Neutrino Masses and Mixings from Continuous Symmetries

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Flavour symmetries are fundamental tools in the search for an explanation to the flavour puzzle: fermion mass hierarchies, the neutrino mass ordering, the differences between the mixing matrices in the quark and lepton sector, can all find an explanation in models where the fermion generations undergo specific geometric relations. An overview on the implementation of continuous symmetries in the flavour sector is presented here, focussing on the lepton sector.

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1. Abelian models

Models based on the Abelian $U(1)$ group are sometimes preferred with respect to other approaches for a series of reasons. First of all, the Abelian $U(1)$ group is an element already present in the Standard Model (SM) and in many beyond SM (BSM) theories. Furthermore, it has been shown much time ago that the quark sector [1] is easily described in this context. In addition, the formulation of a model based on the $U(1)$ symmetry, in the supersymmetric context as the holomorphicity of the superpotential simplifies the construction of the Yukawa interactions, is simple and elegant:

i) The flavour symmetry acts horizontally on leptons and the charges can be written as $e^c \sim (n_1^R, n_2^R, 0)$ for the $SU(2)_L$ lepton singlets and as $\ell \sim (n_1^L, n_2^L, 0)$ for the $SU(2)_L$ lepton doublets. The third lepton charges can be set to zero as only charge differences have an impact on mass hierarchies and on mixing angles. Furthermore, it is not restrictive to assume $n_1^R > n_2^R > 0$ to fix the ordering of the charged leptons. The Higgs fields $H_{u,d}$ are not charged under $U(1)$ to prevent flavour-violating Higgs couplings.

ii) Once leptons have $U(1)$ charges, the Yukawa terms are no longer invariant under the action of the flavour symmetry. To formally recover the invariance, a new scalar field (or more than one in non-minimal models) can be introduced, the flavon θ, that transforms non-trivially only under $U(1)$, with charge n_θ. Then, the Yukawa Lagrangian can be written as

$$-\mathscr{L}_Y = (y_e)_{ij} \ell_i H_d e_j^c \left(\frac{\theta}{\Lambda}\right)^{p_e} + (y_\nu)_{ij} \frac{\ell_i \ell_j H_u H_u}{\Lambda_L} \left(\frac{\theta}{\Lambda}\right)^{p_\nu} + \text{H.c.} \tag{1.1}$$

where Λ is the cut-off of the effective flavour theory and Λ_L the scale of the lepton number violation, in principle distinct from Λ. $(y_e)_{ij}$ and $(y_\nu)_{ij}$ are free parameters: for naturalness, these parameters are taken to be complex and with modulus of order 1. p_e and p_ν are suitable powers of the dimensionless ratio θ/Λ necessary to compensate the $U(1)$ charges for each Yukawa term and therefore recover the invariance under the flavour symmetry. Without loss of generality, we can fix $n_\theta = -1$; consequently, $n_1, n_2 > 0$ to assure that the Lagrangian expansion makes sense. Here and in the following, neutrino masses are described by the effective Weinberg operator, while the extension to ultraviolet completions, such as See-Saw mechanisms, is straightforward (see i.e. Refs. [2–5]).

iii) Once the flavon and the Higgs fields develop non-vanishing vacuum expectation values (VEVs), the flavour and electroweak symmetries are broken and mass matrices arise from the Yukawa Lagrangian. In particular, the ratio of the flavon VEV $\langle\theta\rangle$ and the cut-off Λ of the effective theory defines the expanding parameter of the theory,

$$\varepsilon \equiv \frac{\langle\theta\rangle}{\Lambda} < 1. \tag{1.2}$$

A useful parametrisation for the Yukawa matrices then follows as

$$Y_e = F_{e^c} y_e F_\ell, \qquad Y_\nu = F_\ell y_\nu F_\ell, \tag{1.3}$$

where $F_f = \text{diag}(\varepsilon^{n_{f1}}, \varepsilon^{n_{f2}}, \varepsilon^{n_{f3}})$. Following Ref. [5, 6], the charges will be taken to be integers, since non-integer charges can always be redefined to integers as long as it is accompanied by a suitable redefinition of the parameter ε.

1.1 Specific $U(1)$ models

In the following, focussing on constructions where neutrino masses are described by the Weinberg operator, two specific models will be considered: the model A representative of anarchical constructions and the model H representative of hierarchical ones. The first – see Tab. 1 – encodes the idea that an even structureless mass matrix can lead to a correct description of neutrino data: this mass matrix is characterised by entries that are random numbers which, under the additional requirement of basis invariance, leads to a unique measure of the mixing matrix – the Haar measure [7–11]. It has been claimed that such matrix generically prefers large mixings [7–9] and that the observed sizable deviation from a zero reactor angle seems to favour anarchical models when compared to other more symmetric constructions [10]. However, as discussed in Ref. [12], how much a large value of a parameter is preferred can depend strongly on the definition of "preferred" and of "large".

It has been suggested in Ref. [5, 6] that the performances of anarchical models, formulated in a $U(1)$ context giving no charges to the left-handed fields, in reproducing the 2012 neutrino data are worse than those of models constructed upon the $U(1)$ flavour symmetry. In the latter ones, the small neutrino parameters are due to the built-in hierarchies and not due to chance. The construction H – see Tab. 1 – considered in Ref. [6] has been shown by mean of the Bayesian inference to be the best one to describe the data, among all the possible $U(1)$ models.

Model	e_R	ℓ_L
Anarchy (A)	(3,1,0)	(0,0,0)
Hierarchy (H)	(8,3,0)	(2,1,0)

Table 1: *The flavon charge is -1, while the Higgs charge is zero. (A slightly different notation as been adopted with respect to Ref. [6].)*

The textures for the charged leptons Y_e and neutrino Y_ν Yukawa matrices are as follows:

$$A : \left\{ Y_e = \begin{pmatrix} \varepsilon^3 & \varepsilon & 1 \\ \varepsilon^3 & \varepsilon & 1 \\ \varepsilon^3 & \varepsilon & 1 \end{pmatrix}, \, Y_\nu = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} ; \quad H : \left\{ Y_e = \begin{pmatrix} \varepsilon^{10} & \varepsilon^6 & \varepsilon^2 \\ \varepsilon^9 & \varepsilon^5 & \varepsilon \\ \varepsilon^8 & \varepsilon^4 & 1 \end{pmatrix}, \, Y_\nu = \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon^2 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix} . \right. \right.$$

$$(1.4)$$

In the spirit of $U(1)$ models, the coefficients in front of ε^n are expected to be complex numbers with absolute values of $\mathcal{O}(1)$ and arbitrary phases. As Y_ν is a symmetric matrix, the total number of parameters that should be considered in the analysis is 30, from the Yukawa matrices, plus the unknown value of ε.

While for the details of the analysis we refer to the original publication Ref. [6], here we just comment on the results of the comparison between A and H. Both the models have a χ^2-minimum of zero and therefore a χ^2-analysis can never exclude any of the models or be meaningful to compare them. On the other side, a Bayesian analysis allows instead a quantitative comparison of the models: the ratio between the logarithms of the evidences of H normalised to the evidence

of A, i.e., the Bayes factor between H and A, is given by

$$\log B \simeq 3 \div 4.5\,, \qquad (1.5)$$

depending on the prior on ε. The uncertainty on the logarithms of the Bayes factors is about 0.2. Accordingly to the Jeffreys scale, these values translate in a moderate evidence in favour of H with respect to A.

2. Non-Abelian models

One of the main problematics of dealing with Abelian symmetries is the fact that the three fermion generations are independent from each other, translating in the large number of free parameters. On the other hand, in the context of non-Abelian symmetries, when fermions transform with multidimensional representations, the three families are connected one to the others, reducing the number of free parameters and therefore increasing the predictivity of a model.

Non-Abelian continuous symmetries have also been deeply investigated in the flavour sector, but mainly connected to the Minimal Flavour Violation (MFV) [13, 14] ansatz: i.e. the requirement that all sources of flavour violation in the SM and BSM are described at low-energies uniquely in terms of the known fermion masses and mixings. Several distinct models formulated in this framework [15–35] turn out to be consistent with a new physics scale at the TeV, while comparable models without the MFV hypothesis are forced to have a scale larger than hundreds of TeV [36].

The power of MFV descends from the fact that it exploits the symmetries that the SM itself contains in a certain limit: that of massless fermions. For example, in the case of the Type I Seesaw mechanism with three RH neutrinos added to the SM spectrum, the flavour symmetry of the full Lagrangian, when Yukawa couplings and the RH neutrino masses are set to zero, is:

$$G_f = G_f^q \times G_f^\ell \qquad \begin{cases} G_f^q = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R} \\ G_f^\ell = U(3)_{\ell_L} \times U(3)_{E_R} \times U(3)_N \end{cases} . \qquad (2.1)$$

Under the flavour symmetry group G_f fermion fields transform as

$$Q_L \sim (3,1,1)_{G_f^q}\,, \qquad U_R \sim (1,3,1)_{G_f^q}\,, \qquad D_R \sim (1,1,3)_{G_f^q}\,,$$
$$\ell_L \sim (3,1,1)_{G_f^\ell}\,, \qquad E_R \sim (1,3,1)_{G_f^\ell}\,, \qquad N_R \sim (1,1,3)_{G_f^\ell}\,. \qquad (2.2)$$

The Yukawa Lagrangian for the Type I Seesaw mechanism, then, reads:

$$-\mathscr{L}_Y = \overline{Q}_L Y_D H D_R + \overline{Q}_L Y_U \tilde{H} U_R + \bar{\ell}_L Y_E H E_R + \bar{\ell}_L Y_\nu \tilde{H} N_R + \overline{N}_R^c \frac{M_N}{2} N_R + \text{h.c.} \qquad (2.3)$$

To introduce $\mathscr{L}_Y$ without explicitly breaking G_f, the Yukawa matrices Y_i and the mass matrix for the RH neutrinos M_N have to be promoted to be spurion fields transforming under the flavour symmetry as:

$$Y_U \sim (3,\bar{3},1)_{G_f^q}\,, \qquad Y_D \sim (3,1,\bar{3})_{G_f^q}\,,$$
$$Y_E \sim (3,\bar{3},1)_{G_f^\ell}\,, \qquad Y_\nu \sim (3,1,\bar{3})_{G_f^\ell}\,, \qquad M_N \sim (1,1,\bar{6})_{G_f^\ell}\,. \qquad (2.4)$$

The quark masses and mixings are correctly reproduced once the quark spurion Yukawas get background values as

$$Y_U = V^\dagger \mathbf{y}_U, \qquad\qquad Y_D = \mathbf{y}_D, \tag{2.5}$$

where $\mathbf{y}_{U,D}$ are diagonal matrices with Yukawa eigenvalues as diagonal entries, and V a unitary matrix that in good approximation coincides with the CKM matrix.

When discussing the MFV ansatz in the leptonic sector one has at disposal three different spurions, as can be evinced from the list in Eq. (2.4). The number of parameters that can be introduced in the model through these spurions is much larger than the low energy observables. This in general prevents a direct link among neutrino parameters and FV observables. The usual way adopted in the literature to lower the number of parameters consists in reducing the number of spurions from three to two: for example Ref. [16] takes $M_N \propto \mathbb{1}$; in Ref. [20], a two-family RH neutrino model is considered with $M_N \propto \sigma_1$; finally in Ref. [24] $Y_\nu^\dagger Y_\nu \propto \mathbb{1}$ is assumed.

An unifying description for all these models can be obtained by introducing the Casas-Ibarra parametrisation [37]: in the basis of diagonal mass matrices for RH neutrinos, LH neutrinos and charged leptons, the neutrino Yukawa coupling can be written as

$$Y_\nu = \frac{1}{v} U \sqrt{\hat{m}_\nu} R \sqrt{\hat{M}_N}, \tag{2.6}$$

where v is the electroweak vev, the hatted matrices are light and heavy neutrino diagonal mass matrices, U refers to the PMNS mixing matrix and R is a complex orthogonal matrix, $R^T R = \mathbb{1}$. A correct description of lepton masses and mixings is achieved assuming that Y_E acquires a background value parametrised by a diagonal matrix,

$$Y_E = \mathbf{y}_E \equiv \text{diag}(y_e, y_\mu, y_\tau), \tag{2.7}$$

while the remaining spurion, M_N or Y_ν, accounts for the neutrino masses and the PMNS matrix (see Refs. [16, 20, 24]).

2.1 Dynamical Yukawas

Despite of the phenomenological success, it has to be noticed that, however, MFV does not provide by itself any explanation of the origin of fermion masses and/or mixing, or equivalently does not provide any explanation for the background values of the Yukawa spurions. This observation motivates the studies performed in Refs. [23, 31, 34, 35], where the Yukawa spurions are promoted to dynamical scalar fields: the case in which a one-to-one correlation among Yukawa couplings and fields is assumed, $Y_i \equiv \langle \mathbf{Y}_i \rangle / \Lambda_f$, is discussed at length. The scalar potential constructed out of these fields was studied in Refs. [23, 31, 34, 35], considering renormalisable operators (and adding also lower-order non-renormalisable terms for the quark case): these effective Lagrangian expansions are possible under the assumption that the ratio of the flavon vevs and the cutoff scale of the theory is smaller than 1, condition that is always satisfied but for the top Yukawa coupling. In this case a non-linear description would be more suitable.

We focus here only on the lepton sector, while for the quark sector we refer to the original article in Ref. [23].

2.1.1 Two-family case

It is instructive and interesting to start with a toy model with only two generations [31]. Under the assumption of degenerate RH neutrino masses, $M_1 = M_2 \equiv M$, only the spurion fields Y_E and Y_ν are promoted to dynamical fields, $\mathbf{Y}_E$ and $\mathbf{Y}_\nu$, and the flavour symmetry in this case is

$$G_f^\ell = U(2)_{\ell_L} \times U(2)_{E_R} \times O(2)_N. \tag{2.8}$$

Only five independent invariants can be obtained at the renormalisable level:

$$\text{Tr}\left[\mathbf{Y}_E \mathbf{Y}_E^\dagger\right], \qquad \text{Tr}\left[\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger\right], \qquad \text{Tr}\left[\left(\mathbf{Y}_E \mathbf{Y}_E^\dagger\right)^2\right], \qquad \text{Tr}\left[\left(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger\right)^2\right], \qquad \text{Tr}\left[\left(\mathbf{Y}_\nu \sigma_2 \mathbf{Y}_\nu^\dagger\right)^2\right]. \tag{2.9}$$

All the terms account for the lepton masses, but the last one that fixes the mixing angle. By adopting the Casas-Ibarra parametrisation in Eq. (2.6) and minimising the scalar potential with respect to the angle θ and the Majorana phase α, the following two conditions result:

$$(y^2 - y'^2)\sqrt{m_{\nu_2} m_{\nu_1}} \sin 2\theta \cos 2\alpha = 0, \qquad \text{tg} 2\theta = \sin 2\alpha \frac{y^2 - y'^2}{y^2 + y'^2} \frac{2\sqrt{m_{\nu_2} m_{\nu_1}}}{m_{\nu_2} - m_{\nu_1}}, \tag{2.10}$$

where y and y' are two parameters of Y_ν (see Ref. [31] for details). The first condition implies a maximal Majorana phase,

$$\alpha = \pi/4 \qquad \text{or} \qquad \alpha = 3\pi/4, \tag{2.11}$$

for a non-trivial mixing angle. However, this does not imply observability of CP violation at experiments, as the relative Majorana phase among the two neutrino eigenvalues is $\pi/2$. The second condition above represents a link among the size of mixing angle and the type of the neutrino spectrum: a large mixing angle is obtained from almost degenerate masses, while a small angle follows in the hierarchical case. It is, however, necessary to discuss the full minimisation of the scalar potential in order to identify the angle configuration corresponding to the absolute minimum. In Ref. [31] it was shown that degenerate neutrino masses are a good minimum of the scalar potential and therefore one can conclude that the maximal angle solution is indeed a good minimum.

2.1.2 Generalisation to the three-family case

Moving to the realistic scenario of three families of charged leptons and light neutrinos, it is possible to consider either two or three RH neutrinos (as one of the light neutrino can be massless). In the former case, the interesting case where G_f^ℓ accounts for the factor $O(2)_N$ is not satisfactory anymore, as the large angle would necessarily arise in the solar sector (only degenerate masses in tho case) and would lie in the wrong quadrant (see Ref. [31] for further details). It is then necessary to move to the three RH neutrino case, where N_R (N_R') is a doublet (singlet) of $O(2)_N$. Correspondingly, the neutrino Yukawa field accounts for two components: a doublet and a singlet of $O(2)_N$,

$$\mathbf{Y}_\nu \sim (3, 1, \bar{2}), \qquad \mathbf{Y}_\nu' \sim (3, 1, 1). \tag{2.12}$$

The leptonic flavour Lagrangian is given in this case by

$$-\mathcal{L}_Y = \bar{\ell}_L Y_E H E_R + \bar{\ell}_L Y_\nu' \tilde{H} N_R' + \bar{\ell}_L Y_\nu \tilde{H} N_R + \frac{M'}{2} \overline{N}_R'^c N_R' + \frac{M}{2} \overline{N}_R^c \mathbb{1} N_R + \text{h.c.}. \tag{2.13}$$

Once the Yukawa flavons develop vevs, the light neutrino mass matrix is generated:

$$m_\nu = \frac{v^2}{M'} Y'_\nu Y'^T_\nu + \frac{v^2}{M} Y_\nu Y^T_\nu \,. \tag{2.14}$$

A total of nine independent invariants at the renormalisable level can be constructed in this case, namely

$$\mathrm{Tr}\left[\mathbf{Y}_E \mathbf{Y}_E^\dagger\right], \qquad \mathrm{Tr}\left[\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger\right], \qquad \mathbf{Y}_\nu'^\dagger \mathbf{Y}_\nu', \qquad \mathrm{Tr}\left[\left(\mathbf{Y}_E \mathbf{Y}_E^\dagger\right)^2\right], \qquad \mathrm{Tr}\left[\left(\mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger\right)^2\right],$$

$$\mathrm{Tr}\left[\mathbf{Y}_E \mathbf{Y}_E^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger\right], \qquad \mathrm{Tr}\left[\mathbf{Y}_\nu \mathbf{Y}_\nu^T \mathbf{Y}_\nu^* \mathbf{Y}_\nu^\dagger\right], \qquad \mathbf{Y}_\nu'^\dagger \mathbf{Y}_E \mathbf{Y}_E^\dagger \mathbf{Y}_\nu', \qquad \mathbf{Y}_\nu'^\dagger \mathbf{Y}_\nu \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu' \,. \tag{2.15}$$

When considering the minimisation of the scalar potential, there are four analytical solutions:

$$1)\begin{cases} \tan 2\theta_{12} = z/z' \\ m_{\nu_1} \neq m_{\nu_2} \\ m_{\nu_3} = 0 \end{cases} \quad 2)\begin{cases} \theta_{12} = \pi/4 \\ m_{\nu_1} = m_{\nu_2} \neq m_{\nu_3} \\ \alpha = \pi/4 \end{cases} \quad 3)\begin{cases} \theta_{23} = \pi/4 \\ m_{\nu_1} \neq m_{\nu_2} = m_{\nu_3} \\ \alpha = \pi/4 \end{cases} \quad 4)\begin{cases} \tan 2\theta_{23} = z/z' \\ m_{\nu_2} \neq m_{\nu_3} \\ m_{\nu_1} = 0 \end{cases} \tag{2.16}$$

Case 1 (4) describes an inverse (direct) hierarchical spectrum and only one sizable mixing angle, the solar (atmospheric) one. In case 2, the light neutrinos ν_1 and ν_2 are degenerate and both mass orderings (hierarchical or degenerate) can be accommodated, while a maximal solar angle is predicted. Finally, case 3 corresponds to degenerate ν_2 and ν_3: a realistic scenario points to three almost degenerate neutrinos. Note that cases 2 and 3 encompass two degenerate neutrinos and the relative Majorana phase between the two degenerate states is $\pi/2$.

Cases 1-4 only account for one sizable angle. Realistic configurations with three non-trivial angles, however, follow in a straightforward way when interpolating between these four cases, at the prize of non-exact solutions that depend on the parameters of the of the scalar potential. The setup appears very promising, though, as all three angles can be naturally non-vanishing and moreover the number of free parameters is smaller than the number of observables, leading to predictive scenarios in which mixing angles and Majorana phases are linked to the spectrum.

It is finally interesting to consider the case with three degenerate RH neutrinos [34, 35]. The flavour symmetry is

$$G_f = U(3)_{\ell_L} \times U(3)_{E_R} \times O(3)_N \,, \tag{2.17}$$

and the basis of invariants is composed of the operators in Eq. (2.9). The study of the extrema of these invariants has been presented in Ref. [35] and from the minimisation of the potential it follows an interesting configuration:

$$\theta_{23} = \pi/4 \,, \qquad m_{\nu_1} \neq m_{\nu_2} = m_{\nu_3} \,, \qquad \alpha = \pi/4 \,. \tag{2.18}$$

In the normal or inverse hierarchical case, two of the light neutrinos are degenerate in mass and a maximal angle and a maximal Majorana phase arise in their corresponding sector. On the other hand, if the third light neutrino is almost degenerate with the other two, then the perturbations split the spectrum and a second sizable angle arises [35].

In summary, these results indicate that a realistic solution for the Flavour Puzzle in the lepton sector requires three RH neutrinos, two of which must be degenerate. All three light neutrinos would therefore acquire masses, and the precise values of the mixing angles and Majorana phases are related to the specific light mass spectrum, result that is almost exclusively a feature of continuous non-Abelian symmetries.

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