On the correlation of the angular and lateral deflections of electrons after multiple scattering allowing for energy losses

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Abstract

In the model of multiple small angle scattering the correlation coefficient of a fast particle angle and its lateral deflection from the initial direction is large : $\rho = 0.87$.
For electrons in EAS it is much smaller : $\rho \sim 0.5$. This paper explains why the difference is so large.

Model 1: Multiple scattering (Coulomb) of electron by small angles, electron energy E = const.

Electron makes N steps Δz being scattered by a small angle ϕ in each step, with the dispersion σ_ϕ .

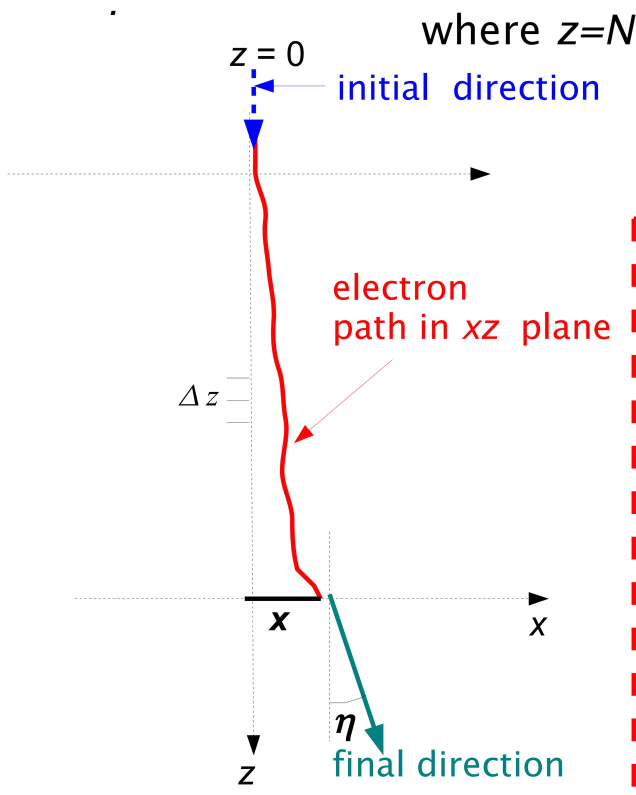
Definition*: Correlation coefficient** ρ between **final** angle η and lateral deflection ***x. $\rho = \frac{\langle \eta * x \rangle}{\sqrt{\langle \eta^2 \rangle * \langle x^2 \rangle}}$.

From

$$\langle \eta^2(z) \rangle = \sigma_\varphi^2 \frac{z}{\Delta z}, \quad \langle x^2(z) \rangle = \frac{1}{3} \sigma_\varphi^2 \frac{z^3}{\Delta z} \quad \text{and} \quad x(z+\Delta z) = x(z) + \eta(z) \Delta z$$

where $z = N\Delta z$, we obtain that $\rho = \frac{\sqrt{3}}{2} \approx 0.866$

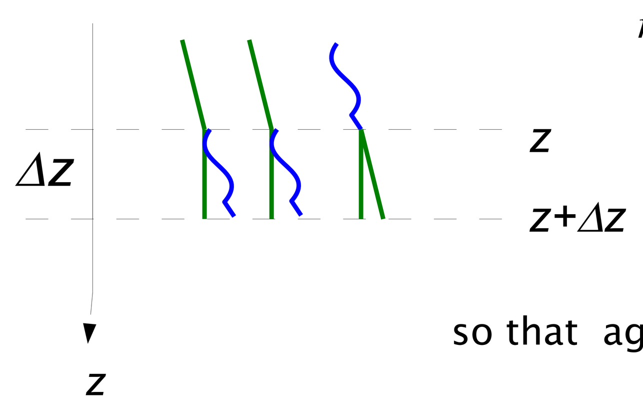
a very strong correlation !



Model 2 : Heitler-like model of electromagnetic cascade – electrons and photons, E= const.

After each step Δz electron emits a photon, photon creates two electrons (e- e+).
Let $\eta_e(z)$ be the angle of an electron at level z and $\eta_{ee}(z)$ $(\eta_{e\gamma}(z))$ - the angle of an electron at z, the parent of which at z-Δz is an electron (photon). We have:

$$n_e(z) = 2 n_\gamma(z), \quad \langle \eta_{ee}^2(z) \rangle = \langle \eta_e^2(z-\Delta z) \rangle + \langle \varphi^2(z) \rangle, \quad \text{and} \quad \langle \eta_{e\gamma}^2(z) \rangle = \langle \eta_e^2(z-2\Delta z) \rangle + \langle \varphi^2(z) \rangle.$$



From this we obtain:

$$\langle \eta^2(z) \rangle = \frac{2}{3} \sigma_\varphi^2 \frac{z}{\Delta z}, \quad \langle x^2(z) \rangle = \frac{2}{9} \sigma_\varphi^2 \frac{z^3}{\Delta z}, \quad \langle \eta(z) x(z) \rangle = \frac{1}{3} \sigma_\varphi^2 \frac{z^2}{\Delta z},$$

so that again $\rho = \frac{\sqrt{3}}{2} \approx 0.866$

parent photons do not influence ρ if E=const.

Model 3: Multiple scattering of electron by small angles, electron loses energy : $E = E(z)$.

After N steps electron angle equals $\eta_N = \Sigma_{i=1}^N \varphi_i = \frac{E_s}{E_1} \varphi_{s1} + \frac{E_s}{E_2} \varphi_{s2} + \dots + \frac{E_s}{E_N} \varphi_{sN}$, where φ_{si} is a scattering angle for some fixed $E = E_s$.

Assuming that $\Delta z \to 0$ so that $N\Delta z$ = const. and E_s = 21 MeV we obtain:

$$\langle \eta^2(z) \rangle = \frac{1}{X_0} \int_0^z \left(\frac{E_s}{E(z')} \right)^2 dz', \quad \langle x^2(z) \rangle = \frac{1}{X_0} \int_0^z (z-z')^2 \left(\frac{E_s}{E(z')} \right)^2 dz' \quad \text{and} \quad \langle \eta(z) x(z) \rangle = \frac{1}{X_0} \int_0^z (z-z') \left(\frac{E_s}{E(z')} \right)^2 dz'$$

Assuming $E(z)$ one can find the above variances and compute the correlation coefficient ρ :

a). Energy losses for Bremsstrahlung only

$$E(z) = E_0 e^{-z/X_0} \qquad u = E/E_0$$

$$\rho_{br} = \frac{1}{\sqrt{2}} \frac{1-u^2+2u^2 \ln u}{\sqrt{(1-u^2)*(1-u^2+2u^2(1-\ln u)\ln u)}}$$

(Fig.1) .

b) Ionisation only

$$E(z) = E_0 - \beta z / X_0 \, ,$$

$$\rho_{ion} = [\ln(1/u) + u - 1] / \sqrt{(1/u-1)(1-u^2+2u\ln u)}$$

(Fig.1)

c). Bremsstrahlung and ionisation

$$E(z) = (E_0 + \beta) \exp(-z/X_0) - \beta$$

β - the critical energy of the medium, X_0 - the radiation unit.
Only $\langle \eta^2(z) \rangle$ can be calculated analytically , so that the correlation coefficient ρ has to be computed numerically **(Fig.2)**

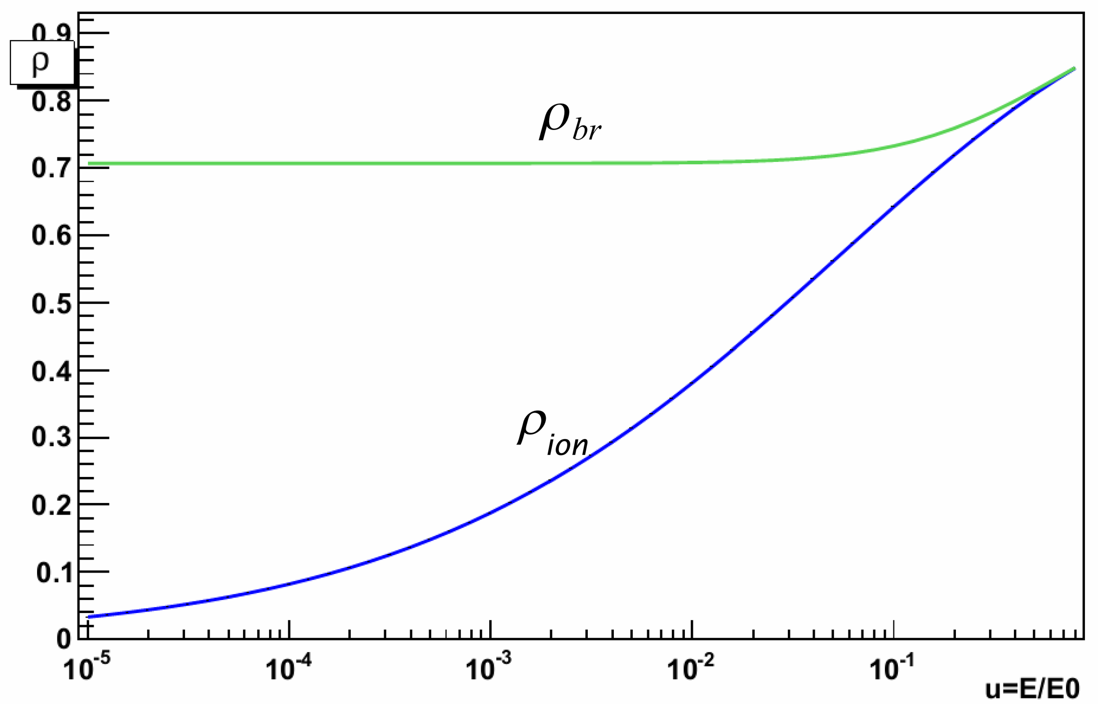
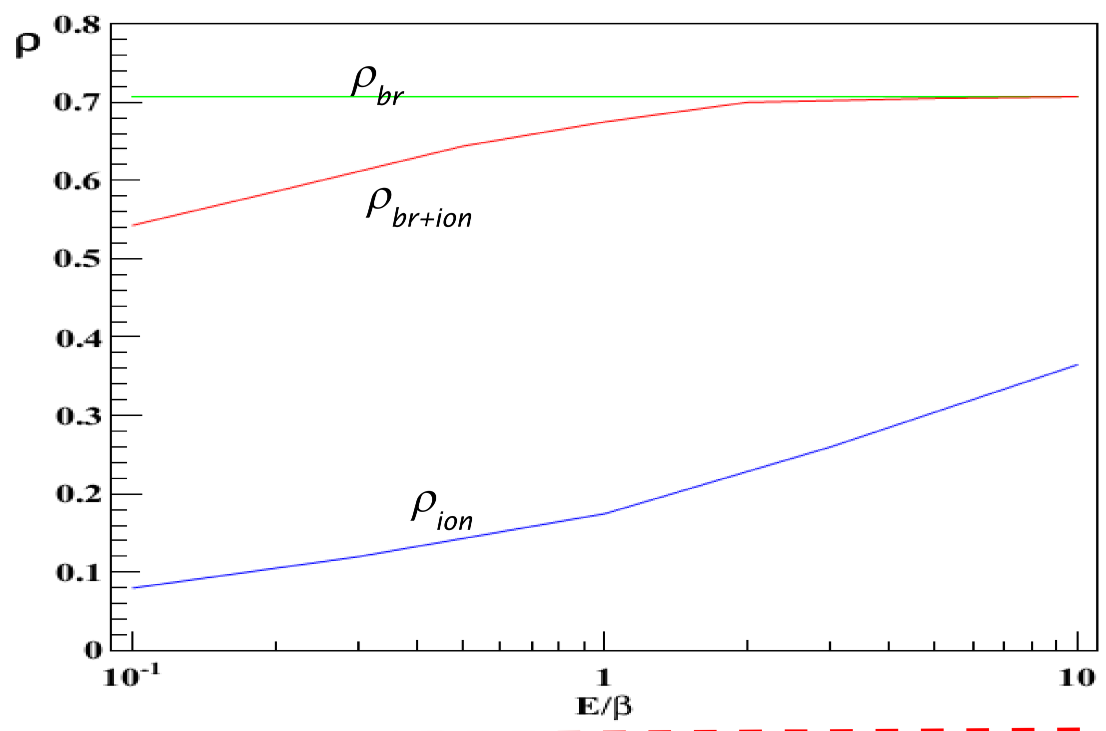


Fig.1.
Correlation coefficient ρ of η and x as function of the ratio of the final to initial electron energy .
ρ_{br} - bremsstrahlung only
ρ_{ion} - ionisation only.

A completely different behaviour of ρ in the two cases can be seen.

Fig.2.
Correlation coefficient r as function of electron energy in units of the critical energy b.
Green curve – bremsstrahlung energy losses only,
blue – ionisation,
red – both processes.

$E_0 = 100 \ GeV.$



So, why is the correlation for electrons in EAS much **smaller** , as seen in **Fig.3** below ?

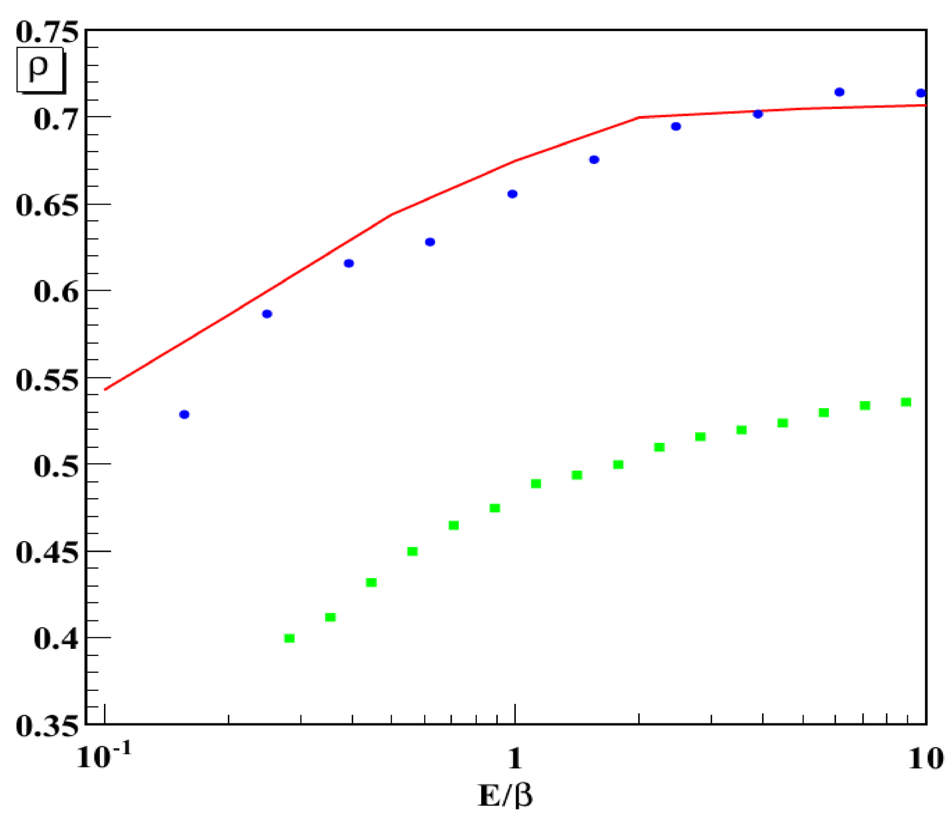


Fig.3. *Comparison of the correlation coefficient ρ in EAS (green points), as function of final electron energy E, (in units of the critical energy β) with ρ for single electron with average energy losses (red line -the same as that in Fig.2). Blue points - MC simulations for single electron with fluctuations in energy losses.*

No effect of fluctuations is seen.
For electrons in EAS ρ is considerably smaller.

Looking for the reason of the difference :

To study the possible effect of photons in the framework of some more realistic models we consider two of them:

Model 4. Multiple scattering of a single electron in steps

At each step electron energy is diminished by a constant factor k < 1, so that $E_i = E_0 k^i$ and $\langle \varphi_i^2 \rangle = U_0 k^{-2i}$. We obtain :

$$\langle \eta_N^2 \rangle = U_0 (k^{-2N} - 1)(1 - k^2)^{-1},$$
$$\langle x_N^2 \rangle = U_0 (\Delta z)^2 k^{-2N} \Sigma_{j=1}^{N-1} j^2 k^{2j},$$
$$\langle \eta_N x_N \rangle = U_0 \Delta z \, k^{-2N} \Sigma_{j=1}^{N-1} j k^{2j}$$

and the correlation coefficient for $N \to \infty$ equals

$$\rho(k) = k^2 / \sqrt{(1-k^2)^3 \Sigma_{j=1}^{\infty} j^2 k^{2j}}$$

For k = ½ ρ **= 0.447.**

Thus, even if the energy - loss "process" considered here is similar to bremsstrahlung we obtain a much smaller value as when continuous , mean bremsstrahlung losses were assumed leading to ρ = 0.707.
Also:
MC simulations of electron propagation were done where emission of high-energy photons was chosen randomly. No mean effect on ρ was found (Fig.3).

Model 5. Heitler-like model of electromagnetic cascade – ~ electrons and photons, as Model 2 but $E_i = E_0 (1/2)^i$

Note: electron energy decrease in each step is the same in both *Model 4* and *Model 5.*

For $N \to \infty$ we obtain

$$\langle \eta_N^2 \rangle \approx 1.184 \, U_0 4^N,$$
$$\langle x_N^2 \rangle \approx 0.448 \, U_0 (\Delta z)^2 4^N,$$
$$\langle \eta_N x_N \rangle \approx 0.217 \, U_0 \Delta z \, 4^N$$

so that ρ = **0.30** independently of N.

Thus, going from *Model 4 (single electron)* to *Model 5 (cascade with photons)* **decreases correlation** by factor ~ 0.30/0.447 = **0.67.**
By analogy, we may expect that going from multiple scattering *Model 3 (single electron)* to EAS *(cascade)* would decrease ρ by roughly the same factor:
at E/β=1 this ratio is ~ **0.73** **(Fig.3)** ,
- what we consider as a reasonable agreement.

Conclusion :

Weak correlation of electron angles and their lateral deflections in EAS, when compared with multiple scattering models, is caused by electron energy losses – mainly those for ionisation - and by the parent photons.