PROCEEDINGS OF SCIENCE

Differences of masses and widths of the charged and neutral $\rho(770)$, $\rho(1450)$, $\rho(1700)$ mesons from data on electro-weak processes

Erik Bartoš*
Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, 845 11 Bratislava, Slovakia
E-mail: erik.bartos@savba.sk

Stanislav Dubnička
Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, 845 11 Bratislava, Slovakia
E-mail: stanislav.dubnicka@savba.sk

Anna-Zuzana Dubničková
Faculty of Mathematics, Physics and Informatics, Comenius University, Mlynská dolina, 842 48 Bratislava, Slovakia
E-mail: dubnickova@fmph.uniba.sk

Hisaki Hayashii
Physics Department, Nara women's University, 630-8506, Nara, Japan
E-mail: hayashii@hepl.phys.nara-wu.ac.jp

The $\rho(770)$, $\rho(1450)$, $\rho(1700)$ mesons exist in three charged states ρ^0, ρ^+ and ρ^-, whereby masses of positively charged mesons are identical with masses of negatively charged mesons, due to the CPT theorem. However, there is no reason for the identity of charged meson masses with neutral meson masses. For determination of differences of masses and decay widths of charged and neutral $\rho(770)$, $\rho(1450)$, $\rho(1700)$ mesons are employed the data on $e^+e^- \to \pi^+\pi^-$ an $\tau^- \to \pi^-\pi^0\nu_\tau$ processes to be analyzed by the Unitary and Analytic models of the electromagnetic and weak pion form factors, respectively.

The European Physical Society Conference on High Energy Physics
22–29 July 2015
Vienna, Austria

*Speaker.

PoS(EPS-HEP2015)450

1. Introduction

Rho meson ρ is strongly interacting particle compound of a mix of two u and d valence quarks. It possess the isospin $I = 1$, so it exists in three charge states denoted by ρ^0, ρ^+ and ρ^-, each with some value of mass and decay width. The small difference in the parameters between charge and neutral states is the consequence of the breaking of isospin symmetry, which is possible due to the difference in the masses of quarks and the effects of the electromagnetic (EM) interaction.

The current value of the difference of the parameters of neutral and charged rho mesons for the ground state $\rho(770)$ has the avarage value $m_{\rho^0} - m_{\rho^\pm} = -0.7 \pm 0.8\,\mathrm{MeV}$, resp. $\Gamma_{\rho^0} - \Gamma_{\rho^\pm} = 0.3 \pm 1.3\,\mathrm{MeV}$ [1]. For charged rho mesons parameters experiments give $m_{\rho^+} - m_{\rho^-} = 1.5 \pm 0.8 \pm 0.7\,\mathrm{MeV}$, resp. $\Gamma_{\rho^+} - \Gamma_{\rho^-} = 1.8 \pm 2.0 \pm 0.5\,\mathrm{MeV}$ [2, 3]. In the past the determination of ρ-meson parameters from processes including the pion-proton interactions $\pi^\pm \mathrm{p} \to \pi\pi\mathrm{n}$ [4, 5], the proton-antiproton annihilation $\mathrm{p\bar{p}} \to \pi^+\pi^-\pi^0$ [6, 7], $\mathrm{e^+e^-} \to \pi^+\pi^-\pi^0$ annihilation [2, 8], τ^--lepton decay processes [9] has been performed predominantly by Breit-Wigner formalism. Such formalism yields high experimental uncertainties (see Tab. 1). Moreover, the using of not the same Breit-Wigner functions for the analyses of the spectra of processes inserts into the results a considerable model dependence.

$\Delta m = m_{\rho^0} - m_{\rho^\pm}$ [MeV]	**Processes**	**Literature**
2.4 ± 2.1	$\pi^\pm \mathrm{p} \to \rho \mathrm{N}$	[4]
-5.0 ± 5.0	$\mathrm{\bar{p}p} \to \rho\pi$	[6]
-4.0 ± 4.0	$\pi^- \mathrm{p} \to \rho \mathrm{N}$	[5]
1.6 ± 1.9	$\mathrm{\bar{p}p} \to \pi^+\pi^-\pi^0$	[7]
1.3 ± 2.3	$\mathrm{e^+e^-} \to \pi^+\pi^-\pi^0$	[8]
0.4 ± 0.9	$\mathrm{e^+e^-} \to \pi^+\pi^-\pi^0$	[2, 3]
-2.4 ± 0.8	$\tau^- \to \pi^-\pi^0\nu_\tau$, $\mathrm{e^+e^-} \to \pi^+\pi^-$	[9, 10]
-0.7 ± 0.8	PDG weighted average	[1]

Table 1: Mass difference of neutral ρ^0 and charged $\rho^\pm$ mesons for the state $\rho(770)$.

In connection with the updated $\mathrm{e^+e^-} \to \pi^+\pi^-$ annihilation data [11, 12] we propose to use the unitary and analytic (U&A) model of EM pion FF to determine precisely the ρ-meson family parameters. On the other hand such approach allows us to use also the accurate weak pion FF data from the measurement of τ^-lepton decay [13] through derived connection between EM and weak pion FFs which is allowed by conserved vector current hypothesis [14].

2. U&A model of EM pion FF

The unitary and analytic model of the pion EM form factor is a phenomenological approach, based on the synthesis of the experimental fact of a creation of neutral $\rho(770)$, $\rho(1450)$, $\rho(1700)$ mesons in the process $\mathrm{e^+e^-} \to$ hadrons. The model has builtin the asymptotic behavior $\mathrm{F}_\pi(t)|_{t\to-\infty} \sim t^{-1}$ as the consequence of QCD asymptotic freedom $\mathrm{F}_\pi(t)|_{t\to-\infty} \sim -\frac{16\pi f_\pi^2 \alpha_s(t)}{t}$, ($f_\pi = 92.4 \pm 0.2$MeV

– weak pion decay constant, $\alpha_s(t)$ – QCD running coupling constant) and all other known its theoretical properties. Due to the possessing analytic properties it is still the most successful way of a reconstruction of the EM pion FF behavior in space-like and time-like regions simultaneously.

This approach leads to U&A model of EM pion FF defined on four-sheeted Riemann surface to be by the expression

$$
\begin{aligned}
F_\pi^{E,I=1}[W(t)] &= \left(\frac{1-W^2}{1-W_N^2}\right)^2 \frac{(W-W_Z)(W_N-W_P)}{(W_N-W_Z)(W-W_P)} \\
&\times\Big\{\frac{(W_N-W_\rho)(W_N-W_\rho^*)}{(W-W_\rho)(W{\_}W_\rho^*)}\cdot\frac{(W_N-1/W_\rho)(W_N-1/W_\rho^*)}{(W-1/W_\rho)(W-1/W_\rho^*)}\Big(\frac{f_{\rho\pi\pi}}{f_\rho}\Big) \\
&+\sum_{v=\rho',\rho''}\Big[\frac{(W_N-W_v)(W_N-W_v^*)}{(W-W_v)(W{\_}W_v^*)}\cdot\frac{(W_N+W_v)(W_N+W_v^*)}{(W+W_v)(W+W_v^*)}\Big]\Big(\frac{f_{v\pi\pi}}{f_v}\Big)\Big\}
\end{aligned} \tag{2.1}
$$

with the conformal mapping

$$
W(t) = \mathrm{i}\frac{\sqrt{q_{in}+q}-\sqrt{q_{in}-q}}{\sqrt{q_{in}+q}+\sqrt{q_{in}-q}}, \quad q=\sqrt{\frac{t-t_0}{4}}, \quad q_{in}=\sqrt{\frac{t_{in}-t_0}{4}} \tag{2.2}
$$

of four-sheeted Riemann surface in t-variable into one W-plane. The complex conjugate poles on unphysical sheets correspond to unstable ρ-meson resonances, $t_0 = 4m_\pi^2$ is the lowest branch point and t_{in} is an effective parameter which takes into account contributions of all higher branch points on the positive real axis. W_Z and W_P are the zero and the pole by means of which the left-hand cut contribution to the pion FF on the II. Riemann sheet is simulated and their values can be found in the interval $(0,1)$ for $t \in (-\infty, 0)$. The ratios $f_{v\pi\pi}/f_v$ consist of two-pion decay coupling constants, $f_{v\pi\pi}$ and the universal vector-meson coupling constants f_v, defined through the photon-vector-meson coupling constants $g_{\gamma^* v}$ as $f_v = em_v^2/g_{\gamma^* v}$. The ratios of coupling constants for higher excited states $\rho(1450)$ and $\rho(1700)$ (in our notation ρ', ρ'') can be expressed by the forms

$$
\begin{aligned}
\Big(\frac{f_{\rho'\pi\pi}}{f_{\rho'}}\Big) &= \frac{1}{\frac{N_{\rho'}}{|W_{\rho'}|^4}-\frac{N_{\rho''}}{|W_{\rho''}|^4}} \\
&\times\Big\{1-\Big(\frac{N_{\rho'}}{|W_{\rho'}|^4}-\Big[1+2\frac{W_ZW_P}{W_Z-W_P}\mathrm{Re}W_\rho(1-|W_\rho|^{-2})\Big]N_\rho\Big)\Big(\frac{f_{\rho\pi\pi}}{f_\rho}\Big)\Big\}, \\
\Big(\frac{f_{\rho''\pi\pi}}{f_{\rho''}}\Big) &= \frac{1}{\frac{N_{\rho'}}{|W_{\rho'}|^4}-\frac{N_{\rho''}}{|W_{\rho''}|^4}} \\
&\times\Big\{-1+\Big(\frac{N_{\rho''}}{|W_{\rho''}|^4}-\Big[1+2\frac{W_ZW_P}{W_Z-W_P}\mathrm{Re}W_\rho(1-|W_\rho|^{-2})\Big]N_\rho\Big)\Big(\frac{f_{\rho\pi\pi}}{f_\rho}\Big)\Big\} \\
N_\rho &= (W_N-W_\rho)(W_N-W_\rho^*)\times(W_N-1/W_\rho)(W_N-1/W_\rho^*), \\
N_v &= (W_N-W_v)(W_N-W_v^*)(W_N+W_v)(W_N+W_v^*), \quad v=\rho',\rho'',
\end{aligned}
$$

which reduce the number of physically interpretable free parameters of our model, i. e., t_{in}, m_ρ, Γ_ρ, $f_{\rho\pi\pi}/f_\rho$, $m_{\rho'}$, $\Gamma_{\rho'}$, $m_{\rho''}$, $\Gamma_{\rho''}$, W_Z and W_P. The non-zero widths of ρ-meson resonances are incorporated in the model by the relations $W_v(t) = W(t)|_{t=(m_v-\mathrm{i}\frac{\Gamma_v}{2})^2}$.

Parameter	ρ^0 (e^+e^- **data**) [MeV]	$\rho^\pm$ (τ^- **data**) [MeV]	$\Delta(\rho^0-\rho^\pm)$ [MeV]
$m_{\rho(770)}$	758.23 ± 0.46	761.60 ± 0.95	$-\ 3.37 \pm 1.06$
$m_{\rho(1450)}$	1342.31 ± 46.62	1373.83 ± 11.37	-31.53 ± 47.99
$m_{\rho(1700)}$	1718.50 ± 65.44	1766.80 ± 52.36	-48.30 ± 83.81
$\Gamma_{\rho(770)}$	144.56 ± 0.80	139.90 ± 0.46	4.66 ± 0.85
$\Gamma_{\rho(1450)}$	492.17 ± 138.38	340.87 ± 23.84	151.30 ± 140.42
$\Gamma_{\rho(1700)}$	489.58 ± 16.95	414.71 ± 119.48	74.87 ± 120.67

Table 2: U&A model values of fitting parameters for the masses and decay widths of ρ^0 (for e^+e^- data) and $\rho^\pm$ (for τ^--lepton decay data) mesons and their excited states.

The experimentally observed ρ–ω meson interference effect was included as the second term in the expression for the total cross section σ_{tot}

$$\sigma_{\text{tot}}(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2\beta_\pi^3}{3t}\left|F_\pi^{E,I=1}(t) + Re^{i\Phi}\frac{m_\omega^2}{m_\omega^2 - t - im_\omega\Gamma_\omega}\right|^2, \tag{2.3}$$

in the form of corrected parametrization by the Breit-Wigner formula where $\beta_\pi = (1-4m_\pi^2/t)^{1/2}$ and R is the ρ–ω interference amplitude (free parameter). The corresponding phase Φ is given through the ρ- and ω-meson parameters by the relation

$$\Phi = \arctan\frac{m_\rho\Gamma_\rho}{m_\rho^2 - m_\omega^2}. \tag{2.4}$$

The expression in Eq. (2.3) was used for the fitting procedure of the experimental points in the region $-9.77\,\text{GeV}^2 \le t \le 13.48\,\text{GeV}^2$. The most important from the experimentaly measured data are the accurate KLOE data [15] at the energy range $0.35\,\text{GeV}^2 \le t \le 0.95\,\text{GeV}^2$ obtained in Frascati by the radiative return method and also the corrected Novosibirsk CMD-2 data [11] at the range $0.36\,\text{GeV}^2 \le t \le 0.9409\,\text{GeV}^2$ and SND data [12] at the range $0.1521\,\text{GeV}^2 \le t \le 0.9409\,\text{GeV}^2$, which can influence the finite results substantially. They are supplemented at the interval $-9.77\,\text{GeV}^2 \le t \le 0.3364\,\text{GeV}^2$ and $0.9557\,\text{GeV}^2 \le t \le 13.48\,\text{GeV}^2$ by other existing data [16, 17, 18].

3. Weak pion FF

Conserved vector current hypothesis allows one to relate the isovector part of the weak spectral function $v_0^{I=1}(s)$ characterizing the $\pi^+\pi^-$ system (s is the invariant mass squared of such system) with the spectral function $v_-(s)$ of $\pi^-\pi^0$ system. Because there also exists the dependence of the spectral function and pion form factor, one can relate $F_\pi^{E,I=1}$ – EM pion FF with F_π^{W} – the weak pion FF through the relation (see [14])

$$F_\pi^{W} = \sqrt{2}F_\pi^{E,I=1}. \tag{3.1}$$

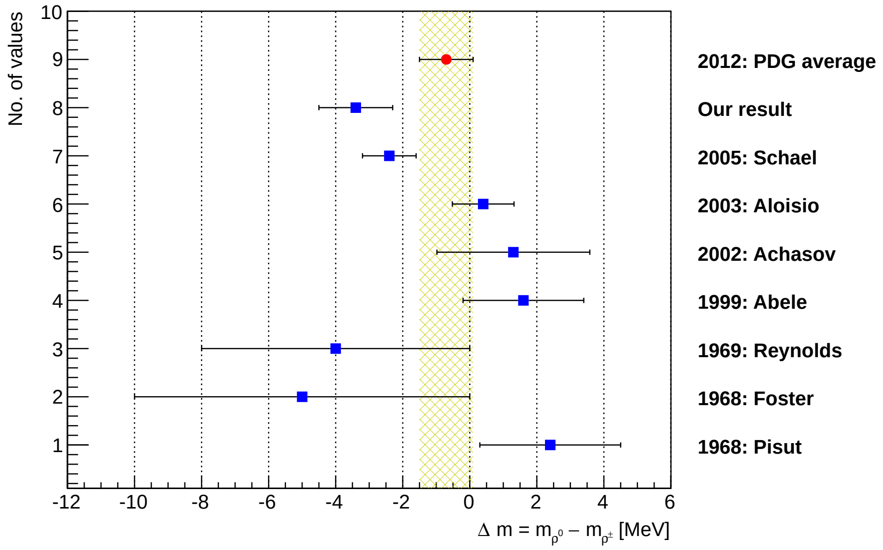

Figure 1: The comparisson of values for mass difference of neutral ρ^0 and charged $\rho^\pm$ mesons for the state $\rho(770)$.

In such a way it was possible to use the same U&A model of EM pion FF for the description of the weak pion FF data from τ^--lepton decay into two pions in the whole kinematic region in contrast to previous model dependent approaches. The fitting procedure was performed only with the high-statistics measurement [13] of the weak pion FF from $\tau^- \to \pi^-\pi^0\nu_\tau$ decay with the Belle detector at the KEK-B asymmetric-energy e^+e^- collider as they are charged by the lowest total errors in comparison with previous measurements done by ALEPH [9] and CLEO [19].

4. Conclusions

In contrast to previous approaches to determine the ρ-meson family parameters, we have applied new phenomenological approach based on $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^-\pi^0\nu_\tau$ data analysis by the advanced pion EM structure model in which ρ-mesons are considered as complex poles on unphysical sheets of the Riemann surface. The complex U&A models of EM and weak pion FF allow one to determine the masses and widths for neutral and charge rho mesons and their excitations precisly. Our result (see Fig. 1) for $m_{\rho^0} - m_{\rho^\pm} = -3.37 \pm 1.06\,\mathrm{MeV}$ is in the coincidence with the last published value [9].

Acknowledgments

This work was supported by the Slovak Grant Agency for Sciences VEGA, Grant No. 2/0158/13 and the Slovak Research and Development Agency, contract No. APVV-0463-12.

References

[1] J. Beringer et al., Phys. Rev. D86 (2012) 010001.

[2] A. Aloisio et al., Phys. Lett. B561 (2003) 55-60.

[3] A. Aloisio et al., Phys. Lett. B609 (2005) 449-450.

[4] J. Pisut and M. Roos, Nucl. Phys. B6 (1968) 325-352.

[5] B. G. Reynolds et al., Phys. Rev. 184 (1969) 1424-1442.

[6] M. Foster, P. Gavillet, G. Labrosse and L. Montanet, Nucl. Phys. B6 (1968) 107.

[7] A. Abele et al., Phys. Lett. B469 (1999) 270-275.

[8] M. N. Achasov et al., Phys. Rev. D65 (2002) 032002.

[9] S. Schael et al., Phys. Rept. 421 (2005) 191-284.

[10] R. R. Akhmetshin et al., Phys. Lett. B527 (2002) 161-172.

[11] R. R. Akhmetshin et al., Phys. Lett. B648 (2007) 28.

[12] M. N. Achasov et al., J. Exp. Theor. Phys. 103 (2006) 380.

[13] M. Fujikawa et al., Phys. Rev. D78 (2008) 072006.

[14] A. Z. Dubnickova et al., Czech. J. Phys. 43 (1993) 1057-1070.

[15] A. Aloisio et al. (KLOE), Phys. Lett. B606 (2005) 12.

[16] R. R. Akhmetshin et al., Phys. Lett. B578 (2004) 285.

[17] V. M. Aulchenko et al., JETP Lett. 82 (2005) 743.

[18] H. Blok et al., Phys. Rev. C78 (2008) 045202.

[19] S. Anderson et al., Phys. Rev. D61 (2000) 112002.

PoS(EPS-HEP2015)450