Universality of the lateral and angular distributions of electrons in large extensive air showers

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Abstract

Based on shower simulations we show that the electron distribution $f(\theta, \varphi, r, E; s)$, describing fully the fraction of electrons with energy E, at shower age s, at the distance from the axis r and having angles θ, φ, is the same for *any* shower independently of the primary energy or mass and shower fluctuations. We find an analytic description of this function fitting it best in most populated regions of the variable space.

Introduction

It was already shown [1, 2] that the electron energy distributions depend on the shower age s only; they do not depend on the energy or mass of the primary particle. In [3] it was also shown that the lateral distributions of electrons with different energies depend only on the shower age, being independent of the primary particle. Moreover, they can be described by a single, universal function for any shower age [5]. We studied there [5] the distributions of electron's radial angles, but integrated over the tangential angles, i.e. those in the plane perpendicular to the lateral vector and gave an analytical description of them for any electron energy E and lateral distance r.

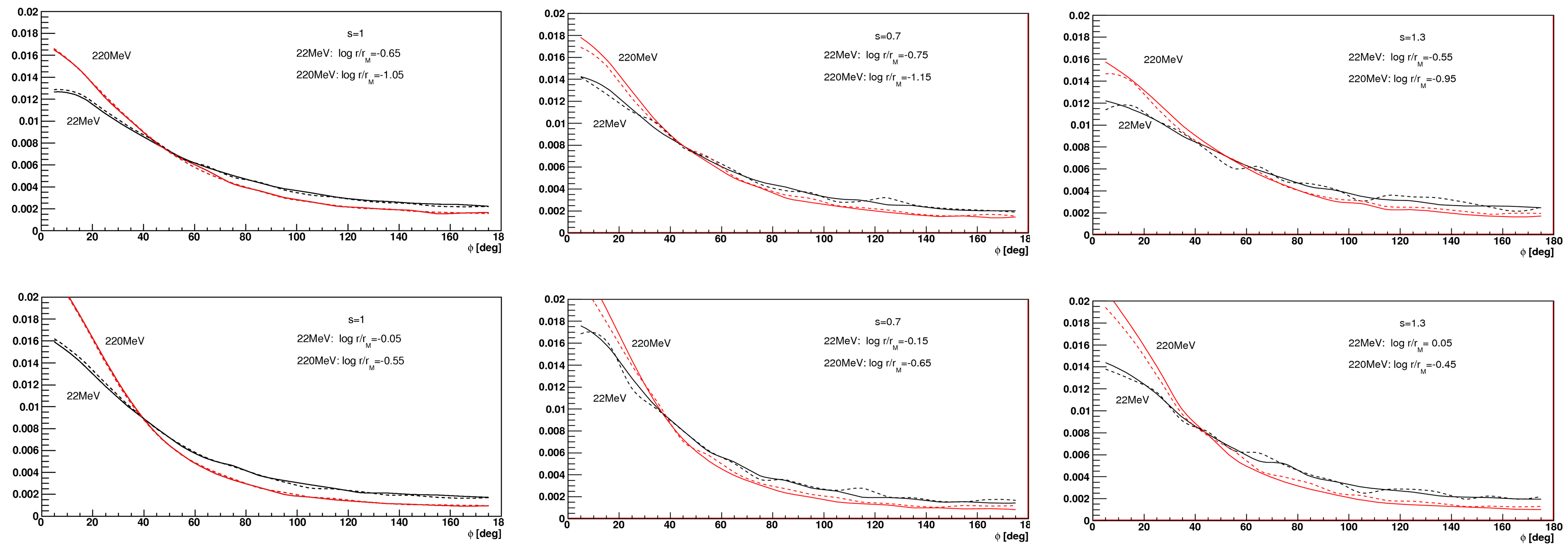

Figure 1. Distributions $f_\varphi(\varphi\,;r/r_M\,, E\,, s)$ of electron azimuth angle φ with respect to the shower axis for two electron energies E within $\Delta E\,/E = 0.15$, at two lateral distances r/r_M within $\Delta log(r/r_M) = 0.1$. Continuous curve - one primary proton with $E_0 = 10^{19}\,eV$, dashed - average of 10 iron showers with $E_0 = 10^{17}\,eV$. Left column: $s = 1$; middle: $s = 0.7$; right: $s = 1.3$. Independence of the primary particle's energy or mass is seen. $\varphi = 0$ means directions away from shower axis.

The Aim of this paper is:

- To show that any electron distribution in a large shower is universal in the sense that it is independent of the primary particle and of the shower to shower fluctuations.
- To give a full description, in a possibly compact form, of the state of electrons in a shower.

Universality of electron distributions

The state of electrons in a shower is uniquely determined by the numbers ΔN_e:

$$\Delta N_e(\theta\,, \varphi\,, r, E\,; s) = N_e(s)\, f(\theta\,, \varphi\,, r, E\,; s)\Delta\theta\,\Delta\varphi\,\Delta r\Delta E$$

where $$f(\theta\,, \varphi\,, r, E\,; s) = f_E(E\,; s)\, f_r(r; E\,, s)\, f_\varphi(\varphi\,; r, E\,, s)\, f_\theta(\theta\,; \varphi, r, E\,, s)$$

Distributions $f_E\,, f_r\,, f_\varphi$ and f_θ are normalised to unity and r is in $g\,cm^{-2}$

The energy distributions at various ages $f_E(E\,; s)$ have been found already [1] and parametrised by Nerling et al [2]. The lateral distribution $f_r(r/r_M\,; E\,, s)$ has also been shown to be universal [3, 4]. Moreover, in [5] it was shown that $f_r(r/r_M\,; E, s)$, a function of three variables, can be represented as a function of only one variable. Yet it remains to be shown that the last two angular distributions are universal.

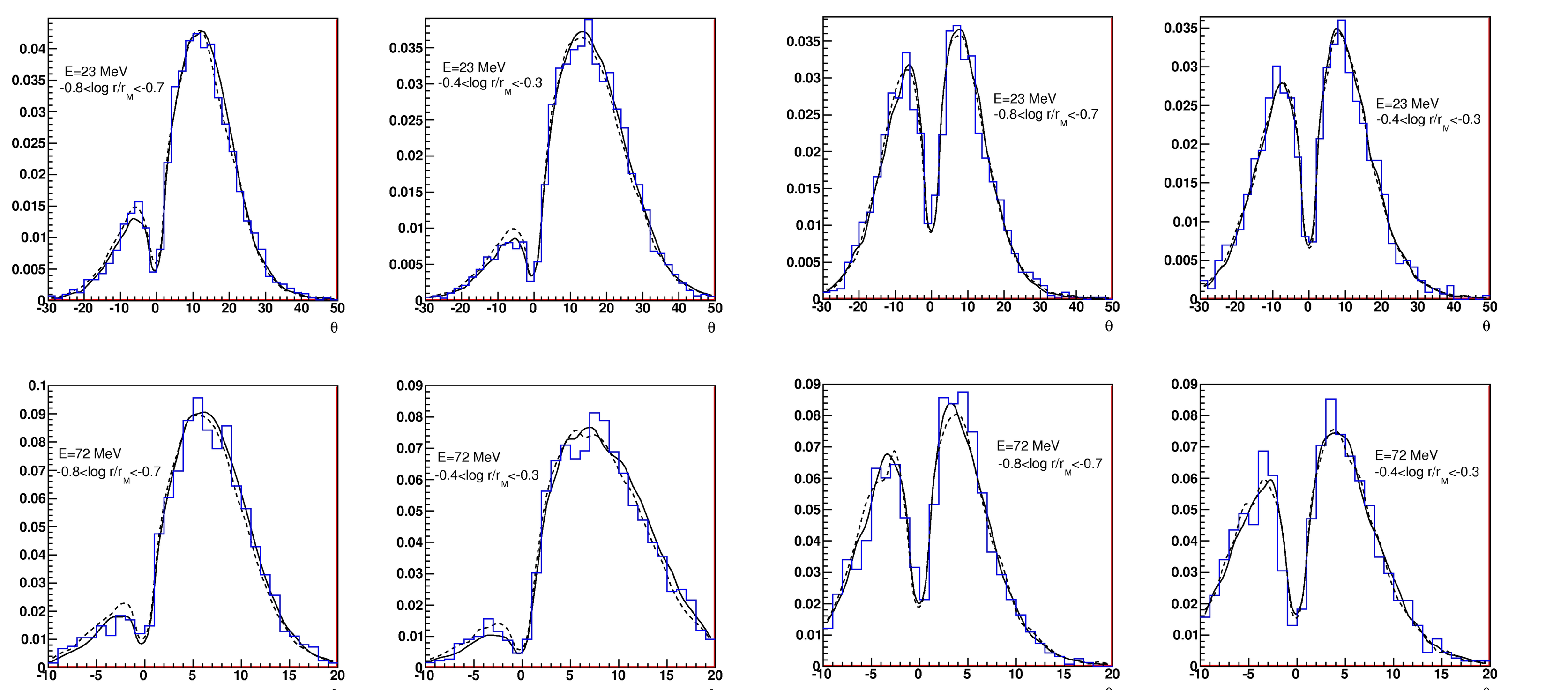

Figure 2. Distributions $f_\theta(\theta\,; \varphi, r/r_M\,, E\,, s)$ of electron angle θ (in deg) for $s = 1$ for two electron energies E and two distances r/r_M. Continous curve - p 10^{19} eV, dashed - averaged of 10x Fe 10^{16} eV, histogram - one Fe 10^{16} eV. Left half: $0 < \varphi < 20^\circ$ - bigger peaks, $160^\circ < \varphi < 180^\circ$ - smaller peaks. Right half: $70^\circ < \varphi < 90^\circ$ - bigger peaks, $90^\circ < \varphi < 110^\circ$ - smaller peaks.

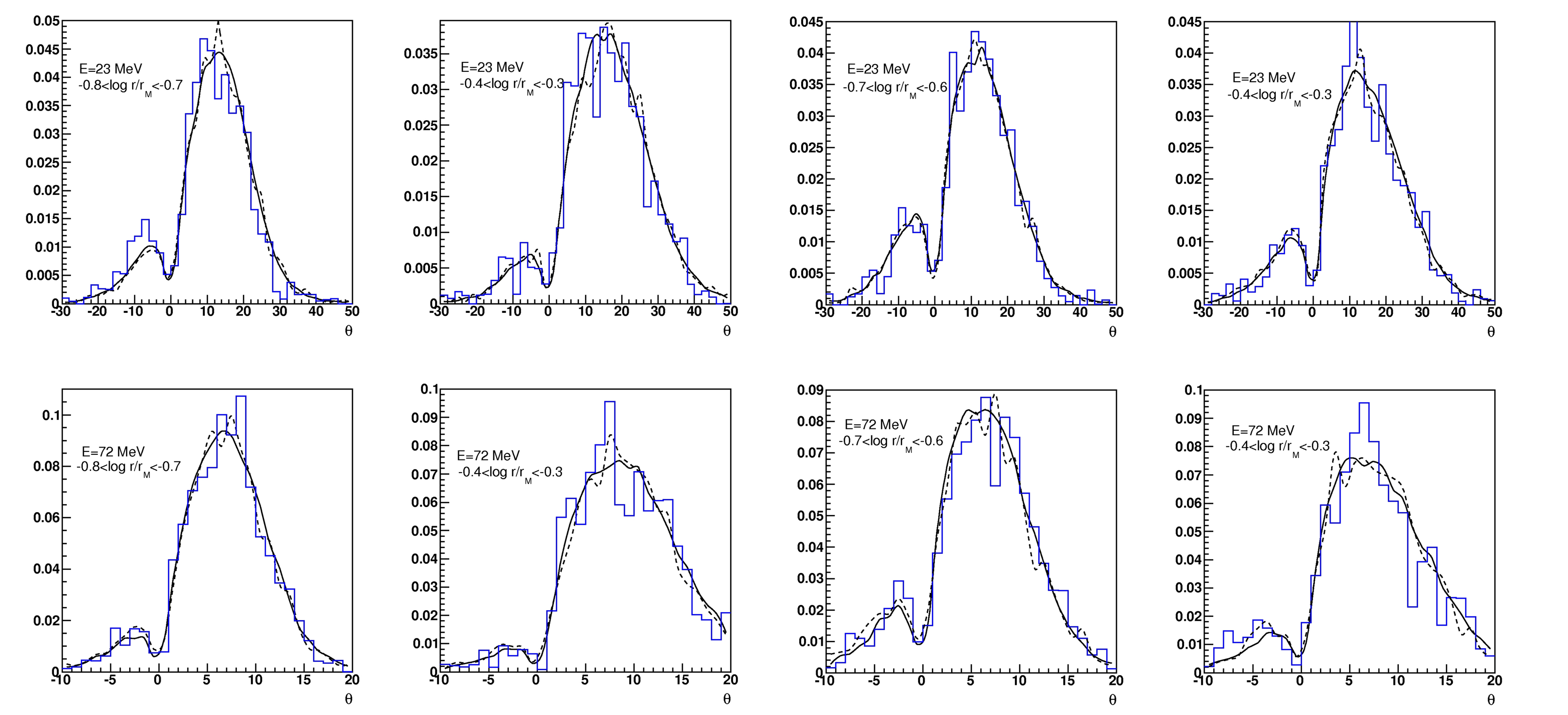

Figure 3. The same as left part of Fig.2: $0 < \varphi < 20^\circ$ - bigger peaks, $160^\circ < \varphi < 180^\circ$ - smaller peaks, but for s=0.7 (left half) and s=1.3 (right half).

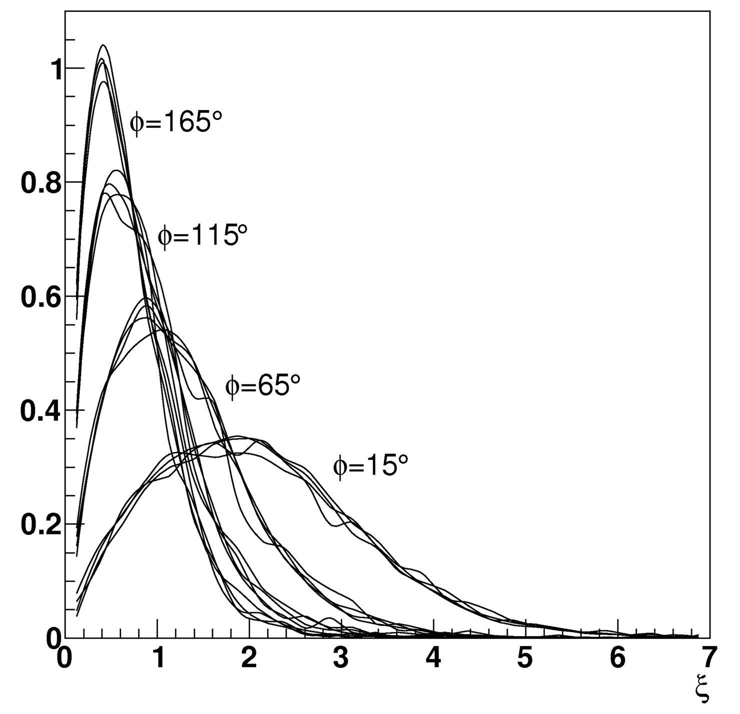

Figure 5. Independence of $F_\xi(\xi\,; \varphi, s = 1)$ of electron energy E or distance r/r_M for fixed ξ. One iron shower with $E_0 = 10^{17}\,eV$ (without thinning).

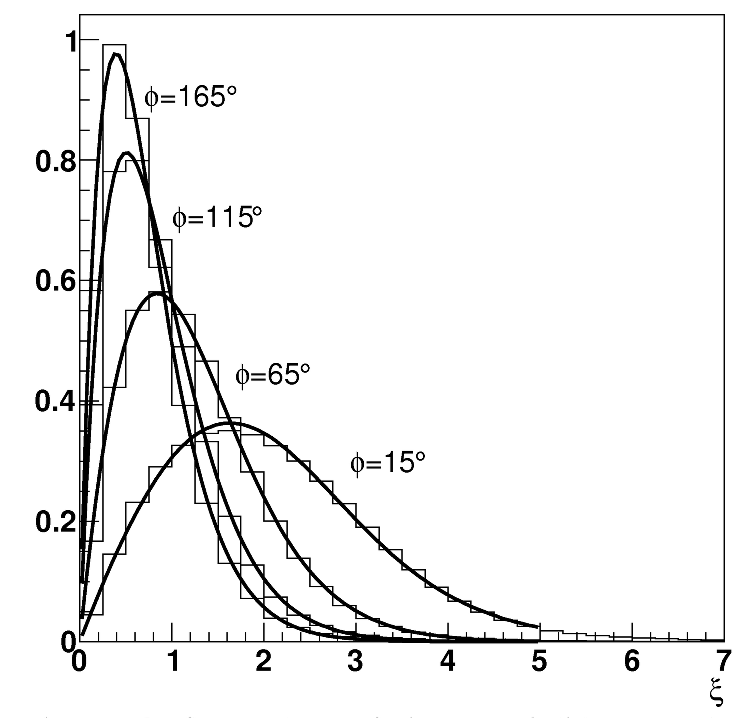

Figure 6. Comparison of the actual (histograms) and parametrised (lines) distributions $F_\xi(\xi\,; \varphi, s = 1)$ referring to those of axial angle θ. The same shower as in Fig.5.

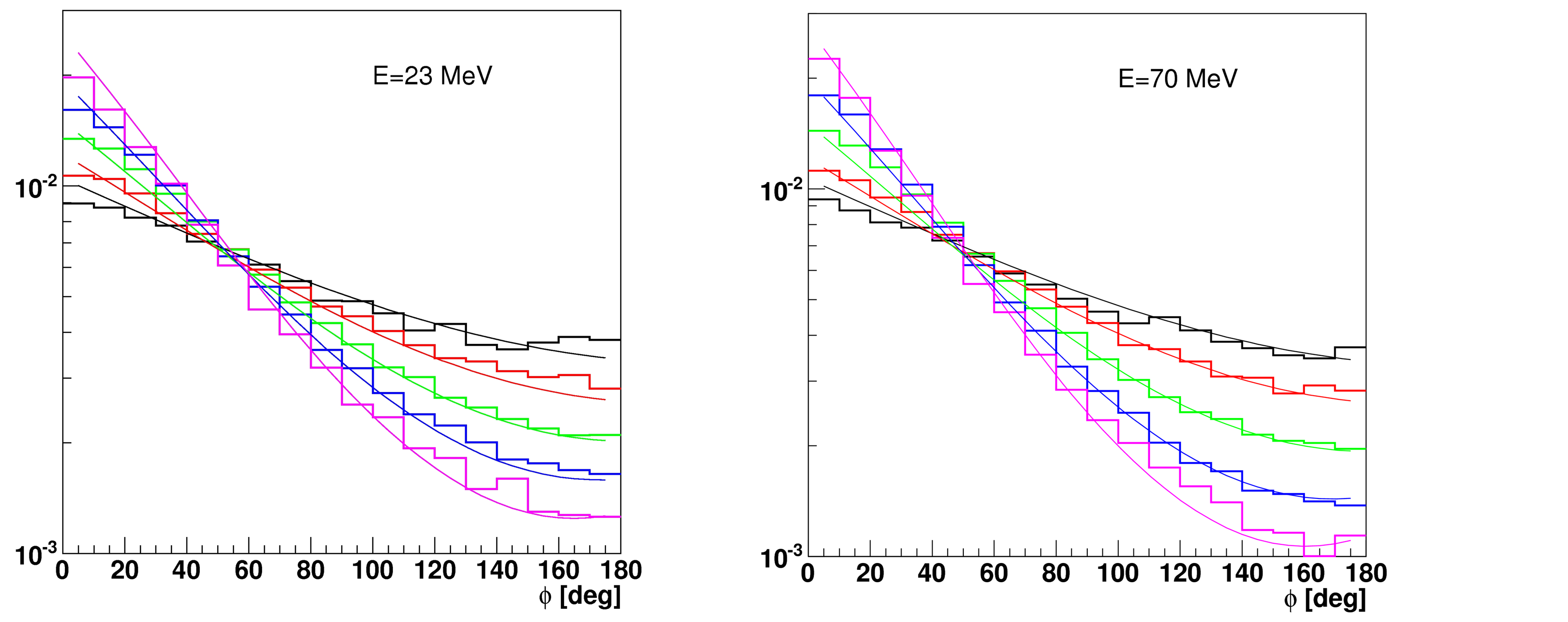

Figure 4. Comparison of actual (histograms) and parametrised (lines) distributions $f_\varphi(\varphi\,; r/r_M\,, E\,, s = 1)$ of azimuth angles φ for two electron energies $E = 23\,MeV$ and $E = 70\,MeV$, each for five values of r/r_M; left graph (starting from the flattest): $log\; r/r_M$ = −1.6, −1.1, −0.6, −0.1, 0.4; right graph: $log\; r/r_M$ = −1.8, −1.3, −0.8, −0.3, 0.2. One iron shower with $E_0 = 10^{17}\,eV$ (without thinning).

Angular distributions $f_\varphi(\varphi\,; r/r_M\,, E\,, s)$ and $f_\theta(\theta\,; \varphi\,, r/r_M\,, E\,, s)$

To obtain the electron distributions we simulate showers with CORSIKA [6] and find the number of electrons $\Delta N_e(\theta\,, \varphi\,, r/r_M\,, E\,, s)$ in variable bins $\Delta\theta\,\Delta\varphi\,\Delta(r/r_M)\Delta E$ at various ages s. (Note that r/r_M is proportional to r in $g\,cm^{-2}$). We restrict ourselves to variable regions where there are most electrons in the shower, i.e. to $0.7 < s < 1.3$ and $20\,MeV < E < 200\,MeV$. Showers with $E_0 = 10^{17}\,eV$ and $10^{16}\,eV$ were fully simulated i.e. without the thinning procedure, whereas at $10^{19}\,eV$ the thinning was used.

Fig.1 demonstrates the universality of the distributions $f_\varphi(\varphi\,; r/r_M\,, E\,, s)$ of the azimuth angle φ, of the primary particle energy and mass. We have parametrised distributions in the form:

$$log\, f_\varphi = A + Bcos(a\varphi + b)$$ (see ICRC paper for details of parametrisation)

The best fit curves are shown in Fig.4 together with the actual distributions.

Figs 2 and 3 illustrate the independence of the angular distributions of electrons $f_\theta(\theta\,; \varphi, r/r_M\,, E\,, s)$, of the axial angles, of the primary particle energy and mass. We have found a new variable ξ, defined as

$$\xi = \frac{E^\alpha\,\theta}{(r/r_M)^\beta} \quad where \quad \alpha > 0 \quad and \quad \beta > 0$$

such that $f_\theta(\theta\,; \varphi, r/r_M\,, E\,, s)$ reduces to a function $F_\xi(\xi\,; \varphi, s)$ of three, instead five, variables: ξ, φ and s. We choose E in GeV and θ in degrees. Fig.5 shows distributions $F_\xi(\xi\,; \varphi, s)$. For each φ there are four curves corresponding to two values of E and two r/r_M. It is seen that the distributions are essentially independent of energy or distance once the value of ξ is fixed.

We fit the distributions $F_\xi(\xi\,; \varphi, s = 1)$ for various φ in bins $\Delta\varphi = 10^\circ$, with a function:

$$F_\xi(\xi\,; \varphi, s{=}1) = \frac{C\xi}{(d + e^\xi)^\gamma}$$

The best fitted curves are shown, together with actual distributions, in Fig. 6. (see ICRC paper for details)

Conclusions

In this work we have demonstrated the universality of both angular distributions: $f_\varphi(\varphi\,; r/r_M\,, E\,, s)$ and $f_\theta(\theta\,; \varphi\,, r/r_M\,, E\,, s)$, of electrons with various energies E, at various lateral distances r/r_M and at various shower ages s. Together with the universality of the electron energy distributions and of their lateral distributions this allows us to claim that any electron distribution in a large shower is universal.

We have also parametrised the angular distributions for $s = 1$. In the case of the distribution of the axial angle θ we have found a variable $\xi = E^\alpha\,\theta\,(r/r_M)^{-\beta}$ such that $f_\theta(\theta\,; \varphi\,, r/r_M\,, E\,, s = 1)$ depends actually on only three, instead of five, variables.

For $s \neq 1$ the distributions do not differ much from those at $s = 1$. However, since the differences reach sometimes more than 10% , an additional dependence of parametrisations on age s needs to be done.

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References

[1] M. Giller, G. Wieczorek, A. Kacperczyk, H. Stojek , W. Tkaczyk, J. Phys. G: Nucl. Part. Phys. 30 (2004) 97
[2] F. Nerling, J. Blümer, R. Engel and M. Risse, Astroparticle Phys, 24 (2006) 421
[3] M. Giller, H. Stojek, G. Wieczorek, Int. Journal of Modern Phys. A, Vol 20, (2005), No 29, p. 6821
[4] S. Lafebre, R. Engel et al, Astroparticle Phys. 31 (2009), 243
[5] M. Giller, A. Smiałkowski and G. Wieczorek, Astroparticle Phys. Vol 60, (2015), p. 92-104
[6] D. Heck, J. Knapp et al., Technical Report 6019 (1998) Forchungszentrum, Karlsruhe