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Quantum structure of the minimal calculable unified model

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The present accuracy of the proton lifetime predictions is not sufficient to allow us to rule out particular GUT models. To overcome large theoretical uncertainties we would need to know the mediator mass at better than just the leading order and have a robust flavour structure of the BLV currents not sensitive to generic $\mathscr{O}(1)$ Planck-scale effects. In that respect a minimal realistic perturbative GUT at NLO is the renormalizable non-SUSY $SO(10)$ model with **45** and **126** Higgses, where the leading Planck-scale operators are under control, so we can compute radiative corrections to masses and τ_p. It is a genuinely quantum theory with no available tree level description due to tachyons in the spectrum and where robustness in the neutrino channel is a prerequisite for setting a proton decay upper limit.

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1. Introduction and motivation

The question we ultimately wish to answer in our work is whether it is possible to improve the existing theoretical predictions for proton lifetime and if so, how can it be achieved. It's well established by now that despite its tremendous phenomenological success the Standard Model suffers from various theoretical and experimental issues (neutrino oscillations, dark matter, charge quantization, hierarchy problem, to mention just a few) that seem to indicate the need to go beyond the Standard Model framework. One of the most promising ways to address the open questions was proposed by the Grand Unification paradigm and its undoubtedly best feature is that it's offering the possibility of a distinct experimental signature since any Grand Unified model predicts the proton to decay. Unfortunately though the proton lifetime predictions are burdened by large uncertainties. With the new generation of experiments now under way (Hyper-K, DUNE, ...) an order of magnitude increase in the sensitivity of such searches is expected in the next decade or two. To take advantage of these huge experimental efforts, one would - on the theory side - need to improve the accuracy of the proton lifetime predictions, which have until now only been made in the leading order of perturbation theory. But going to the next-to-leading order in calculations requires first overcoming at least some of the largest uncertainties involved. We wish to comment on the origin of those uncertainties, propose a specific model where those are suppressed and give some future prospects for this project.

2. Theoretical uncertainties

There are three main types of theoretical uncertainties in the proton lifetime estimates. The first source are the hadronic matrix elements $\langle \pi^{+,0}, K^{+,0}, \eta | \ldots | p^+ \rangle$ which are computed on lattice and are typically accurate to some 20–40 %. The second category is due to the unknown Yukawa sector and flavour structure of the BLV currents. The problem there is that partial decay widths $\Gamma(p^+ \to \text{final state})$ in general depend on the unknown rotation matrices. Fortunately some decay channels are more robust with respect to uncertainties in the mass matrices - for example the decay width of the channels $p^+ \to K^+ \bar{\nu}, \pi^+ \bar{\nu}$ with neutrinos in the final state is completely independent of the rotation matrices for symmetric quark-sector mass matrices (which is the case for the $SO(10)$ model with $\mathbf{10} \oplus \mathbf{126}$ representations governing the Yukawa sector). Consequently the decay channels are governed solely by mediator masses, which brings us to the third and most pronounced source of uncertainties. The proton lifetime depends very strongly on the masses of the heavy fields mediating its decay (in non-supersymmetric case it goes approximately as $\tau_p \propto \alpha_{GUT}^{-1} \frac{M_{GUT}^4}{m_p^5}$). In the non-SUSY case those mediators are predominantly the heavy gauge bosons $(3, 2, -5/6)$ or $(3, 2, 1/6)$ with the masses of the order of $g_{GUT} \langle \phi \rangle$, where unified symmetry breaking vev of the field ϕ is identified with the unification scale $\langle \phi \rangle \sim M_{GUT}$. The difficulty lies in determining the scale where the gauge couplings unify. How accurately can we really determine it without access to the high energy physics, i.e. the unknown Planck scale effects? At the NLO order one should consider the 2-loop running of all the couplings and on top of that use the 1-loop threshold corrections to account for the splitting of particles' masses around the matching scales of different effective theories. However, at this level of accuracy, one cannot avoid the Planck suppressed operators, which are in general always present. To illustrate that point, let us consider the "gauge kinetic form" operators

$$\mathscr{L} \ni \frac{\kappa}{\Lambda} F^{\mu\nu} \langle\phi\rangle F_{\mu\nu} \tag{2.1}$$

which are particularly dangerous terms that can cause an inhomogeneous and uncontrolled shift in the matching condition for the gauge couplings. That can be easily understood: this additional term changes the normalization of the gauge field A_μ and hence the matching condition for the gauge coupling. This change then inflicts a theoretical uncertainty of several orders of magnitude on the prediction of M_{GUT} due to logarithmically slow running of the couplings. The moral of the story is that in the presence of this operator, there is no point of ever trying to compute proton decay at better than the leading order.

3. The minimal NLO-calculable model

There is a rare exception to this case. If we wanted to overcome this issue, we need to consider Higgs fields in the coset of the adjoint representation of the gauge fields since then the term from eq. 2.1 becomes exactly 0. In the $SU(5)$ case where the rank of the group needs to be preserved there is no such possibility (at least involving reasonably small representations). But such a possibility exists in the $SO(10)$ group spontaneously broken by **45** (there such a term is exactly zero due to the antisymmetry of the adjoint representation A: $\mathrm{Tr} AA'A'' = 0$ as long as two of the antisymmetric representations are equal.) But of course only the adjoint representation is not enough since the rank of the unification group must be reduced at the same time as well. Another irreducible representation - either **16** or **126** - needs to be added to the Higgs sector of the model. Focusing on the renormalizable models, one can immediately discard the supersymmetric case since there the breaking necessarily has to proceed through the $SU(5)$ intermediate symmetry, which is ruled out by phenomenology. The non-SUSY case is much less restrictive in that sense. Having neutrino masses in the right ball park discards the option with **16** that doesn't give us enough freedom in the Yukawa sector. But even the minimal renormalizable non-supersymmetric $SO(10)$ model with the **45** and **126** Higgses we will consider henceforth has long been considered excluded due to tachyonicity of the tree-level spectrum or otherwise admitting only the phenomenologically non-viable $SU(5)$ vacuum [1, 2, 3, 4]. There are states in the spectrum whose tree level masses are proportional to

$$m^2_{(8,1,0)} \propto (\omega_{BL} - \omega_R)(\omega_R + 2\omega_{BL}) \tag{3.1}$$

$$m^2_{(1,3,0)} \propto (\omega_R - \omega_{BL})(\omega_{BL} + 2\omega_R) \tag{3.2}$$

where ω_{BL} and ω_R are the vevs of Standard Model singlet fields from the **45** which break the $SO(10)$ to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and $SU(4)_C \times SU(2)_L \times U(1)_R$, respectively. Therefore one of them becomes tachyonic unless

$$\frac{1}{2} < \left|\frac{\omega_{BL}}{\omega_R}\right| < 2 \tag{3.3}$$

in which case the breaking pattern is forced towards the $SU(5)$ direction ($\omega_{BL} = \omega_R$). But at the quantum level the masses receive substantial corrections and can become non-tachyonic [5, 6, 7].

$$\Delta m^2_{(8,1,0)} = \frac{1}{(4\pi)^2}\Big[70\,\tau^2 + \frac{1}{3}\beta^2(49\omega_R^2 - 15\omega_R\omega_{BL} + 172\omega_{BL}^2 + \frac{432\omega_R^4\omega_{BL}^2}{\omega_R^4 + 34\omega_R^2\omega_{BL}^2 + \omega_{BL}^4}) + \beta\beta'(60\omega_R\omega_{BL} + 60\omega_R^2) + \beta'^2(220\omega_R^2 - 100\omega_R\omega_{BL} + 480\omega_{BL}^2) + g^4(52\omega_R^2 + 4\omega_R\omega_{BL} + 88\omega_{BL}^2)\Big] + \ldots \tag{3.4}$$

$$\Delta m^2_{(1,3,0)} = \frac{1}{(4\pi)^2}\Big[70\,\tau^2 + \beta^2(40\omega_R^2 - 5\omega_R\omega_{BL} + 31\omega_{BL}^2) + \beta\beta'(60\omega_R\omega_{BL} + 60\omega_{BL}^2) + \beta'^2(320\omega_R^2 - 100\omega_R\omega_{BL} + 380\omega_{BL}^2) + g^4(64\omega_R^2 + 4\omega_R\omega_{BL} + 76\omega_{BL}^2)\Big] + \ldots \tag{3.5}$$

where τ, β and β' are just the coefficients in the scalar potential and g is the unified gauge coupling. This insight revives model, which has been abandoned long ago. Naively one would expect to be sufficient to insert the tree level masses into the 1-loop threshold corrections, but due to the tachyonic character of the tree level spectrum which is changed in the loop calculation, we are forced to use 1-loop masses.

4. Outlook

Our goal is then to use the effective potential approach to compute the whole spectrum at 1-loop, show that it's realistic, the vacuum state is long-lived and provide the first ever NLO computation of the corresponding proton lifetime. This is an ongoing project.

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