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Radiative corrections to polarization observables of elastic electron-proton scattering

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We consider radiative corrections to polarization observables in elastic electron-proton scattering, in particular, for the polarization transfer measurements of the proton form factor ratio $R = \mu G_E/G_M$. The corrections are of two types: two-photon exchange (TPE) and bremsstrahlung (BS). TPE corrections are calculated within dispersion approach taking into account elastic and inelastic parts. The elastic part includes pure nucleon intermediate state only. The inelastic part is saturated by πN intermediate states. The advantages of this approach w.r.t. considering contributions of resonances are (i) automatically having correct resonance width, (ii) automatically having correct resonance shape, (iii) including not only resonances but background as well. Among different πN states we concentrate on the P_{33} channel (with quantum numbers of $\Delta(1232)$ resonance). BS corrections are calculated assuming small missing energy or missing mass cut-off. It was shown that such correction can be represented in a model-independent form, with both electron and proton radiation taken into account. Numerical calculations show that the contribution of the proton radiation is not negligible. Overall, at high Q^2 and energies the total correction to R grows, but is dominated by TPE. At low energies both TPE and BS may be significant; the latter amounts to ~ 0.01 for some reasonable cut-off choices.

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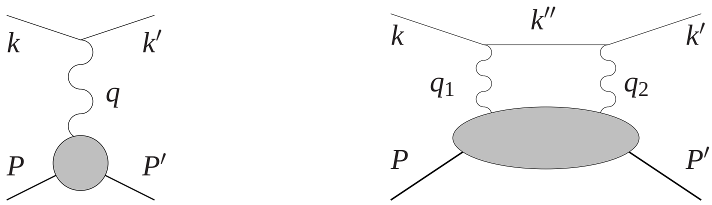

Figure 1: One-photon exchange (left) and two-photon exchange (right) diagrams.

The experimental study of the electron-nucleon scattering gives important information about the electromagnetic structure of the nucleon. Experimental data for the elastic scattering are usually expressed in terms of two fundamental observables, the electric and magnetic form factors (FFs), G_E and G_M, which parametrize the γNN vertex with two on-mass-shell nucleons

$$\Gamma_\mu = \gamma_\mu F_1(q^2) - [\gamma_\mu, \gamma_\nu]\frac{q^\nu}{4M} F_2(q^2), \tag{1}$$
$$G_E(q^2) = F_1(q^2) - \tau F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2), \qquad \tau = -q^2/4M^2,$$

where M is the nucleon mass.

At small $Q^2 = -q^2$ the form factors G_E and G_M are related to so-called electric and magnetic radii of the nucleon. At high Q^2 they give information about quark structure of a nucleon. Knowledge of nucleon form factors is also needed for understanding electromagnetic structure of more complicated hadron systems, for example, the deuteron, ^{3}He, ^{3}H, ^{4}He, etc.

The differential cross section and double polarization observables are simply connected with FFs in the framework of one photon exchange (or the Born approximation), Fig. 1, left. Nevertheless, the precision level of present-day electron-proton scattering experiments makes it necessary to take into account effects beyond Born approximation [1], which are usually called radiative corrections. They are of two types: two-photon exchange (TPE, see Fig. 1, right) and bremsstrahlung (BS). Here we will consider contribution of such corrections to the double polarization observables.

First let us consider TPE. There are two mainline approaches to the theoretical evaluation of the TPE amplitude: "quark" and "hadronic" ones. In the "quark" approach, as its name suggests, the nucleon is viewed as an ensemble of quarks (partons), interacting according to QCD [2, 3, 4, 5]. Naturally, the applicability of this approach is limited to the high-Q^2 region. Despite all its advantages, the serious drawback is that it is hard to calculate the TPE correction to the electric form factor G_E in this approach, while this is surely needed for the correct interpretation of G_E/G_M measurements.

In the "hadronic" approach TPE is mediated by the production of virtual hadrons and/or hadronic resonances. The TPE amplitudes are broken into different contributions according to the intermediate state involved. The most important and well-established one is the elastic contribution, which corresponds to pure nucleon intermediate state. In turn, all other contributions are called inelastic. Among them, the contributions of some prominent resonances [$\Delta(1232)$ and others] were studied in Refs. [6, 7, 8]. In Refs. [6, 7] it was shown that their overall effect *on the cross-section* is smaller than that of the elastic contribution, with $\Delta(1232)$ yielding its main part and the contributions of other resonances partially cancelling each other.

Later, it was found [8] that $\Delta(1232)$ yields relatively large correction to the G_E/G_M form factor ratio at high Q^2 (far exceeding that of the elastic intermediate state), and that the correction grows with Q^2. This result suggests that the contributions of other inelastic states may also be important and at least should be estimated carefully. Unfortunately, all the above-mentioned papers use "zero-width" approximation, i.e. widths of resonances are assumed to be negligibly small. This approximation seems rather crude, especially for $\Delta(1232)$, since its width ($\Gamma_\Delta \sim 110$ MeV) is comparable to the distance from the threshold ($M_\Delta - M - m_\pi \sim 160$ MeV).

To overcome this issue, in Ref. [9] the inelastic contribution to the TPE amplitude was estimated from the πN (pion+nucleon) intermediate states. This may be viewed as a significant improvement of the previous "resonance" calculations, since most resonances have dominant πN content. Consequently, the advantages of our approach are

- automatically having correct resonance width
- automatically having correct resonance shape
- including not only resonances but background as well

The πN contribution may further be split into the contributions of different partial waves of the πN system. Though, in principle, all partial waves may be taken into account in our method, it is particularly useful for the P_{33} channel, where Δ resides. The Δ resonance has almost 100% πN content, thus we will get pure improvement w.r.t. previous works. The situation is not so simple for other resonances, such as S_{11} and D_{13}, since they have significant $\pi\pi N$ branching ratio; the corresponding contribution will be missing in the present approach. Later on we will be consider the contribution of the P_{33} channel only, following Ref. [9].

The idea of the calculation is the following. The πN system is fully described by its isospin, spin-parity, and invariant mass. No other internal quantum numbers exist. Thus, with respect to the calculation of the TPE amplitudes, the πN system in the intermediate state is fully equivalent to the single particle with the same isospin, spin-parity and mass (and properly defined transition amplitudes). If we are able to calculate the TPE contribution of the resonance with given quantum numbers, we can do precisely the same thing for the πN system of fixed invariant mass and then integrate over invariant masses. The full contribution of the πN partial wave with the same quantum numbers will be

$$\delta\mathscr{G}^{\pi N} = \int \delta\mathscr{G}[W, A^{\pi N}(q^2, W)]\, dW^2, \tag{2}$$

where the integration variable W is the invariant mass of the πN system and $A^{\pi N}$ is appropriately defined transition form factor. Note that here q^2 stands for the square of virtual photon momentum and is not the same as the total momentum transfer in the elastic process.

In calculations the transition form factors $A^{\pi N}$ were taken from the unitary isobar model MAID2007 [10]; the numerical values were downloaded from the dedicated website [11].

As usual, TPE are described by three invariant amplitudes (generalized form factors) $\delta\mathscr{G}_E$, $\delta\mathscr{G}_M$, and $\delta\mathscr{G}_3$. The corrections to the cross-section or polarization observables can be expressed in terms of these amplitudes; for all relevant formulae see Refs. [8, 12].

The TPE amplitudes in the resonance region are shown in Fig. 2. Just as it was expected, there are smooth bumps at the resonance position, instead of the sharp peaks, which are seen in the zero-width approximation [8] (dashed lines). In Fig. 3 we plot the TPE correction to the polarization

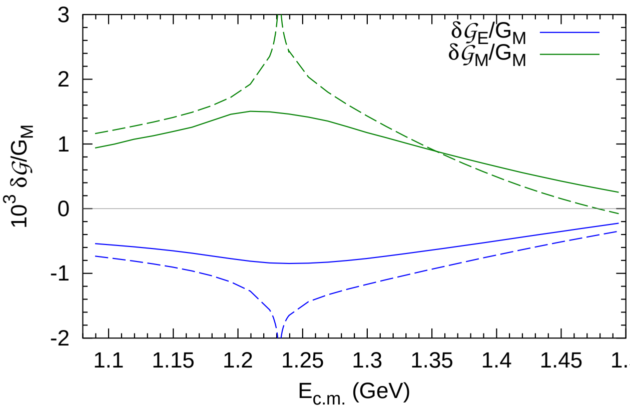

Figure 2: The TPE amplitudes near the Δ resonance, $\theta_{\text{c.m.}} = 90^\circ$, πN contribution from Ref. [9] (solid), zero-width Δ [8] (dashed).

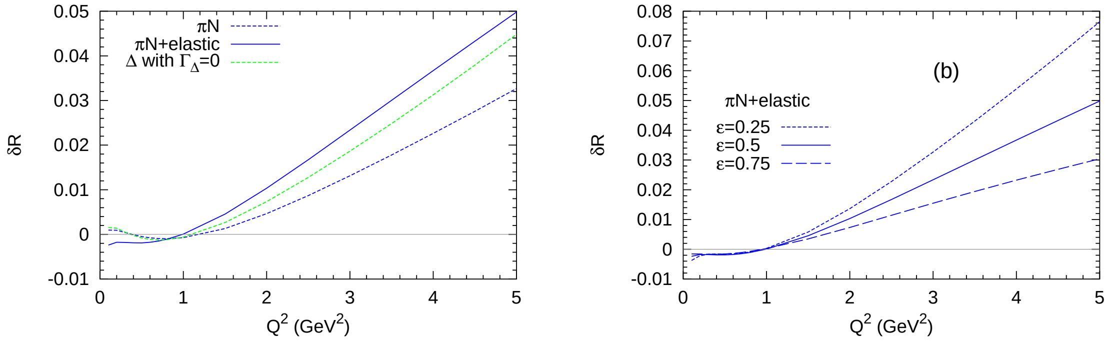

Figure 3: The TPE correction to the proton form factor ratio $R = \mu G_E/G_M$, as measured in polarization experiments, various contributions at fixed $\varepsilon = 0.5$ (left) and total at different values of ε (right). Here ε is the virtual photon polarization parameter $\varepsilon = [1+2(1+Q^2/4M^2)\text{tg}^2\,(\theta/2)]^{-1}$, where θ is lab. scattering angle.

ratio. At high Q^2 we see the same behaviour which was found in Ref. [8], namely the correction grows rapidly with Q^2.

Numerically we obtain the following results:

- at small Q^2 this contribution is small (negligible w.r.t. the elastic one)
- the TPE amplitudes have smooth maxima at the resonance position ($E_{\text{c.m.}} \approx M_\Delta$)
- at high Q^2 we confirm the findings of Ref. [8], obtained with the zero-width Δ. The main correction comes to the generalized electric form factor. This correction (and, consequently, the correction to the polarization ratio) is relatively large and grows with Q^2. Its numerical value is somewhat smaller than in Ref. [8]

We see, that (contrary to the common belief) the TPE corrections to the polarization ratio are not negligible at high Q^2.

Now let us discuss the BS corrections. The corrections coming from the radiation by the electrons were considered in an exact model-independent calculations in Refs. [13, 14]. The neglection of the proton radiation seems well-justified at low momentum transfer, when the proton remains practically at rest. However, at typical experimental conditions in JLab [15] the final proton is relativistic, thus the electron and the proton are on an equal footing and their contributions to the BS should be of the same order of magnitude.

Certainly exact analytical and model-independent calculation of the proton radiation is impossible (still this is not needed for practical applications). However, we are able to obtain the result of such sort after the expansion in powers of photon energy, in the first non-vanishing order. Such program was performed in Ref. [16].

In Ref. [16] a simple idealized experiment was considered, in which the final proton is detected in a fixed direction, that is, the angular acceptance of the proton detector is very small. Both electron and proton energies are measured to determine missing energy ΔE, and the event is counted as the elastic one if $\Delta E < r_m$, where r_m is some cut-off. This is the way the elastic events were selected in the real experiments [15]. Authors of Refs. [13, 14] use a cut on the missing mass, which was not applied in Ref. [15]. This case is also considered and compared with the "missing energy" approach in Ref. [16].

To reduce inelastic background, one must choose reasonably small r_m. For example, to exclude pion production, r_m should be restricted by $r_m < m_\pi \approx 140$ MeV. Therefore we have a small parameter r_m or, more precisely, r_m/M. The radiative correction was calculated in the first non-vanishing order in r_m. To this order, the low-energy theorem [17] allows us to obtain a model-independent result in the sense that it is expressed solely through on-shell proton FFs and their derivatives.

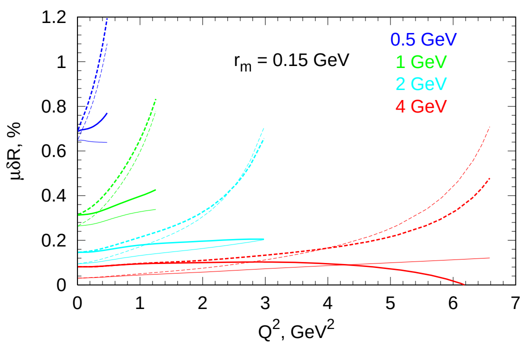

Figure 4: Bremsstrahlung correction to $\mu G_E/G_M$ ratio vs. Q^2 at different beam energies, as labelled on the plot. Solid — missing energy cut-off, dashed — missing mass cut-off; thick — full radiation, thin — electron only.

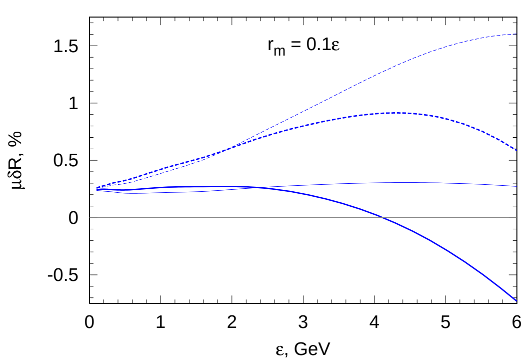

Figure 5: Bremsstrahlung correction to $\mu G_E/G_M$ ratio vs. beam energy at fixed scattering angle 90^o. Curve types are the same as in Fig. 4.

In all numerical calculations we use proton FF parameterization by Arrington *et al.* [18]. Everywhere below ε is initial electron energy (not to be confused with virtual photon polarization parameter).

Fig. 4 displays the BS correction to G_E/G_M ratio, as measured via polarization transfer, at four different beam energies. The missing energy cut-off is $r_m = 0.15$ GeV. Since in our approximation the BS correction is proportional to r_m, the transition to another r_m value is straightforward. The quantity shown in the figure is $\mu\,\delta R = \delta R(Q^2)/R(Q^2=0)$. It is more convenient to plot than the relative correction $\delta R/R$, since R approaches zero at $Q^2 \sim 7$ GeV^2; therefore the relative correction strongly grows, even while δR itself does not. The dashed curves are obtained in the "missing mass" approach with the cut-off $(p'+r)^2 - M^2 \leq 2Mr_m = u_m$. Thick curves results from the full calculation thin ones — including electron radiation only.

The energy dependence of the BS correction at fixed lab. scattering angle 90° is shown in Fig. 5. Here the missing energy cut-off is taken proportional to the incident electron energy: $r_m = 0.1\varepsilon$. The meaning of different curve types is the same as in Fig. 4. All four curves become close at $\varepsilon \to 0$; this is clear, since at $\varepsilon \to 0$ the final proton remains practically at rest ($p' \approx p$) and thus does not radiate. At $\varepsilon \gtrsim M$ full and "electron only" calculations give very different results, as expected.

In summary, our calculation shows that:

1. The proton radiation yields a significant part of the BS correction at $\varepsilon \gtrsim M$ in both "missing energy" and "missing mass" approaches.

2. In the "missing mass" approach the correction strongly grows at large angles, whereas in the "missing energy" approach it does not.

3. The BS correction is small at high energies ($\varepsilon \gtrsim M$), where the TPE correction is much larger. However there is no final reliable estimate of the TPE amplitude in this region; this is an important open problem. The significance of the BS correction at low energies depends on experimental details; thus it should be checked separately for each case.

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