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Correlation of the angular and lateral deflections of electrons in extensive air showers

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The aim of this paper is to explain the weak correlation of the angular and lateral deflections of electrons in extensive air showers, when compared with that in some models of electron propagation. We derive analytical formulae for the correlation coefficient in the multiple scattering model with energy losses and show a strong role of the ionisation in diminishing the correlation. By considering a Heitler-like model of a cascade we show that also the presence of photons, parent to electrons, causes a decrease of the correlation, roughly explaining quantitatively the small correlation in air showers.

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1. Introduction

In the effort to determine the angular distributions of electrons with various energies, at different distances from the shower axis, at various shower ages [1, 2] we have been puzzled by a rather weak dependence of electron angles on their lateral distances. The correlation coefficient of the two for shower electrons is 0.46 at the critical energy for air, whereas, as we will derive, in the model of multiple scattering of electrons by small angles it is as big as 0.87. This work is devoted to studies of the relation of the angular and lateral deflections of electrons in some theoretical models and in the extensive air showers, aiming at a clarification of the above difference.

2. Models with constant particle energy

2.1 Multiple scattering of electrons by small angles (MS)

In this model it is assumed that a fast particle (electron in our case) is moving along the z-axis in a medium where it is being scattered by a small angle φ_i every small step Δz many times. The question is what is the correlation coefficient (CC) between final angles η_n and lateral deflections x_n of electrons after n scatterings, where $n >> 1$ (here we restrict ourselves to the 2-dim. case). Since φ_i are independent of each other, then, after n collisions, the variance of the final angle η_n equals

$$\sigma_\eta^2 = \sigma_\varphi^2 n = \sigma_\varphi^2 z/\Delta z \tag{2.1}$$

where σ_φ is the dispersion of the scattering angle φ_i in a single collision. To obtain the distribution of lateral deflections x we note that

$$x_n = \varphi_1(n-1)\Delta z + \varphi_2(n-2)\Delta z + \ldots + \varphi_{n-1}\Delta z. \tag{2.2}$$

Thus, x_n is a sum of $n-1$ independent variables with variance

$$\sigma_x^2 \approx \sigma_\varphi^2 z^3/(3\Delta z) \tag{2.3}$$

for $n >> 1$. If the particle has some lateral deflection $x(z)$ at depth z then at depth $z+\Delta z$ it is $x(z+\Delta z) = x(z) + \eta(z)\Delta z$. From the above expressions we obtain that

$$\rho = \frac{\langle \eta \cdot x \rangle}{\sigma_\eta \sigma_x} = \sqrt{3}/2 \tag{2.4}$$

This correlation is much stronger that that for electrons in a shower, as we shall see later. We note that the value $\sqrt{3}/2$ does not depend on the scattering process, i.e. on σ_φ, so it does not depend on the electron energy. Thus, the model MS in the above version does not explain the weak angle-distance correlation of electrons in EAS.

2.2 A Heitler model of electromagnetic cascade

Electrons in a shower have, however, different history than those considered in MS. Tracking a shower electron back one arrives at a parent photon, then again at an electron as a parent of the photon, and so on. The question arises whether the presence of photons can spoil the electron angle-distance correlation. The presence of photons does not change the electron angles but it does affect final lateral distances. To study the effect of photons we shall consider a scenario based on the Heitler model of an electromagnetic cascade [3]. We expand the Heitler model by considering separately electrons and photons and by adding a second dimension, the x-axis perpendicular to z. Let $\eta_{ee}(z)\,[\eta_{e\gamma}(z)]$ be the angle of an electron at z, the parent of which at $z-\Delta z$ is an electron (photon). Then we have

$$\eta_{ee}(z) = \eta_e(z-\Delta z) + \varphi(z) \tag{2.5}$$

where $\eta_e(z)$ is an angle of any electron at $z-\Delta z$ and $\varphi(z)$ is the electron scattering angle gained between depths $z-\Delta z$ and z. From this we get

$$\langle \eta_{ee}^2(z)\rangle = \langle \eta_e^2(z-\Delta z)\rangle + \langle \varphi^2(z)\rangle \tag{2.6}$$

and for electrons originating from photons:

$$\langle \eta_{e\gamma}^2(z)\rangle = \langle \eta_e^2(z-2\Delta z)\rangle + \langle \varphi^2(z)\rangle \tag{2.7}$$

Since the number of electrons originating from electrons is the same as that originating from photons we obtain that the mean square angle of all electrons at z equals

$$\langle \eta_e^2(z)\rangle = \Big(\langle \eta_e^2(z-\Delta z)\rangle + \langle \eta_e^2(z-2\Delta z)\rangle\Big)/2 + \langle \varphi^2(z)\rangle \tag{2.8}$$

After some lengthy calculations we obtain for $\langle \varphi^2(z)\rangle = \sigma_\varphi^2 = const$ that,

$$\langle \eta_n^2\rangle \approx 2\sigma_\varphi^2 n/3\,,\ \langle x_n\rangle \approx 2\sigma_\varphi^2 n^3(\Delta z)^2/9\,,\ \langle \eta_n x_n\rangle \approx \sigma_\varphi^2 n^2\Delta z/3 \text{ and } \rho_n = \sqrt{3}/2 \tag{2.9}$$

for large n, so that CC does not depend on n and it is the same in the Heitler model as that for a single particle in MS. Thus, considering a cascade with added photons does not affect CC for large n. Of course, this case is not a physical one since one must assume that the energy of electron decreases as the cascade develops.

3. Models with energy losses included

3.1 Multiple scattering of electron by small angles (MS)

Now we assume that the scattering angle gained in each step is, on average, inversely proportional to the electron energy, what takes place in the Coulomb scattering process. We have:

$$\langle \eta^2(z)\rangle = \langle \varphi_S^2\rangle \sum_{i=1}^{n}(E_S/E_i)^2 \rightarrow X_0^{-1}\int_0^z [E_S/E(z')]^2 dz'\,,$$

$$\langle x^2(z)\rangle = \langle \varphi_S^2\rangle(\Delta z)^2\sum_{i=1}^{n-1}(N-i)^2(E_S/E_i)^2 = X_0^{-1}\int_0^z (z-z')^2[E_S/E(z')]^2 dz'$$

$$\text{and } \langle \eta(z)x(z)\rangle = X_0^{-1}\int_0^z (z-z')[E_S/E(z')]^2 dz'\,, \tag{3.1}$$

where X_0 is the radiation unit of the medium, $E_S = 21MeV$, and $\langle \varphi_S^2\rangle$ refers to E_S. Thus, we have obtained general formulae enabling one to calculate the correlation coefficient ρ for any $E(z')$, providing that the function exists.

We shall consider now the main processes governing the behaviour of relativistic electrons in EAS: bremsstrahlung and ionisation of the atmosphere.

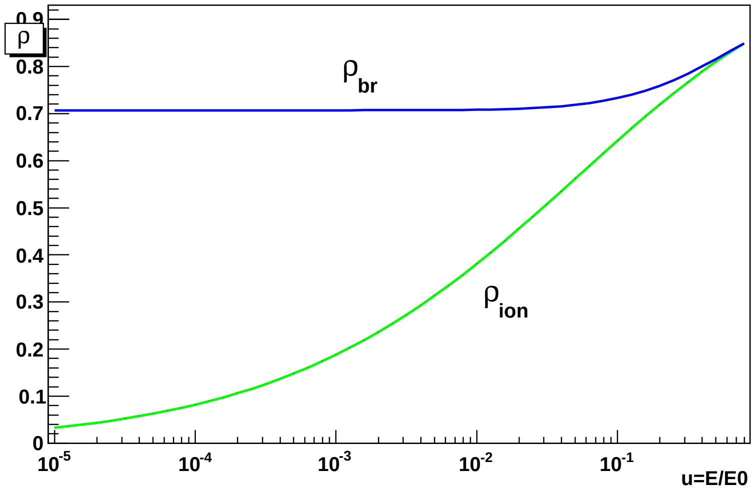

Figure 1: Correlation coefficient ρ of η, x as function of the ratio of the final to initial electron energy E/E_0. Upper curve - bremsstrahlung losses only, lower curve - ionisation losses only.

a) Bremsstrahlung. We assume that the energy loss rate of an electron with energy E equals to its mean energy loss, so that $-dE/dz = E/X_0$ and $E(z) = E_0 e^{-z/X_0}$, where E_0 is the initial electron energy. From the above general formulae we get for CC

$$\rho_{br} = \frac{1 - u^2 + 2u^2 ln u}{\sqrt{2(1-u^2)\cdot[1-u^2+2u^2(1-ln u)ln u]}} \text{ where } u = E/E_0 \tag{3.2}$$

ρ_{br} depends only on the ratio u (Fig.1). The formula gives $\rho_{br}(u=1) = 0.87$, as should be expected. For $u \to 0$ $\rho_{br} \to 1/\sqrt{2} \approx 0.707$. Thus, allowing for energy loss causes a decrease of CC, although in this case this change is small.

b) Ionisation. A good approximation is now $-dE/dz = \beta/X_0$, where β is the critical energy of the medium, with the solution $E(z) = E_0 - \beta z/X_0$ for $z \le E_0 X_0/\beta$. We obtain for CC:

$$\rho_{ion} = \frac{ln(1/u) + u - 1}{\sqrt{(1/u-1)(1-u^2+2u\, ln u)}} . \tag{3.3}$$

It is a completely different behaviour of CC, which goes to 0 as $u \to 0$ (Fig. 1), than in the previous case.

c) Bremsstrahlung and ionisation. For both processes at work we have

$$E(z) = (E_0 + \beta)exp(-z/X_0) - \beta \tag{3.4}$$

Now only the variance $\langle\eta^2(z)\rangle$ can be solved analytically. For $E_0 \to \infty$ all the variances depend only on the ratio E/β. The resulting CC is shown in Fig.2. We have also drawn there ρ_{br}, and ρ_{ion} for $E_0 = 100\,GeV$, the values of which follow from Fig.1. We conclude that if a particle loses energy while being Coulomb-scattered CC decreases mainly due to the losses for ionisation.

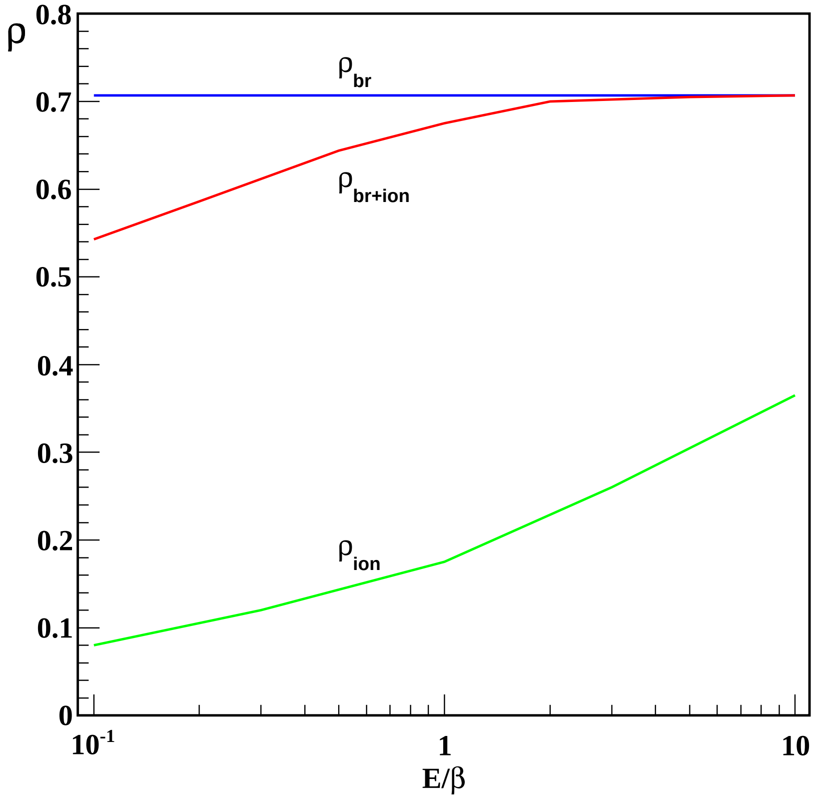

Figure 2: Correlation coefficient as function of electron final energy E in units of the critical energy β for $E \ll E_0$. Upper curve - bramsstrahlung losses only, lower - ionisation losses for $E_0 = 100\,GeV$, middle - both processes.

3.2 Correlation of electron angles with lateral distances in EAS

Coulomb scattering is actually the main cause of angular and lateral deflections of electrons in EAS. We have simulated one iron shower with $E_0 = 10^{17}\,eV$ with CORSIKA [5]. Since the angular distribution of electrons with some fixed energy E stays practically the same at any shower age s [4], we chose the shower maximum level $(s = 1)$ for our study of electron distributions.
Comparing CC in EAS and that from MS with energy losses (Fig.3), the latter shown already in Fig.2, we see that in both cases the correlation increases with the electron energy although it is considerably weaker in EAS than that for a single electron of the same final energy.

3.3 Attempts to explain the low angle-distance correlation in EAS

One of the reasons of the difference between the two curves in Fig.3 may be that electrons in a cascade go through a stage of photons on their way from some initial energy to the final one, whereas the curve obtained analytically corresponds to a propagation of a single electron. In section 2 we showed that in the Heitler model of a cascade the correlation coefficient was the same as that for a single electron when no energy losses were allowed for. To study the possible effect of photons in the framework of some more realistic models we will now consider and compare two of them.

a) Multiple scattering with steps. We start again with a consideration of an electron moving along the $z-$axis, being scattered by a small angle along any step Δz, as in Section 3, but assuming that at each step its energy is diminished by a constant factor $k < 1$. After i steps the electron energy E_i equals $E_i = E_0 k^i$. The variance of scattering angle *gained* in the $i-th$ step equals $\langle \varphi_i^2 \rangle = U_0 k^{-2i}$, where $U_0 = \langle \varphi^2(E_0) \rangle$. We obtain

$$\begin{aligned}
\langle \eta_n^2 \rangle &= U_0 \sum_{i=1}^{n} k^{-2i} = (k^{-2n} - 1)(1 - k^2)^{-1} U_0 \,, \\
\langle x_n^2(z) \rangle &= U_0 (\Delta z)^2 k^{-2n} \sum_{j=1}^{n-1} j^2 k^{2j} \,, \\
\langle \eta_n x_n \rangle &= U_0 \Delta z k^{-2n} \sum_{j=1}^{n-1} j k^{2j} \quad \text{and} \\
\rho(k) &= \frac{k^2}{\sqrt{(1-k^2)^3 \sum_{j=1}^{\infty} j^2 k^{2j}}} \quad \text{for} \quad n \to \infty
\end{aligned} \tag{3.5}$$

For $k = 1/2$ we get $\rho(k = 1/2) = 0.447$. Thus, even if the energy loss process considered here is similar to bremsstrahlung we have obtained a much smaller value as in the case from Section 3.1, where the *continuous*, mean bremsstrahlung losses were assumed, leading to $\rho_{br} \approx 0.707$.

b). The Heitler model. We assume that $\langle \varphi_i^2 \rangle = U_0/4^i$ since at each step the energy of each particle decreases by 2 and $U_i \sim E_i^{-2}$. Taking this into account we obtain for large n

$$\begin{aligned}
\langle \eta_n^2 \rangle &\approx 1.184 \cdot 4^n U_0 \,, \\
\langle x_n^2 \rangle &\approx 0.448 \cdot 4^n U_0 (\Delta z)^2 \,, \\
\langle \eta_n x_n \rangle &\approx 0.217 \cdot 4^n U_0 \Delta z \,,
\end{aligned} \tag{3.6}$$

so that $\rho \approx 0.30$. This is $\sim 67\%$ of the value 0.447 for the step model above. Note that the energy decrease rate is the same in both cases a) and b). Thus, whereas the presence of photons does not affect the studied correlation when $\langle \varphi_i^2 \rangle = const$ (section 2), it does so when one takes into account electron energy losses. Since the energy losses considered here are bremsstrahlung-like we compare the ratio of the actual correlation in EAS and its analytical value at $E/\beta > 1$ (Fig.3). It is $\sim 75\%$, being in a reasonable agreement with 67% from the models.

c). Fluctuations of the energy loss. To check a possible effect of fluctuations in the bremsstrahlung energy loss process we have performed a simple Monte - Carlo simulation of high energy electron Coulomb scattering with energy losses. Starting with some high energy E_0 an electron was followed for t radiation units every small step $\Delta t = 0.01$. Energy losses for ionisation and for emitting low energy photons with $E_\gamma/E = \nu < \nu_0 = 0.1$ were treated as continuous, while emission of a photon with $\nu_0 < \nu < 1$ was randomly chosen, corresponding to the mean free path $\lambda(\nu > \nu_0) = 1/ln(1/\nu_0)$. After $100 \cdot t$ steps the electron energy E, its angle and lateral distance x were recorded. This procedure was repeated for 10^4 electrons. For electrons in given final

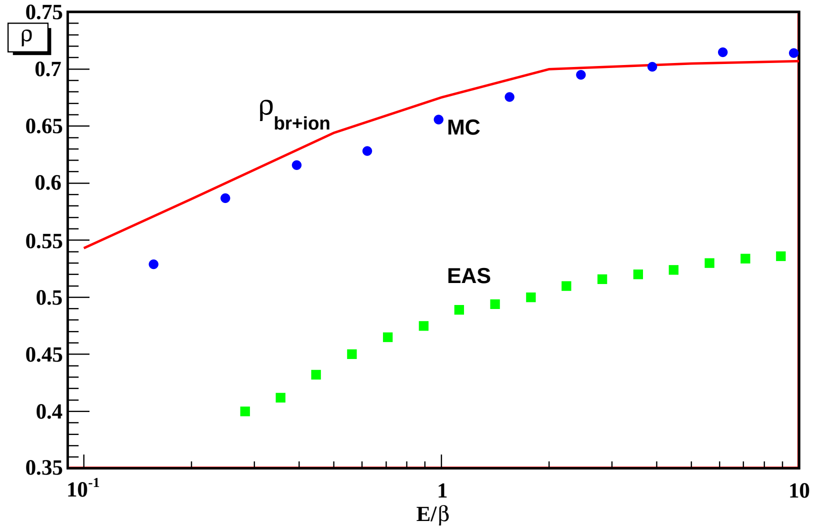

Figure 3: Comparison of the correlation coefficient in EAS (lower points) with that for single electron (upper smooth curve) with average energy losses. Upper points - MC simulations for single electron with fluctuations in energy losses.

energy intervals $(E, E+\Delta E)$ the variances $\langle\eta^2(E)\rangle, \langle x^2(E)\rangle$, the covariance $\langle\eta(E)\cdot x(E)\rangle$ and $\rho(E)$ were calculated. The obtained dependence $\rho(E)$ is shown in Fig.3 together with the analytical calculations refering to average energy losses (no fluctuations). It is seen that essentially there is not much difference between the two curves. Thus, allowing for the fluctuations in the bremsstrahlung process does not affect the correlation coefficient averaged over a large number of electrons.

4. Summary

In this work we have studied the correlation between the angular and lateral deflections of relativistic electrons multiply scattered by Coulomb forces. We had noticed that the correlation coefficient (CC) for electrons in extensive air showers was considerably smaller than that predicted by models for a single electron propagation, such as the small angle multiple scattering model. We have shown that allowing for energy losses causes a decrease of the angle-lateral deflection correlation. We have derived analytically exact expressions for the variances of the electron angle, its lateral deflection and the correlation coefficient allowing for bremsstrahlung and ionisation energy losses. A dramatic difference in the dependence of CC on the final electron energy was shown when each of the two processes was considered separately: ionisation leads to a total decorrelation while the electron energy decreases, whereas bremsstrahlung keeps the correlation only slightly diminished. However, when both processes are at work, the correlation coefficient stays still higher that that for electrons in EAS.

We have studied two possible reasons of this. First is the fact that an electron in EAS has, as parents, also photons, each keeping its angle unchanged through a cascade unit or so. To check the role of photons in the decorellation we have compared two models: one, the step model, where a single electron loses energy by a constant fraction in steps, while the second is a Heitler-like cascade, with electrons and photons, where the energy loss rate is the same as in the previous model. The result was that the correlation coefficient in the Heitler model with photons was smaller (at $E > \beta$) by roughly the same fraction as that needed to go from the analytical results to those for EAS. Secondly, we have considered fluctuations in the energy transfer to a photon in the bremsstrahlung process by Monte-Carlo simulations of electron propagation. The obtained dependence of CC on the final electron energy agrees pretty well with the analytical results where average energy losses were assumed.

Our final conclusion is thus that it is the energy losses, mainly for ionisation, together with photons as electron parents, that affect (considerably diminish) the correlation of the angles and lateral distances of electrons in EAS, when compared with a simple MS model.

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