Observational Characteristics of the Final Stages of Evaporating Primordial Black Holes

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Many early universe theories predict the creation of Primordial Black Holes (PBHs). The PBHs could have masses ranging from the Planck mass to 10^5 solar masses or higher depending on the formation scenario. Hawking showed that any Black Hole (BH) has a temperature which is inversely proportional to its mass. Hence a sufficiently small BH will thermodynamically radiate particles at an ever-increasing rate, continually decreasing its mass and raising its temperature. The final moments of this evaporation phase should be explosive. In this work, we investigate the final few seconds of the BH burst using the Standard Model of particle physics and calculate the energy dependent burst time profiles in the GeV/TeV range. We use the HAWC (High Altitude Water Cherenkov) observatory as a case study and calculate PBH burst light curves which would be observed by HAWC.

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1. Introduction

Primordial black holes (PBHs) are black holes created in the early universe. Depending on the formation scenario, the PBHs could have masses ranging from the Planck mass to more than million solar masses [1]. In 1974, Hawking showed by convolving general relativity, thermodynamics and quantum field theory that a Black Hole (BH) has a temperature inversely proportional to its mass and radiates with thermal spectra photons and massive particles [2]. As the BH emits this radiation, its mass decreases and hence its temperature and emission rate increase. A PBH that formed with an initial mass of $\sim 5.0 \times 10^{11}$ kg in the early universe should be expiring today [3] with a burst of high-energy particles, including gamma-rays in the MeV to TeV energy range. Thus PBHs are candidate gamma-ray burst (GRB) progenitors [4].

Confirmed detection of a PBH evaporation event would provide valuable insights into many areas of physics including the early universe, high energy particle physics and the convolution of gravitation with thermodynamics. Conversely, non-detection of PBH evaporation events in sky searches would place important limits on models of the early universe.

The properties of the PBH final burst depend on the physics governing the production and decay of high-energy particles. In the Standard Evaporation Model (SEM) which incorporates the Standard Model of particle physics, a BH should directly Hawking-radiate the fundamental Standard Model particles whose Compton wavelengths are of the order of the black hole size [5]. As the BH evaporates and loses mass over its lifetime, its temperature surpasses the rest mass thresholds of further fundamental particle species. When the temperature exceeds the Quantum Chromodynamics (QCD) confinement scale (~ 300 MeV), the BH should directly emit quarks and gluons which fragment and hadronize (analogous to jets seen in high-energy collisions in terrestrial accelerators) as they stream away from the BH into the particles which are stable on astrophysical timescales [5, 3]. Thus according to the SEM, the evaporating black hole will be seen astronomically as a burst of photons, neutrinos, electrons, positrons, protons and anti-protons.

2. Photons from a PBH Burst

2.1 Hawking Radiation

Hawking showed that a black hole radiates fundamental particle species at an emission rate of [2, 1]

$$\frac{d^2N}{dEdt} = \frac{\Gamma/2\pi\hbar}{e^x - (-1)^{2s}} \, n_{\text{dof}}, \tag{2.1}$$

where s is the particle spin and n_{dof} is the number of degrees of freedom of the particle species (e.g. spin, electric charge, color and flavor). The dimensionless quantity x is defined by

$$x \equiv \frac{8\pi GME}{\hbar c^3} = \frac{E}{kT_{BH}} \tag{2.2}$$

where E is the energy of the emitted particle, M is the black hole mass, $T_{BH} \propto 1/M$ is the black hole temperature, and G, c, and $\hbar$ are the universal gravitational constant, speed of light, and the reduced Planck constant, respectively. The absorption coefficient Γ depends on M, E and s. For an

emitted species of rest mass m, Γ at $E \gg mc^2$ has the form

$$\Gamma(M,E,s) = 27\left(\frac{x}{8\pi}\right)^2 \gamma_s(x) \tag{2.3}$$

such that $\gamma_s(x) \to 1$ for large x.

To calculate the spectrum of the final photon burst from the PBH, two important relations pertaining to the final phase of BH evaporation are needed. The first relation is the black hole mass M expressed as a function of remaining lifetime. The mass loss rate can be written as [4]

$$\frac{dM}{dt} \equiv -\frac{\alpha(M)}{M^2}, \tag{2.4}$$

where the factor $\alpha(M)$ incorporates all emitted particle species and degrees of freedom. As the BH evaporates, the value of M is reduced by an amount equal to the total mass-energy of the emitted particles. By conservation of energy,

$$\frac{d(Mc^2)}{dt} = -\sum_i \int_0^\infty \frac{d^2N_i}{dEdt} E dE \tag{2.5}$$

where the summation over i is over all radiated species. Therefore,

$$\alpha(M) = \frac{M^2}{c^2}\sum_i \int_0^\infty \frac{d^2N_i}{dEdt} E dE. \tag{2.6}$$

Assuming the Standard Model including the top quark, 125 GeV Higgs, and 3 families of massive Majorana neutrinos, we find that the asymptotic value α_{SM} of $\alpha(M)$ as M decreases is

$$\alpha_{SM} = 8.40 \times 10^{17}\,\mathrm{kg^3 s^{-1}}. \tag{2.7}$$

For the current and future generations of high energy gamma-ray observatories, we are interested in bursts generated by black holes of temperature $T_{BH} \gtrsim 1\,\mathrm{TeV}$. Thus for $T_{BH} \gtrsim 1\,\mathrm{TeV}$ BHs (corresponding to $M \lesssim 10^7$ kg and a remaining evaporation lifetime of $\tau \lesssim 500\,\mathrm{s}$), $\alpha(M) \approx \alpha_{SM}$ and the BH mass as a function of remaining evaporation lifetime τ is

$$M(\tau) \approx (3\alpha_{SM}\tau)^{1/3} = 1.36 \times 10^6 \left(\frac{\tau}{1s}\right)^{1/3}\,\mathrm{kg}. \tag{2.8}$$

The second relation that we require for the final evaporation phase is the temperature T_{BH} expressed as a function of τ. Combining Equations 2.2 and 2.8, we have

$$kT_{BH} = \frac{\hbar c^3}{8\pi GM} = 7770 \left(\frac{1s}{\tau}\right)^{1/3}\,\mathrm{GeV}. \tag{2.9}$$

for $kT_{BH} \gtrsim 1\,\mathrm{TeV}$.

2.2 QCD Fragmentation

According to the SEM, Equation 2.1 describes the direct Hawking radiation of the fundamental particle species of the Standard Model: the leptons, quarks, and the gauge bosons [5, 4]. As they stream away from the BH, these particles will then evolve by Standard Model processes,

ultimately into the particles which are stable on astrophysical timescales: photons, neutrinos, electrons, positrons, protons, and antiprotons. In particular, quarks and gluons will undergo fragmentation and hadronization into intermediate states which will eventually decay into the astrophysically stable particles.

For application to PBH searches at gamma-ray observatories, we seek the total photon emission rate from the BH. The photon spectrum has several components: (i) the "direct photons" which are directly Hawking radiated by the BH: this component peaks at a few times T_{BH} and is most important at the highest photon energies at any given T_{BH}; (ii) the "fragmentation photons" arising from the fragmentation and hadronization of the directly Hawking radiated quarks and gluons: this component is the dominant source of photons at energies below T_{BH}; and (iii) the photons produced by the decays of other Hawking-radiated fundamental particles, e.g. the tau lepton, W and Z gauge bosons, and Higgs bosons: this component is small compared to the component produced by the directly Hawking radiated quarks and gluons and is neglected here. (We note that, because the W, Z, and Higgs bosons decay predominantly via hadronic channels, the main effect of component (iii) will be to enhance the fragmentation photon component (ii) somewhat.)

2.3 The Pion Fragmentation Model

In jet fragmentation and hadronization, most of the photons arise from the decays of π^0 states, which is π^0 decays to 2γ's with a branching fraction 98.8%. We proceed assuming that the QCD fragmentation of quarks and gluons may be approximated entirely by pion production. In our pion fragmentation model, two questions must be addressed: what is the pion distribution generated by the partons (the initial quarks and gluons) and what is the photon spectrum generated by the pion decays?

To answer the first question, we utilize a heuristic fragmentation function

$$D_{\pi/i}(z) = \frac{15}{16} z^{-3/2} (1-z)^2 \qquad (2.10)$$

where $z \equiv E_\pi/E$ is the energy fraction carried by a pion generated by a parton of energy E [4]. We assume the same form of $D_{\pi/i}(z)$ for all partons and that all of the parton energy goes into pions i.e. the function in Equation 2.10 is normalized such that $\int_0^1 z D_{\pi/i}(z)dz = 1$. The instantaneous BH pion production rate per pion energy interval is then

$$\frac{d^2 N_\pi}{dE_\pi dt} = \sum_i \int_{m_\pi c^2}^\infty \int_0^1 \frac{d^2 N_i}{dE dt} D_{\pi/i}(z) \, \delta(E_\pi - zE)dzdE. \qquad (2.11)$$

2.4 Photon Flux from Pion Fragmentation

To answer the second question, we obtain the photon flux from the $\pi^0 \to 2\gamma$ decay of the pion distribution. Because the fragmentation function $D_{\pi/i}(z)$ includes all three pion charge states π^+, π^-, and π^0 as equal components, and each π^0 decays into two photons, we must multiply by 2/3 to get the γ multiplicity. In the π^0 rest frame, the two photons have equal energies, $m_\pi c^2/2$ and equal but opposite momenta. In the reference frame of the gamma-ray detector, the energies of the

two photons are unequal but complementary fractions of the π^0 energy in the detector frame, E_π. We assume that only one of the photons in each pair is detected.[1]

Let θ' be the angle between the momentum of the observed photon in the π^0 rest frame and the momentum of the pion in the detector frame. In the detector frame, the photon energy $E_\gamma = (E_\pi/2)(1 + \beta \cos \theta')$ where the π^0 velocity $\beta = v/c \approx 1$ and $E_\pi = m_\pi/\sqrt{1 - \beta^2}$. Because the angular distribution of the photons is isotropic in the π^0 rest frame, the distribution of photon energy in the detector frame is

$$\frac{d^2N_\gamma}{dE_\gamma dt} = \frac{2}{3} \int_{-1}^{1} \frac{2\pi d\cos\theta'}{4\pi} \int_{m_\pi}^{\infty} \frac{d^2N_\pi}{dE_\pi dt} \delta[E_\gamma - (E_\pi/2)(1 + \beta\cos\theta')]dE_\pi. \tag{2.12}$$

2.5 Parametrization of the Direct and the Fragmentation Contribution

To further simplify our analysis and subsequent calculations, we parameterize the directly Hawking radiated and the pion fragmentation components of the instantaneous PBH burst photon spectrum using the variable

$$\xi_\gamma = 1.287 \times 10^{-4} \left(\frac{E_\gamma}{1\text{GeV}}\right) \left(\frac{1s}{\tau}\right)^{1/3} \tag{2.13}$$

as follows:

$$\left(\frac{d^2N_\gamma}{dE_\gamma dt}\right)_{\text{direct}} = 1.13 \times 10^{19} (\xi_\gamma)^6 (\exp(\xi_\gamma) - 1)^{-1} \times F(\xi_\gamma) \text{ GeV}^{-1} \text{ s}^{-1} \tag{2.14}$$

where

$$F(\xi_\gamma) = 1.0 \text{ for } \xi_\gamma \leq 2$$
$$F(\xi_\gamma) = \exp([-0.0962 - 1.982(\ln\xi_\gamma - 1.908)][1 + \tanh[20(\ln\xi_\gamma - 1.908)]]) \text{ for } \xi_\gamma > 2$$

and

$$\left(\frac{d^2N_\gamma}{dE_\gamma dt}\right)_{\text{frag.}} = 6.339 \times 10^{23} (\xi_\gamma)^{-3/2}[1 - \Theta_S(\xi_\gamma - 0.3)]$$
$$+ 1.1367 \times 10^{24} \exp(-\xi_\gamma)[\xi_\gamma(\xi_\gamma + 1)]^{-1} \Theta_S(\xi_\gamma - 0.3) \text{ GeV}^{-1} \text{ s}^{-1} \tag{2.15}$$

where

$$\Theta_S(\lambda) = 0.5[1 + \tanh(10\,\lambda)].$$

These parametrizations are valid for energies above $E_\gamma \sim 1$ GeV and are accurate to $\pm$ 15 % for ξ_γ in the range from 0.1 to 5.0, and to $\pm$ 3 % for smaller and larger ξ_γ. If greater accuracy is required, we use a table of the ratios of the exact integration of Equation 2.11 using the fragmentation function 2.12 to the parameterized value, to correct the parameterized values.

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[1]The angle between the 2 photon trajectories in the detector frame will be very small because of the large Lorentz boost. However, if the BH is at a distance of order 1 parsec from the detector, then only one of the photons from each π^0 decay will hit the detector.
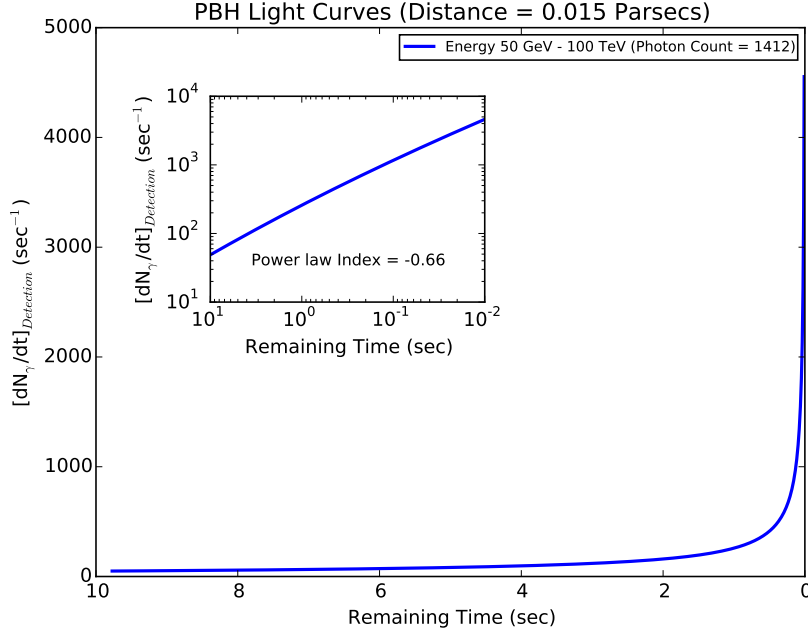
Figure 1: Simulated PBH Burst lightcurve observed by HAWC (at a distance of 0.05 light years) obtained by convolving with the HAWC effective area published in Ref [7]. This shape is also well described by a power law with a index of -0.66.

2.6 PBH Burst Light Curve

To find the time evolution of the PBH burst, we integrate the total instantaneous photon flux $d^2N_\gamma/(dE_\gamma dt)$ over photon energy E_γ while retaining the time dependence:

$$\left[\frac{dN_\gamma}{dt}\right]_{\text{Emission}} = \int_{E_{\min}}^{E_{\max}} \frac{d^2N_\gamma}{dE_\gamma dt}dE_\gamma. \tag{2.16}$$

In general $E_{\min}$ and $E_{\max}$ are set by the energy range of the detector. As a case study, we use here the High Altitude Water Cherenkov (HAWC) observatory [6] for which the relevant energy range is $E_{\min}$=50 GeV to $E_{\max}$=100 TeV.

The time profile (the *light curve*) of the PBH burst at the detector can be calculated as follows:

$$\left[\frac{dN_\gamma}{dt}\right]_{\text{Detection}} = \frac{1}{4\pi r^2}\int_{E_{\min}}^{E_{\max}} A(E_\gamma)\frac{d^2N_\gamma}{dE_\gamma dt}dE_\gamma \tag{2.17}$$

where $A(E_\gamma)$ is the effective area of the HAWC detector and r is the distance to the PBH. Figure 1 shows the time profile of a PBH burst at a distance of 0.05 light years in the HAWC energy range and for the HAWC effective area published in Ref [7].

It is also interesting to investigate the energy dependence of the PBH burst time profile. In Figure 2, we show dN_γ/dt calculated using various energy bands in the HAWC energy range. As seen in Figure 2, the low energy bands show similar emission profiles. However, above $E_\gamma \sim 10$
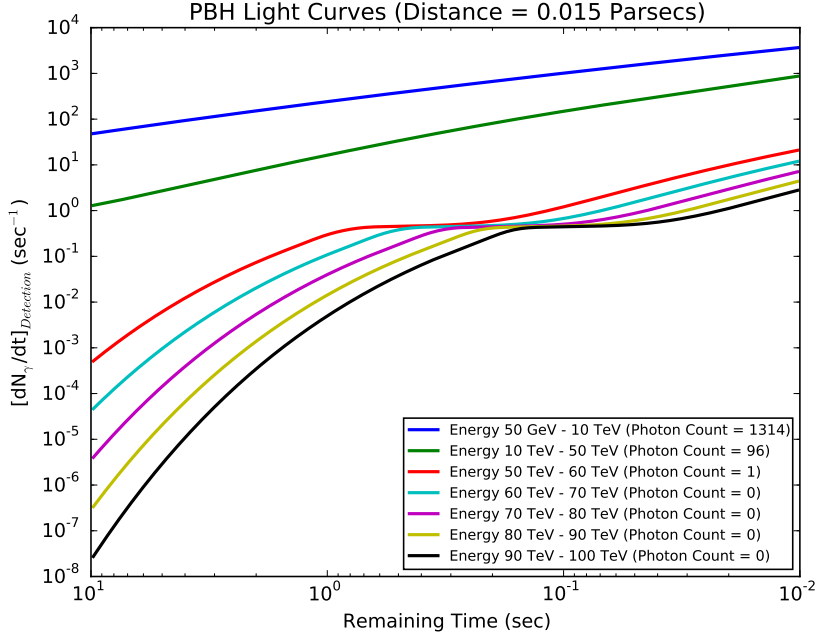
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Figure 2: Simulated detection time profiles of a PBH burst at a distance of 0.05 light years observed by HAWC in multiple energy bands. Photon numbers detected in each energy band is shown in the legend.

TeV the burst emission time profile is energy-dependent and exhibits an inflection around $\tau \sim 0.1$ seconds. This occurs due to the domination of the direct photon component at the highest energies.

3. Summary and Conclusions

In this paper, we have briefly reviewed the theoretical framework of the PBH standard emission model and calculated the PBH burst light curves which would be observed by HAWC. The main findings and conclusions of our paper are:

1. We have developed approximate analytical formulae for the instantaneous PBH spectrum which includes both the directly Hawking radiated photons and the photons arising from the other directly Hawking radiated species.

2. For the first time, we have calculated the PBH burst light curve and studied its energy dependence at a detector.

3. For low energies ($E_\gamma < 10$ TeV) the light curve profile does not show much evolution with energy and is well described by a power law $\tau^{-0.66}$. However, at high energy the light curve displays significant energy dependence that may be used as an unique signature of PBH bursts. The HAWC observatory is sensitive in this high energy range and potentially can be used to uniquely identify PBH bursts.

Acknowledgments

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