Computations have been performed using two ensembles of gauge link configurations generated by the European Twisted Mass Collaboration (ETMC) with dynamical u/d quarks. Information on these ensembles can be found in Table 1 and [17, 18].

β	lattice size	μ_l	a in fm	m_π in MeV	# configurations
3.90	$24^3 \times 48$	0.00400	0.079	340	480
4.35	$32^3 \times 64$	0.00175	0.042	352	100

Table 1: Ensembles of gauge link configurations (β: inverse gauge coupling; μ_l: bare u/d quark mass in lattice units; a: lattice spacing; m_π: pion mass).

4. $qq\bar{b}\bar{b}$ tetraquarks in the Born-Oppenheimer approximation

To determine an analytical expression for the $\bar{Q}\bar{Q}$ potential or equivalently $\bar{b}\bar{b}$ potential, we fit the ansatz

$$V(r) \;=\; -\frac{\alpha}{r}\exp\left(-\left(\frac{r}{d}\right)^2\right)+V_0 \tag{4.1}$$

with respect to α, d and V_0 to the lattice QCD results obtained in the previous section. The constant V_0 accounts for twice the mass of the ground state static-light meson.

We insert the the analytical expression (4.1) in the Schrödinger equation for the radial coordinate of the two $\bar{b}$ quarks (which we assume to be in an s-wave),

$$\left(-\frac{1}{2\mu}\frac{d^2}{dr^2}+U(r)\right)R(r) \;=\; E_B R(r) \tag{4.2}$$

with $U(r) = V(r)|_{V_0=0}$ and $\mu = m_b/2$ and determine the lowest eigenvalue E_B. If $E_B < 0$, the four quarks $qq\bar{b}\bar{b}$ can form a tetraquark. If $E_B > 0$, there is no binding, i.e. the four-quark system will always be a system of two unbound B mesons. Notice that this so-called Born-Oppenheimer approximation is valid for $m_q \ll m_b$, which is certainly the case for $q \in \{u\,,d\,,s\}$ and at least crudely fulfilled for $q = c$.

To quantify the systematic errors of different channels (scalar isosinglet and vector isotriplet, different light flavors $q \in \{u\,,d\,,s\,,c\}$), we perform a large number of fits varying the range of temporal separations $t_{\text{min}} \leq t \leq t_{\text{max}}$ of the correlation function $C(t,r)$ (cf. eq. (3.1)), at which the lattice potential is read off, as well as the range of spatial $\bar{b}\bar{b}$ separations $r_{\text{min}} \leq r \leq r_{\text{max}}$ considered in the χ^2 minimizing fit of eq. (4.1) to the lattice potential. Details on this parameter variation can be found in [2]. For each set of input parameters $(t_{\text{min}}, t_{\text{max}}, r_{\text{min}}, r_{\text{max}})$ we determine α, d and E_B. Then we generate histograms for α, d and E_B weighted according to the corresponding χ^2/dof. The widths of these histograms are taken as systematic errors of α, d and E_B [19], while the statistical errors are obtained via a jackknife analysis. In Figure 1 example histograms for the scalar isosinglet for $qq = ud$ are shown.

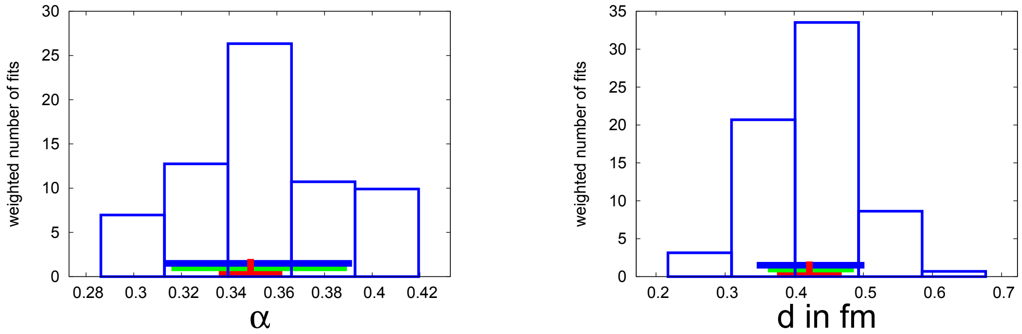

Figure 1: Histograms for the scalar isosinglet for $qq = ud$. The red/green/blue bars indicate the statistical/systematic/combined errors.

The resulting potentials fits for different channels, i.e. eq. (4.1) with corresponding values for α and d, are collected in Figure 2. The error bands represent the combined systematic and statistical errors. One can observe that the potentials are wider and deeper for lighter qq quark masses. Moreover, the scalar channels are more attractive than the respective vector channels. Correspondingly, it turns out that there is a bound state only for the scalar isosinglet with $qq = ud$ with binding energy $-E_B = 93^{+47}_{-43}\,\text{MeV}$, i.e. a bound state with around 2σ confidence level.

In Figure 3 we present our results in an alternative graphical way. The three plots correspond to u/d, s and c light quarks qq, respectively. Each fit of eq. (4.1) to lattice potential results is represented by a dot (red: scalar channels; green: vector channels; crosses: $r_{\text{min}} = 2a$; boxes: $r_{\text{min}} = 3a$). The extensions of the point clouds represent the systematic uncertainties with respect to α and d. If a point cloud is localized above or left of the isoline with $E_B = -0.1\,\text{MeV}$ (essentially the binding threshold), the corresponding four quarks $qq\bar{b}\bar{b}$ cannot form a bound state. A localization below or right of that isoline is a strong indication for the existence of a tetraquark. Again there is clear evidence for a tetraquark state in the scalar u/d channel. The scalar s channel is slightly above, but rather close to the binding threshold. The scalar c and all vector channels clearly do not host bound four-quark states.

5. Summary and outlook

We have found a $ud\bar{b}\bar{b}$ tetraquark with quantum numbers $I(J^P) = 0(1^+)$ (i.e. in the scalar isosinglet channel with $qq = ud$) with a confidence level of around 2σ. There seem to exist no

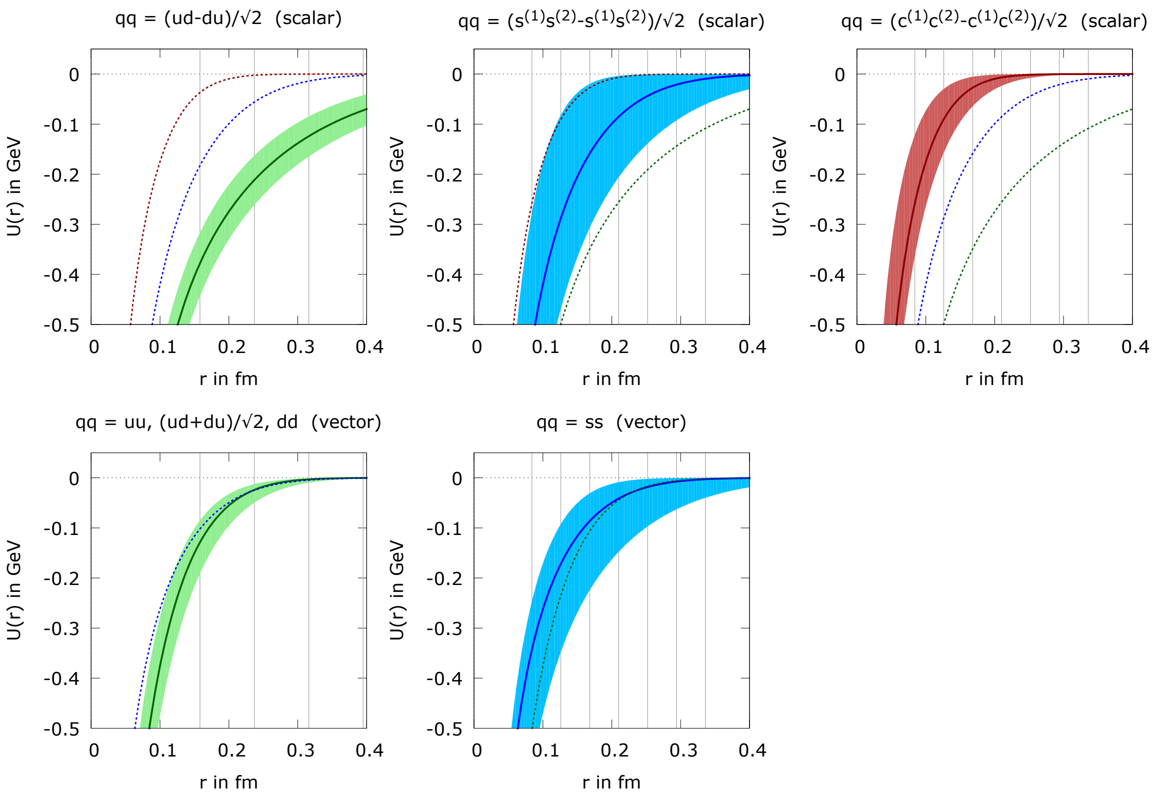

Figure 2: Potentials fits for different channels (upper line: scalar isosinglet; lower line: vector isotriplet). The curves without an error band are copied from the respective other plots in the same line for easy comparison. Vertical lines indicate the available lattice $\bar{b}\bar{b}$ separations.

tetraquarks for the other channels.

In this work lattice QCD computations have been performed for light u/d quarks corresponding to $m_\pi \approx 340\,\text{MeV}$. We plan to repeat the analysis for at least another pion mass and then extrapolate to the physical point. It will then be most interesting to check, whether a bound state will also appear in the vector isotriplet channel with $qq = ud$. Another aspect is to investigate the structure of the found $I(J^P) = 0(1^+)$ tetraquark, i.e. to explore, whether it is rather a mesonic molecule or a diquark-antidiquark pair. We also plan to include corrections due to the heavy quark spins (for first preliminary results cf. [14]). Finally, one should study the experimentally more accessible, but theoretically more challenging case of $q\bar{q}b\bar{b}$ systems.

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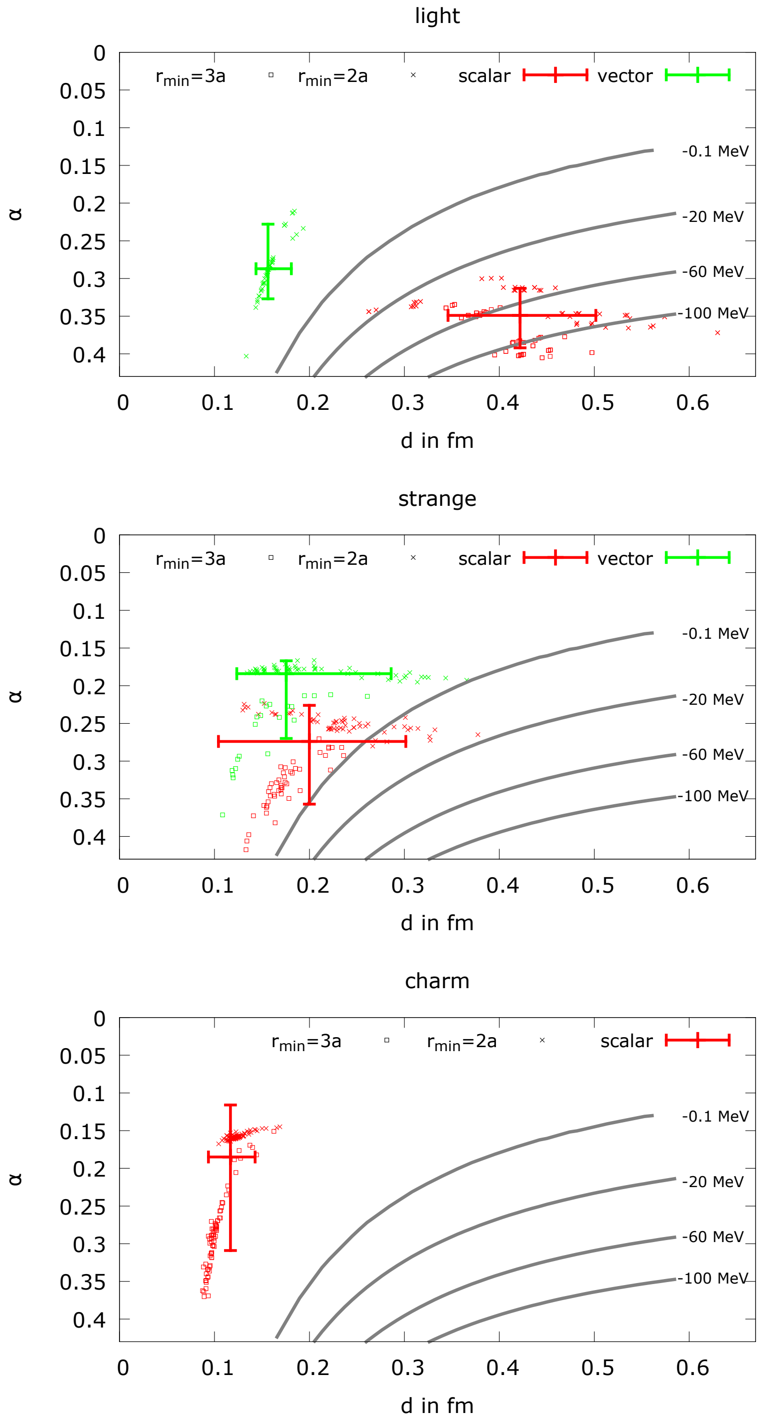

Figure 3: Binding energy isolines $E_B =$ constant in the α-d-plane for u/d, s and c light quarks qq together with the fit results of eq. (4.1) to lattice potentials.

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