Non-Zero θ_{13} and δ_{CP} in a Neutrino Mass Model with A_4 Symmetry

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We consider a neutrino mass model based on A_4 symmetry. The spontaneous symmetry breaking in this model is chosen to obtain tribimaximal mixing in the neutrino sector. We introduce $Z_2 \times Z_2$ invariant perturbations in this model which can give rise to acceptable values of θ_{13} and δ_{CP}. Perturbation in the charged lepton sector alone can lead to viable values of θ_{13}, but cannot generate δ_{CP}. Perturbation in the neutrino sector alone can lead to acceptable θ_{13} and maximal CP violation. By adjusting the magnitudes of perturbations in both sectors, it is possible to obtain any value of δ_{CP}.

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1. Introduction

The tribimaximal form [1] of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix in the leptonic sector provides a close approximation to two of the three mixing angles. A number of attempts are made to obtain this form from discrete symmetries. The group of symmetric permutations, A_4, is a popular choice for the discrete symmetry because it has a unique triplet irreducible representation. The three families of fermions are usually assumed to transform as this triplet under A_4. The review [2] gives a good overview of the various models constructed on the basis of A_4. Here we consider a particular A_4 model [3] with the following interesting property. The fermion and the Higgs content of the model is chosen such that the tribimaximal (TBM) form of the PMNS matrix arises purely from the symmetry, without any dependence on the neutrino or charged lepton masses.

The TBM form of PMNS matrix constrains θ_{13} to be zero. This in turn implies that the CP violating phase δ_{CP} of the PMNS matrix also has to be zero. The measurement that θ_{13} is moderately large [4, 5, 6] means that the PMNS matrix is not of the exact TBM form. For $\theta_{13} \neq 0$, it is possible to consider non-zero values of δ_{CP}. One must consider perturbations to the A_4 symmetry which can lead to non-zero θ_{13} and δ_{CP}. Here we consider a restricted set of perturbations, which are invariant under $Z_2 \times Z_2$ symmetry. We further restrict the form of the perturbations so that they can be parametrized by one parameter each in the charged lepton and neutrino sectors. We search values of these parameters which can lead to viable values of θ_{13} and large δ_{CP}.

2. A_4 Symmetric Neutrino mass model

In this section, we briefly describe the model proposed in ref. [3]. The lepton and the scalar fields and their group charges are given in table 1. One can write down the most general $SU(2)_L \times U(1)_Y \times A_4$ invariant Yukawa Lagrangian for the leptonic sector in terms of the above fields [7]. An additional $U(1)_X$ symmetry is imposed on this Lagrangian [3] to forbid some unwanted neutrino mass terms which spoil the TBM form of the leptonic mixing matrix. TBM mixing requires a special vacuum alignment

$$v_1 = v_2 = v_3 = v,\ w_1 = w_3 = 0 \text{ and } h_\chi w_2 = M^{'}. \tag{2.1}$$

With these vacuum expectation values, the charged lepton mass matrix can be put in diagonal form by the transformation $U_\omega M_l^0 I$ and the Majorana mass matrix of the neutrinos is transformed to diagonal form by $U_\nu M_R U_\nu^\dagger$. The matrices U_ω and U_ν are given by

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad U_\nu = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{2.2}$$

The PMNS matrix $U = U_\omega U_\nu$ is in the tribimaximal form upto phases on both the sides.

3. $Z_2 \times Z_2$ invariant perturbations in the charged lepton sector

U_{e3} element of the tribimaximal form is zero because the 11 and 13 elements of U_ω are equal. By disturbing this equality we can get non-zero U_{e3}. A multiplicative factor in the i^{th} row of

	$SU(2)$	$U(1)$	A_4	
D_{iL}	$\underline{\frac{1}{2}}$	Y=-1	$\underline{3}$	left-handed doublets
l_{iR}	$\underline{0}$	Y=-2	$\underline{1}\oplus\underline{1}'\oplus\underline{1}''$	right-handed charged lepton singlets
ν_{iR}	$\underline{0}$	Y= 0	$\underline{3}$	right-handed neutrino singlets
ϕ_i	$\underline{\frac{1}{2}}$	Y= 1	$\underline{3}$	complex scalar SU(2) doublet
ϕ_0	$\underline{\frac{1}{2}}$	Y= 1	$\underline{1}$	complex scalar SU(2) doublet
χ_i	$\underline{0}$	Y= 0	$\underline{3}$	real scalar SU(2) singlet

Table 1: Assignments of lepton and scalar fields to various irreps of $SU(2)$, $U(1)$, and A_4.

the charged lepton mass matrix leads to reciprocal factor in the i^{th} column of U_ω. We introduce perturbations in the first and the third rows of the charged lepton mass matrix so that corresponding changes occur in the first and the third column of U_ω and its 11 and 13 elements will no longer be equal and U_{e3} will be non-zero. This can be done in a simple way by introducing $Z_2 \times Z_2$ perturbations in the charged lepton sector.

$Z_2 \times Z_2$ invariant perturbation, for the charged lepton mass matrix, is [8]

$$h_1 \bar{\mathrm{D}}_{\mathrm{L}} M_1 \phi l_{1\mathrm{R}} + h_2 \bar{\mathrm{D}}_{\mathrm{L}} M_2 \phi l_{2\mathrm{R}} + h_3 \bar{\mathrm{D}}_{\mathrm{L}} M_3 \phi l_{3\mathrm{R}}, \tag{3.1}$$

where the matrices M_1, M_2 and M_3 are diagonal. To keep the discussion simple, we choose a particular form of $M_i = \mathrm{diag}(\bar{\mathrm{z}}, 0, \omega^{\mathrm{i}-1}\mathrm{z})$, where all the matrix elements are parametrized by a single complex number z. This leads to the following perturbation in the charged lepton mass matrix:

$$\Delta M_l = \begin{pmatrix} h_1 v\bar{z} & h_2 v\bar{z} & h_3 v\bar{z} \\ 0 & 0 & 0 \\ h_1 vz & h_2 vz\omega & h_3 vz\omega^2 \end{pmatrix}. \tag{3.2}$$

Such a ΔM_l can arise from higher scale operators of the theory [3].

Requiring the diagonalizing matrix of $M_l^0 + \Delta M_l$ to be unitary leads to the constraint $z = -1 + \sqrt{1-s^2} + is$, where s is a small real number. Parametrizing $s = \sin\alpha$, we get the modified PMNS matrix to be

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\alpha} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{3.3}$$

We can obtain the following expressions for the modified mixing angles

$$\sin^2\theta_{13} = \frac{2}{3}\sin^2\alpha$$

$$\sin^2\theta_{12} = \frac{1}{2+\cos 2\alpha} \tag{3.4}$$

$$\sin^2\theta_{23} = \frac{2+\cos 2\alpha + \sqrt{3}\sin 2\alpha}{2(2+\cos 2\alpha)} \tag{3.5}$$

The variation of $\sin^2\theta_{13}$ with s and the variation of $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ with $\sin^2\theta_{13}$ are shown in figure 1. To obtain a good fit with the measured value of $\sin^2\theta_{13}$, we need the perturbation parameter $s \sim 0.2$. For this value of s, the predicted values of $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$ are just above the measured 1σ upper bounds.

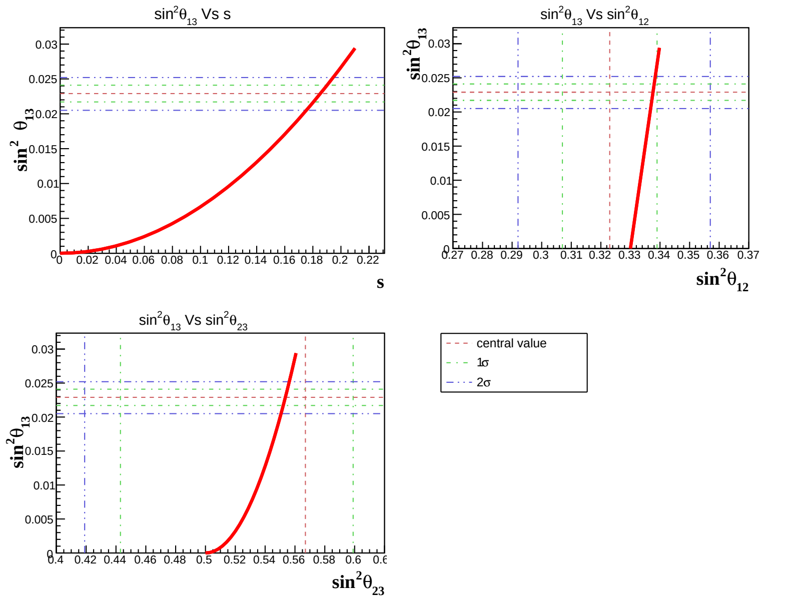

Figure 1: The plots of sine squared values of the modified mixing angles due to a $Z_2 \times Z_2$ invariant perturbation in the charged lepton sector. Lines demarcating the central values and the 1σ and 2σ allowed regions are shown explicitly.

4. $Z_2 \times Z_2$ perturbation in Neutrino Sector

Another reason for the vanishing of U_{e3} is because the 13 and 33 elements of U_ν are $\mp 1/\sqrt{2}$ respectively. This is due to the degeneracy of the 11 and 33 elements of the Majorana mass matrix M_R. If these diagonal elements of M_R are not degenerate, then the diagonalizing matrix is parametrized by an arbitrary mixing angle and U_{e3} will be non-zero. These perturbations, being diagonal, are again $Z_2 \times Z_2$ invariant. Since the diagonal entrees of the Majorana mass matrix are soft terms in the Lagrangian, these perturbations can also be introduced through the same method.

The perturbed Majorana mass matrix becomes

$$\begin{pmatrix} M+aM & 0 & M' \\ 0 & M & 0 \\ M' & 0 & M-aM \end{pmatrix}, \tag{4.1}$$

where a is a small real number characterizing the perturbation. This matrix can be diagonalized by a rotation matrix of angle x where $\tan 2x = M'/aM$. We define a dimensionless parameter $\zeta = aM/M'$. The modified PMNS matrix, under the combined perturbations in the charged lepton and neutrino sectors, is

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i\alpha} \end{pmatrix} \begin{pmatrix} \cos x & 0 & -\sin x \\ 0 & 1 & 0 \\ \sin x & 0 & \cos x \end{pmatrix}. \tag{4.2}$$

From the PMNS matrix, we obtain the expressions for the modified mixing angles due to perturbations in both charged lepton and neutrino sectors.

$$\begin{aligned}
\sin^2\theta_{13} &= \frac{1}{3}(1-\cos 2\alpha \sin 2x), \\
\sin^2\theta_{12} &= \frac{1}{2+\cos 2\alpha \sin 2x}, \\
\sin^2\theta_{23} &= \frac{2+\cos 2\alpha \sin 2x + \sqrt{3}\sin 2x \sin 2\alpha}{4+2\cos 2\alpha \sin 2x}.
\end{aligned} \tag{4.3}$$

The variation of $\sin^2\theta_{13}$ with ζ and the relation between $\sin^2\theta_{12}$ and $\sin^2\theta_{13}$ for various values of ζ are shown in figure 2. Note that $\sin^2\theta_{23}$ remains unperturbed at 0.5, for perturbations purely in the neutrino sector.

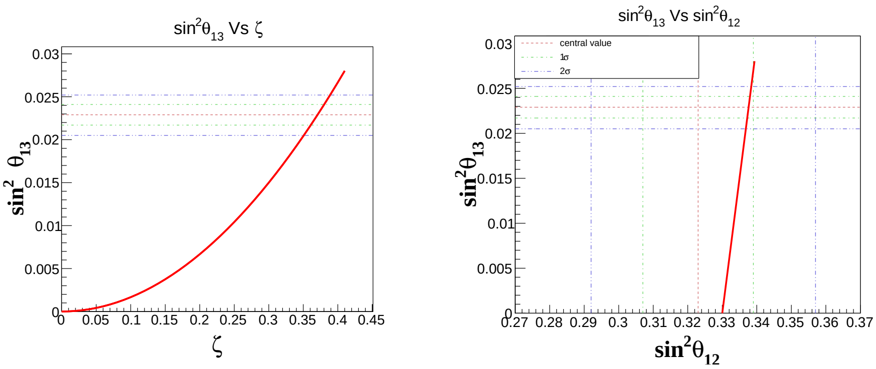

Figure 2: The plot of sine squared values of mixing angles for a $Z_2 \times Z_2$ invariant perturbation in the neutrino sector. The lines for the allowed 1σ and 2σ ranges are indicated in the plots.

The expression for $\sin\delta_{CP}$, as a function of these perturbations is

$$\sin\delta_{CP} = \frac{\cos 2x(2+\cos 2\alpha \sin 2x)}{\sqrt{(1-\cos^2 2\alpha \sin^2 2x)\left[4+4\cos 2\alpha \sin 2x + (-1+2\cos 4\alpha)\sin^2 2x\right]}}. \tag{4.4}$$

$\sin\delta_{CP}$ vanishes when ζ is zero (or $x=\pi/4$), *i.e.* when there is no perturbation in the neutrino sector. $\sin\delta_{CP} = \pm\pi/2$ when $s=0$, *i.e.* when the perturbation is only in the neutrino sector, we have **maximal** CP violation. For perturbation purely in neutrino sector, we need $\zeta \approx 0.4$ to obtain $\sin^2\theta_{13}$ consistent with experiment. Eventhough ζ seems a bit large, the perturbation parameter in the Majorana mass matrix $a = \zeta M'/M$ will be quite small because $M' \ll M$. Arbitrary values of $\sin\delta_{CP}$ can be obtained by choosing appropriate values for s and ζ, as shown in figure 3. We see that if $|\zeta| > 2s$, the value of $|\delta_{CP}| > \pi/4$ from this figure.

5. Conclusions

We considered a neutrino mass model with A_4 symmetry, in which the tribimaximal form for the PMNS matrix arises purely from the symmetry, without depending on the mass pattern in any way. We introduced $Z_2 \times Z_2$ perturbations in the charged lepton sector and in the neutrino sector. With perturbation only in the charged lepton sector, we obtained realistic values for θ_{13} and for the

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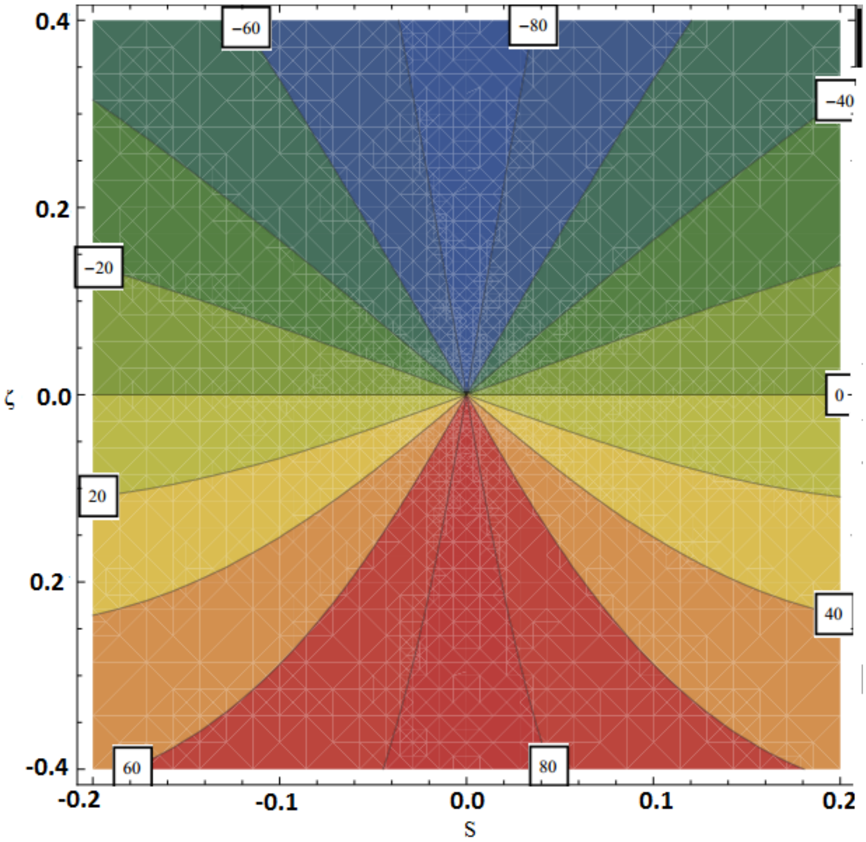

Figure 3: The value of δ_{CP} for different regions in the $s-\zeta$ space.

other mixing angles but no CP violation. With perturbation only in the neutrino sector, we obtained maximal CP violation along with realistic values for θ_{13} and the other mixing angles. Any desired value of δ_{CP} can be obtained by adjusting the perturbations in the charged lepton and the neutrino sectors.

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