

## Self-similarity of Proton Spin

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Origin of spin is one of fundamental mysteries of nature. Spin of proton itself and of point-like quarks is equal to  $1/2\hbar$ . Composition of the proton spin from spins of quarks, gluons and processes including formation and annihilation of the quark-antiquark pairs indicate on a repeated pattern in the internal spin structure over a wide scale range. New hypothesis of self-similarity of the proton spin composition is suggested and discussed. The concept of  $z$ -scaling previously developed for analysis of inclusive reactions is applied for description of processes with polarized particles. Generalized characteristics of proton spin structure, namely the spin-dependent fractal dimensions, are suggested. Possibilities to extract information on these quantities from double spin asymmetries are discussed. New high-precision RHIC data on the asymmetries in polarized proton-proton collisions are analyzed in the framework of the  $z$ -scaling. Information on spin-dependent fractal dimensions of proton is obtained. A microscopic scenario of constituent interactions developed within the  $z$ -scaling approach is used for study of the interactions with polarized protons. Spin dependence of the constituent energy loss as a function of the momentum of the produced hadron and the energy of collision is estimated.

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## 1. Introduction

Spin, along with the electric charge, mass and intrinsic symmetries, is one of the most fundamental properties of elementary particles. It is important to understand the spin of hadrons in terms of the underlying degrees of freedom, i.e. the spins of quarks and gluons and their orbital motion. The axial vector nature of spin is useful in testing symmetries such as parity and time reversal invariance in fundamental interactions. The importance of physics with polarized program can be understood in two ways. One is elucidation of the spin structure of hadrons and the other one is utilizing known spin composition to test symmetries in different reactions. In the framework of QCD, the basic elements of hadron structure are quarks and gluons. The non-linear Yang-Mills equations taking into account gauge invariance and Lorentz covariance regulate dynamics of the constituent interactions both at hard and soft regimes. The subsequent hadronization and property of confinement of color constituents within colorless hadrons represent persistent problems which are not solved up to now.

The spin structure of the nucleon is studied for a long time in processes with longitudinally and transversally polarized leptons and protons [1]-[8]. The goal is to obtain a complete picture of the nucleon spin in terms of the constituent degrees of freedom. Various microscopic scenarios of the proton composition imply knowledge of momentum and spin distributions of the proton constituents at different scales. There are indications that structure of the unpolarized proton reveals self-similarity and fractality over a wide scale range [9, 10]. One can expect therefore that also spin content of proton has similar distribution in terms of the polarized quarks and gluons. Both cases should reflect existence of a subtle structure of geometrical carrier of proton properties in the momentum space. An anisotropy of the momentum space may occur due to spontaneous symmetry breaking at small scales [11]. This property has relevance to scale invariance of fractal composition of the proton spin. The scale invariance of such properties like spin, mass, and, by extension, other quantities would imply existence of topological invariants.

The idea of self-similarity of hadron interactions is a fruitful concept to study collective phenomena in the hadron and nuclear matter [12]-[17]. Important manifestation of such a concept is existence of scaling itself [18]-[21]. Scaling in general means self-similarity at different scales. The physical content meant by behind it can be of different origin. Some of the scaling features constitute pillars of modern critical phenomena. Other category of scaling laws (self-similarity in point explosion, laminar fluid flow, etc.) reflects features not related to phase transitions. The  $z$ -scaling in inclusive reactions, which in a sense pertains to both mentioned groups, has properties reviewed in [9, 10]. It is treated as manifestation of self-similarity of the structure of colliding objects (hadrons, nuclei), interaction mechanism of their constituents, and fragmentation of hadron constituents into real hadrons. The validity of  $z$ -scaling is confirmed in the region which can be far from a boundary of a phase transition. Nevertheless, the parameters of the scaling characterize quantities which are sensitive to the critical phenomena [14]. There are three parameters  $c$ ,  $\delta_A$  and  $\varepsilon_F$  which have physical interpretation of heat capacity of the produced matter, fractal dimension of the structure of hadrons or nuclei and fractal dimension of the fragmentation process, respectively. The  $z$ -scaling shows itself as an effective tool for sophisticated data analysis and provides a basis in searching for new physics phenomena. It gives strong constraints on various phenomenological ingredients of theoretical models used for predictions of the inclusive cross section.

Here we propose extension of the method for phenomenological analysis of the polarization phenomena and verification of the self-similarity and fractality of the constituent interactions for spin-dependent inclusive production in proton-proton collisions. Such an approach could give new insight into the origin of the proton spin at small scales. The corresponding spin-dependent fractal dimensions are new parameters of the  $z$ -scaling theory. They are considered as characteristics of polarization properties of proton structure, constituent interactions and process of hadronization. The possibilities of using the  $z$ -scaling for this type of investigations are discussed below.

## 2. Scaling and universality as general concepts

The concepts developed to understand the critical phenomena are "scaling" and "universality". Scaling means that the system near the critical point exhibiting self-similar properties is invariant under transformation of scale. According to universality, quite different systems behave in a remarkably similar way near the respective critical point [18]-[20]. The universality hypothesis reduces the great variety of critical phenomena to a small number of equivalence classes, the so-called "universality classes", which depend only on few fundamental parameters (critical exponents). The universality has its origin in the long range character of the fluctuations. Close to the transition point the behavior of the cooperative phenomena becomes independent of the microscopic details of the considered system. The fundamental parameters determining the universality class are the symmetry of the order parameter and the dimensionality of space. The concept of universality remains the major tool to study the great variety of non-equilibrium phase transitions as well (see [21] and references therein). It is known that the scaling functions vary more widely between different universality classes than the exponents. Thus, universal scaling functions offer a sensitive and accurate test for the system universality class. We assume that inclusive cross sections observed in the collisions of polarized protons can be described by universal scaling functions with parameters which are the spin-dependent fractal dimensions of proton and fragmentation processes.

## 3. $z$ -Scaling

The  $z$ -scaling belongs to the scaling laws with applications not limited to the regions near a phase transition. The scaling regularity concerns hadron production in the high energy proton (antiproton) and nucleus collisions [9]-[17]. It manifests itself in the fact that the inclusive spectra of various types of particles are described with a universal scaling function. The function  $\Psi(z)$  depends on a single variable  $z$  in a wide range of the transverse momentum, registration angles, collision energies and centralities. The scaling variable has the form:

$$z = z_0 \cdot \Omega^{-1}. \quad (3.1)$$

Here  $z_0$  and  $\Omega$ ,

$$z_0 = \frac{\sqrt{s_{\perp}}}{(dN_{ch}/d\eta|_0)^c m_N}, \quad (3.2)$$

$$\Omega = (1-x_1)^{\delta_1} (1-x_2)^{\delta_2} (1-y_a)^{\epsilon_a} (1-y_b)^{\epsilon_b}, \quad (3.3)$$

are functions of some kinematic and dynamic variables. The quantity  $z_0$  is proportional to the transverse kinetic energy of a selected binary constituent sub-process responsible for production of

the inclusive particle with mass  $m$  and its partner (antiparticle). The multiplicity density  $dN_{ch}/d\eta|_0$  of charged particles in the central interaction region, the nucleon mass  $m_N$  and the parameter  $c$  completely determine the dimensionless value of  $z_0$ . The function  $\Omega$  is hereafter referred to in the following abbreviated notation:

$$\Omega \equiv \Omega_{0000} =: \{\delta_1, \delta_2, \varepsilon_a, \varepsilon_b\}. \quad (3.4)$$

The low index (0000) corresponds to the unpolarized particles in the initial and final states. The quantity  $\Omega$  is proportional to the relative number of the configurations at the constituent level which include the binary sub-processes corresponding to the momentum fractions  $x_1$  and  $x_2$  of colliding hadrons (nuclei) and to the momentum fractions  $y_a$  and  $y_b$  of the secondary objects produced in these sub-processes. The parameters  $\delta_1$  and  $\delta_2$  are fractal dimensions of the colliding objects, whereas  $\varepsilon_a$  and  $\varepsilon_b$  stand for the fractal dimensions of the fragmentation process in the scattered and recoil direction, respectively. For unpolarized processes, we assume the later to have the same value  $\varepsilon_a = \varepsilon_b = \varepsilon_F$  which depends on the type of the inclusive particle. The selected binary sub-process, which results in production of the inclusive particle and its recoil partner (antiparticle), is defined by the maximum of  $\Omega(x_1, x_2, y_a, y_b)$  with the kinematic constraint

$$(x_1 P_1 + x_2 P_2 - p/y_a)^2 = M_X^2. \quad (3.5)$$

Here  $M_X = x_1 M_1 + x_2 M_2 + m/y_b$  is the mass of the recoil system in the sub-process. The 4-momenta of the colliding objects and the inclusive particle are  $P_1, P_2$  and  $p$ , respectively. Equation (3.5) accounts for the locality of the interaction at the constituent level and sets a restriction on the momentum fractions  $x_1, x_2, y_a, y_b$  of particles via the kinematics of the constituent interactions. The microscopic scenario of constituent interactions developed in the framework of  $z$ -scaling is based on dependencies of the momentum fractions on the collision energy, transverse momentum and centrality. The scaling variable  $z$  has property of a fractal measure. It grows in a power-like manner with the increasing resolution  $\Omega^{-1}$  defined with respect to the constituent sub-processes satisfying (3.5). The scaling function  $\Psi(z)$  is expressed in terms of the inclusive cross section  $Ed^3\sigma/dp^3$ , multiplicity density  $dN/d\eta$ , and total inelastic cross section  $\sigma_{in}$ . All these quantities are measurable for the inclusive reaction  $P_1 + P_2 \rightarrow p + X$ . The function  $\Psi(z)$  is determined by the following expression:

$$\Psi(z) = \frac{\pi}{(dN/d\eta)\sigma_{in}} J^{-1} E \frac{d^3\sigma}{dp^3}. \quad (3.6)$$

Here  $J$  is Jacobian for the transition from the variables  $\{p_T^2, y\}$  to  $\{z, \eta\}$ . The function  $\Psi(z)$  satisfies the normalization condition:

$$\int_0^\infty \Psi(z) dz = 1. \quad (3.7)$$

Equation (3.7) allows us to interpret  $\Psi(z)$  as probability density of the production of the inclusive particle with the corresponding value of the variable  $z$ .

#### 4. Scaling in unpolarized pp collisions

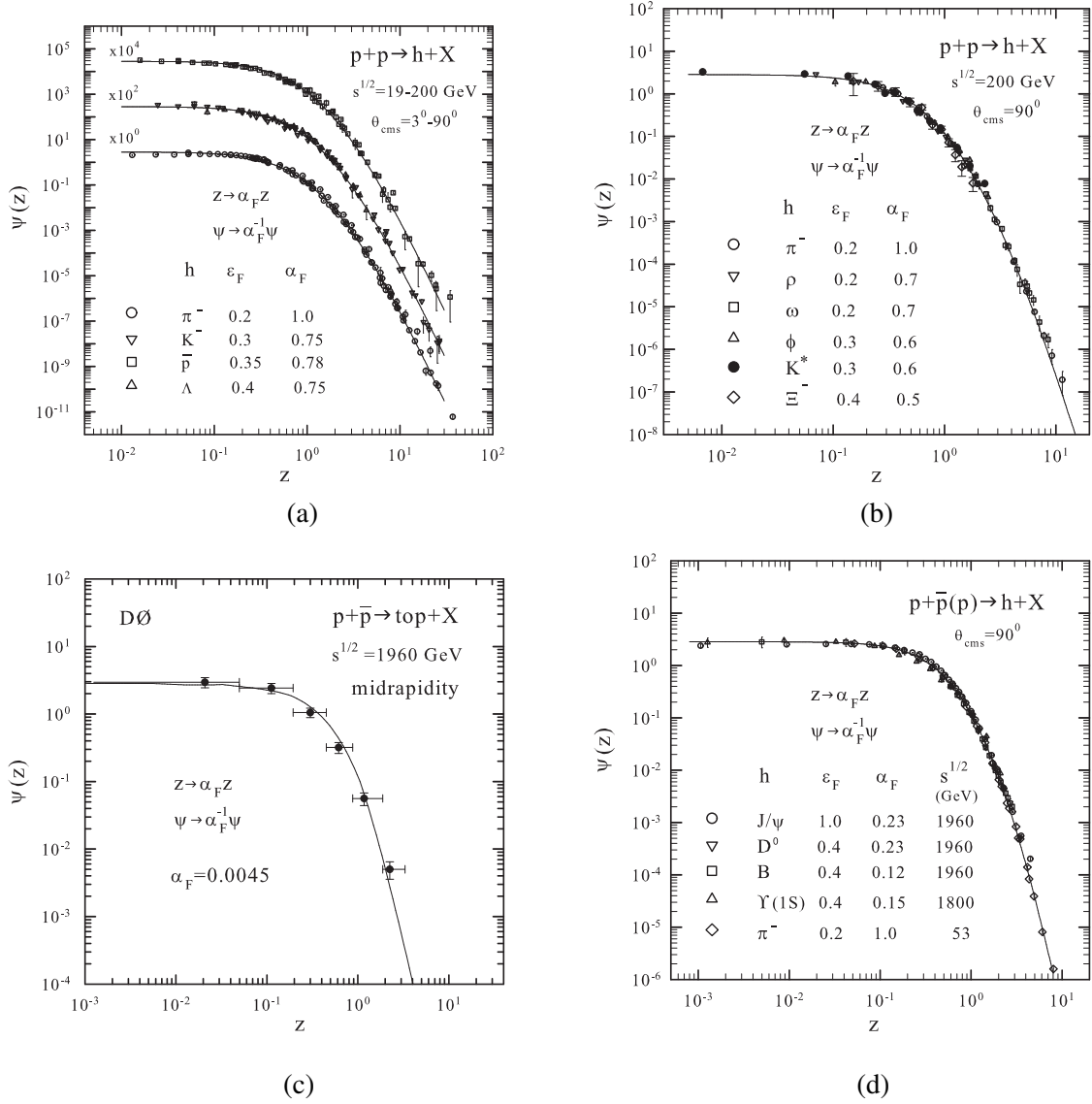
The property of self-similarity of hadron interactions at high energies provides the basis for analyzing inclusive spectra of particles produced in proton and nuclei collisions in the framework of

the  $z$ -scaling approach. Figure 1 shows spectra of hadrons produced in proton-proton interactions in  $z$ -presentation. The kinematic region covers a wide range of the collision energies, registration angles and transverse momenta. The scale factors ( $10^0, 10^2, 10^4$ ) in Fig.1(a) are introduced to split the data into different groups. We observe a collapse of the experimental data points onto a single curve. The solid line is a fitting curve for these data. The derived representation shows the universality of the shape of the scaling curve  $\Psi(z)$  for different types of hadrons. The regularity (universal shape of the function  $\Psi(z)$  and its scaling behavior in the wide kinematic range found at constant values of the parameters  $\delta, \varepsilon_F$  and  $c$ ) is treated as manifestation of the self-similarity of the structure of colliding objects, interaction mechanism of their constituents, and processes of fragmentation into registered particles. The scale transformations  $z \rightarrow \alpha_F z, \Psi \rightarrow \alpha_F^{-1} \Psi$  shown in the figure result in compatibility of the corresponding scaling curves in the plane  $\{z, \Psi\}$ . The normalization condition (3.7) is conserved by the transformations.

As is seen from Fig. 1, the scaling function  $\Psi(z)$  exhibits two kinds of behavior: one in the low- $z$  and the other one in the high- $z$  region. The low- $z$  region corresponds to saturation of the scaling function with the typical flattening out. The behavior of  $\Psi(z)$  at low  $z$  depends mainly on the parameter  $c$ . This parameter is determined from the multiplicity dependence of the spectra. The region of low values of  $z$  (low transverse momenta) and high multiplicity density is preferable (even in  $pp$  interactions) to study the collective effects and search for a phase transition in hadron matter. The region is best suited for studying collective phenomena in the systems of hadrons and their constituents. The scaling function at high  $z$  (high transverse momenta) is characterized by the power behavior  $\Psi(z) \sim z^{-\beta}$  with a constant value of the slope  $\beta$ . At high  $z$ , the observed power shape of the scaling function reflects self-similarity of the constituent interactions at small scales. The asymptotic form of  $\Psi(z)$  imposes restrictions on the cross sections at high  $p_T$ . It can be used to perform the global QCD fit for construction of quark and gluon distribution functions in the regions where the experimental data are still missing.

The parameters  $\delta, \varepsilon_F$  and  $c$  introduced in the definition of the variable  $z$  are determined from analyses of many different sets of experimental data (see [9, 10] and references therein). They are found to be constant and independent of the multiplicity density and of the kinematic quantities such as collision energy, detection angle and transverse momentum of the inclusive particle. A possible change of the parameters can be used as a signature of new phenomena in the kinematic regions not yet explored experimentally. This is primarily true for the low ( $z < 0.01$ ) and high ( $z > 10$ ) regions of the variable  $z$ . In the intermediate region, ( $0.01 < z < 10$ ), the shape of  $\Psi(z)$  is well determined from data obtained in the kinematic range which is now accessible for experiments at the current accelerators. Note that extension of the  $z$ -range does not require obligatory increase in the collision energy. This is possible when rare events are specially selected at low  $z < 0.01$  or high  $z > 30$ . A more stringent restriction on the scaling behavior of  $\Psi(z)$  at high  $z$  would bear witness to self-similarity at scales smaller than  $10^{-4}$  fm related with the notion of fractal space-time.

New accelerator facilities NICA at JINR and FAIR at GSI are planned to be commissioned at the energy around  $\sqrt{s} = 30$  GeV or below. In this range, extreme regime of hadron structure can be reached for  $z > 30$ . A test of the self-similarity of spin-dependent and spin-independent constituent interactions at small scales is of interest for the high values of  $z$ . New information expected from the experiments with polarized proton beams would be complementary to measurements with polarized particles in deep-inelastic, semi-inclusive deep-inelastic and deep-virtual Compton scattering.



**Figure 1:** Inclusive spectra of hadrons produced in proton-proton and proton-antiproton collisions in the  $z$ -presentation. The symbols denote the experimental data obtained in the experiments performed at CERN, FNAL and BNL. Plots are taken from [9, 10, 16].

## 5. Scaling hypothesis for polarized pp collisions

In this section we discuss possibility to use the  $z$ -scaling approach for study of spin-dependent inclusive cross sections of particle production in  $\vec{p} + \vec{p}$  collisions to extract information on spin-dependent fractal dimensions of proton. As it was pointed out in [22], the E581 and E704 collaborations at Fermilab reported data on hadron production in transversely polarized proton-proton scattering that showed large unanticipated transverse spin asymmetries up to 30-40%. Qualitatively, the asymmetries were consistent with zero for mid-rapidities, but increased rapidly in the forward scattering direction.

More recently, the PHENIX, STAR and BRAHMS collaborations at the Relativistic Heavy Ion Collider (RHIC) have studied the transverse spin asymmetries over a wide kinematic range at  $\sqrt{s} = 200$  GeV. The obtained data confirmed and extended the Fermilab results and indicated non-monotonic dependence of  $A_N$  on the transverse momentum  $p_T$  of the produced hadron (see [7, 8, 23] and references therein). The measurements of the non-zero asymmetries give us strong motivation to study fractal properties of proton spin structure.

For processes with one polarized proton, the fractal dimensions should depend on the direction of the proton spin relative to its momentum (left ( $\leftarrow$ ) and right ( $\rightarrow$ ) helicity) or (up ( $\uparrow$ ) and down ( $\downarrow$ )) relative to the reaction plane. The opposite proton polarizations in the inclusive cross section for particle production in the reaction  $\vec{p} + p \rightarrow h + X$  or  $p^\uparrow + p \rightarrow h + X$  are generally denoted by  $\sigma_+$  and  $\sigma_-$ . The single longitudinal ( $A_L$ ) or transversal ( $A_N$ ) spin asymmetries of the processes are written as follows:

$$A_{L,N} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}. \quad (5.1)$$

The unpolarized cross section is equal to  $\sigma_0 = (\sigma_+ + \sigma_-)/2$ . Using the notation as in (3.4), the corresponding quantities  $\Omega$  can be expressed as follows:

$$\Omega_{+000} =: \{\delta - \Delta\delta/4, \delta + \Delta\delta/4, \varepsilon_F - \Delta_F, \varepsilon_F\} \quad (5.2)$$

$$\Omega_{-000} =: \{\delta + \Delta\delta/4, \delta - \Delta\delta/4, \varepsilon_F + \Delta_F, \varepsilon_F\}. \quad (5.3)$$

We know from experiment that  $\sigma_+ \neq \sigma_-$  when  $x_1 \rightarrow 1, x_2 \rightarrow 0$  and  $\sigma_+ = \sigma_-$  for  $x_1 \rightarrow 0, x_2 \rightarrow 0$ . This should result in  $\Psi_{+000} \neq \Psi_{-000}$  and  $\Psi_{+000} = \Psi_{-000}$  in these kinematic regions, respectively. The experimental observation represents restriction on interplay between the corrections  $\Delta\delta/4$  and  $\Delta_F$  which for various particles reflect different production and fragmentation mechanisms. This can give e.g. zero asymmetry for  $\pi^{0,\pm}$  mesons and non-zero one for  $W^\pm$ 's. The values of  $\Omega_{+000}$  and  $\Omega_{-000}$  are expected to be modified differently for longitudinally and transversely polarized protons generating the asymmetry  $A_L$  and  $A_N$  due to different spin-dependent corrections  $\Delta\delta$  and  $\Delta_F$  for the longitudinal and transversal polarizations.

The reaction  $\vec{p} + \vec{p} \rightarrow h + X$  with two longitudinal polarized protons in the initial state is described by spin-dependent cross sections  $\sigma_{++}, \sigma_{--}, \sigma_{+-}, \sigma_{-+}$ . The symbols (+) and (-) denote positive and negative helicities of the protons, respectively. The double spin asymmetry  $A_{LL}$  of the process is expressed via combination of the cross sections in the form:

$$A_{LL} = \frac{\sigma_{++} + \sigma_{--} - \sigma_{+-} - \sigma_{-+}}{\sigma_{++} + \sigma_{--} + \sigma_{+-} + \sigma_{-+}}. \quad (5.4)$$

The corresponding notation for the functions  $\Omega$  is written as follows:

$$\Omega_{++00} =: \{\delta - \Delta\delta/2, \delta - \Delta\delta/2, \varepsilon_F, \varepsilon_F\} \quad (5.5)$$

$$\Omega_{--00} =: \{\delta - \Delta\delta/2, \delta - \Delta\delta/2, \varepsilon_F, \varepsilon_F\} \quad (5.6)$$

$$\Omega_{+-00} =: \{\delta, \delta + \Delta\delta, \varepsilon_F, \varepsilon_F\} \quad (5.7)$$

$$\Omega_{-+00} =: \{\delta + \Delta\delta, \delta, \varepsilon_F, \varepsilon_F\}. \quad (5.8)$$

The measurements of  $A_{LL}$  at RHIC [1]-[4] performed by the STAR collaboration for jet production and by the PHENIX collaboration for  $\pi^0$  meson production showed that the double spin asymmetry is small but non-zero in the central rapidity region and increases with the transverse momentum. Similar analysis is expected to be performed for the double transversal spin asymmetry  $A_{NN}$  in  $p^\uparrow + p^\uparrow$  collisions. The measurements of  $A_{LL}$  and  $A_{NN}$  over a wide range of  $x_1$  and  $x_2$  could give more detailed information regarding polarized constituent interactions and provide complementary restriction on the parameters of the scaling variable  $z$ .

The inclusive reaction  $\vec{p} + p \rightarrow \vec{h} + X$  with one longitudinal polarized proton in the initial state and one longitudinal polarized particle (e.g. lambda hyperon) in the final state is described by the transfer of polarization [4, 24]. The coefficient of polarization transfer is written in the following form:

$$D_{LL} = \frac{\sigma_{++} + \sigma_{--} - \sigma_{+-} - \sigma_{-+}}{\sigma_{++} + \sigma_{--} + \sigma_{+-} + \sigma_{-+}}. \quad (5.9)$$

Here the symbols (+) and (-) denote the cross sections corresponding to the parallel and antiparallel spin orientations relative to the respective momenta of the polarized particles (positive and negative helicities). The polarization in the initial state is related to the spin-dependent correction ( $\Delta\delta$ ) of the proton fractal dimension. Let us consider (+) helicity in the initial state only and denote  $\varepsilon_F$  as the corresponding fractal dimension for hadronization of the unpolarized particle ( $h$ ) in the final state. If the inclusive particle is polarized, ( $\vec{h}$ ), the spin-dependent correction  $\Delta\varepsilon_F$  to the value of  $\varepsilon_F$  is included. The notation for  $\Omega$  concerning the process is as follows:

$$\Omega_{+0+0} =: \{ \delta - \Delta\delta/4, \delta + \Delta\delta/4, \varepsilon_F - \Delta\varepsilon_F/2, \varepsilon_F \} \quad (5.10)$$

$$\Omega_{+0-0} =: \{ \delta - \Delta\delta/4, \delta + \Delta\delta/4, \varepsilon_F + \Delta\varepsilon_F/2, \varepsilon_F \}. \quad (5.11)$$

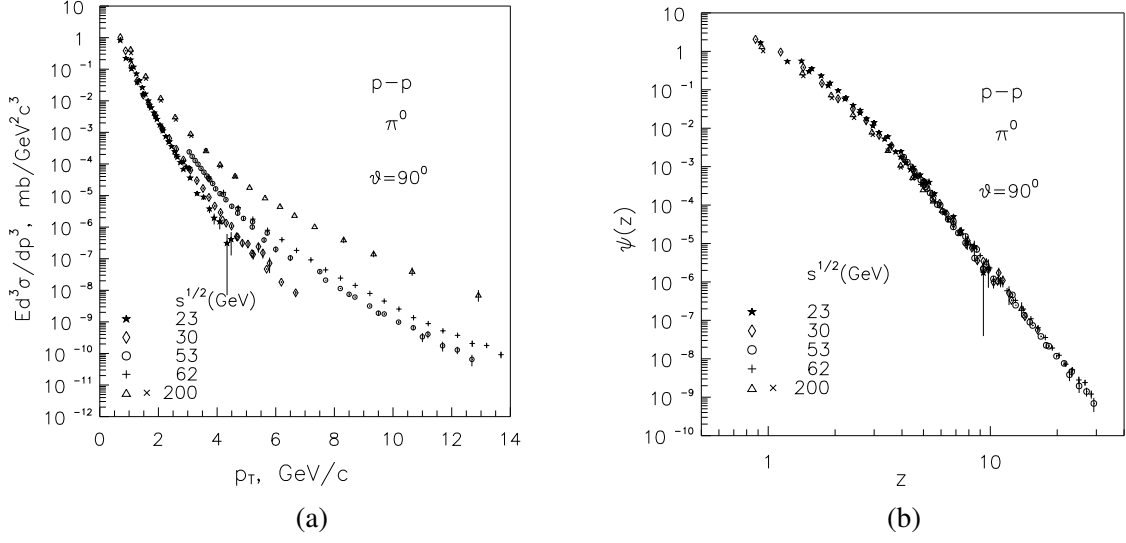
The coefficient of polarization transfer should be zero in the region where  $\Omega_{+0+0} = \Omega_{+0-0}$  and non-zero in the region  $x_1 \rightarrow 1, y_a \rightarrow 1$ , where  $\Omega_{+0+0} \neq \Omega_{+0-0}$ . In the similar way, the transversal spin transfer coefficient  $D_{NN}$  for the production of vector mesons and baryons is of interest to verify the hypothesis of self-similarity for polarized meson and baryon production and to determine the transversely spin-dependent fractal dimensions.

Let us consider the process  $\vec{p} + \vec{p} \rightarrow h + X$  in more detail. The inclusive particle can be a photon, pion, kaon, lambda, or Drell-Yan pairs, heavy quarkonia and jets. The measured cross sections for different polarizations of protons allow obtaining the double spin asymmetry  $A_{LL}$  and the unpolarized cross section  $\sigma_0$  as a function of the transverse momentum  $p_T$  at some angle  $\theta_{cms}$ . Using the information on the asymmetry and the unpolarized cross section, the spin-dependent functions  $\Psi_{++}, \Psi_{--}, \Psi_{+-}, \Psi_{-+}$  can be constructed. The functions have different arguments which we denote as  $z_{++}, z_{--}, z_{+-}, z_{-+}$ , respectively. They depend on spin-dependent fractal dimensions in a way shown above. Based on existence of the  $z$ -scaling in unpolarized proton-proton collisions, we assume self-similarity of polarization processes at a constituent level expressed in the following form:

$$\Psi_{++} = \Psi(z_{++}), \quad \Psi_{+-} = \Psi(z_{+-}), \quad \Psi_{00} = \Psi(z_{00}). \quad (5.12)$$

The relations include corrections  $\Delta\delta$  and  $\Delta\varepsilon_F$  to the fractal dimensions  $\delta$  and  $\varepsilon_F$  established for the unpolarized reactions. The corrections can be determined using data on inclusive cross section and asymmetry of the processes under consideration. Information on both polarized and unpolarized



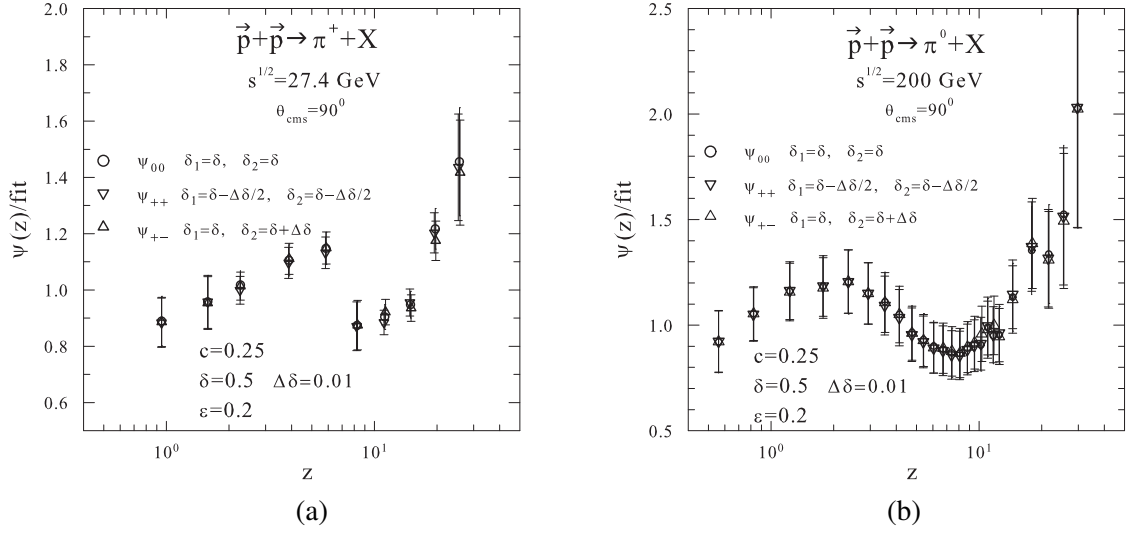


**Figure 2:** Spectra of pion production in the proton-proton collisions at  $\sqrt{s} = 23 - 200$  GeV and  $\theta_{cms} = 90^\circ$  in  $p_T$  (a) and  $z$  (b) presentations [12, 13].

cross sections are necessary to extract spin-dependent fractal dimensions from the polarization characteristics ( $A_{LL}, D_{LL}$ ) of a given process. Such data allows us to obtain restrictions on the parameters  $\Delta\delta$  and  $\Delta\epsilon_F$  of the model.

Figure 2 shows transverse momentum spectra of  $\pi^0$  mesons produced in unpolarized  $p + p$  collisions at  $\sqrt{s} = 23 - 200$  GeV. One can see strong decrease of the cross sections as  $p_T$  increases. The high- $p_T$  part of the spectra is transformed to the asymptotic power behavior of  $\Psi(z)$  for large values of  $z$  [12, 13]. New measurements of the cross sections at high  $p_T$  are needed to verify the power law for the scaling function  $\Psi(z)$ . At high energies, the asymptotic behavior at very high  $z$  is hard to reach. It is therefore of interest to perform measurements of inclusive cross sections at energies lower than typically  $\sqrt{s} = 30$  GeV. If however the energy is too low, fractal dimensions  $\delta_1$  and  $\delta_2$  begin to decrease. They are equal to zero for particles which structures cannot be discerned. Such behavior reflects smearing of the fractal structure of hadrons which are seen as structureless ( $\delta_1 = \delta_2 = 0$ ) at very low  $\sqrt{s}$ . In this regard, it is desirable to perform measurements over a selected energy range, say  $\sqrt{s} = 10 - 30$  GeV. In this range, spin effects are expected to be considerable.

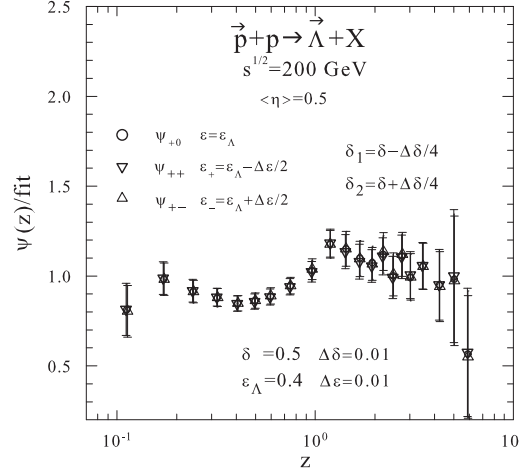
In order to get some estimates concerning the spin dependencies of spectra in  $z$ -presentation at lower energies, we start with analysis of data on double longitudinal spin asymmetry of  $\pi^0$ -mesons measured by the STAR collaboration at  $\sqrt{s} = 200$  GeV [3]. We have used the transverse momentum dependence of the asymmetry and the corresponding unpolarized cross section in the analysis. We have constructed the functions  $\Psi_{++}$  and  $\Psi_{+-}$  exploiting hypothesis on universality of their shape written in the form (5.12) to estimate the spin dependent correction  $\Delta\delta$  to the proton fractal dimension  $\delta$  from the asymmetry  $A_{LL}$  at  $\sqrt{s} = 200$  GeV and  $\theta_{cms} = 90^\circ$ . As follows from  $z$ -scaling for unpolarized interactions, the proton fractal dimension  $\delta = 0.5$  does not depend on energy for  $\sqrt{s} > 20$  GeV. This brings us to an assumption that the spin-dependent fractal dimensions are independent of the collision energy in this kinematic region as well. The hypothesis of self-similarity of



**Figure 3:** The scaled spin-dependent  $\Psi_{++}$ ,  $\Psi_{+-}$  and spin-independent  $\Psi_{00}$  functions of pion production in proton-proton collisions at  $\sqrt{s} = 27.4, 200$  GeV and  $\theta_{cms} = 90^0$  in  $z$ -presentation.

spin-dependent structure of proton encoded in  $\delta, \Delta\delta$  and in the functions  $\Psi_{++}, \Psi_{+-}$  is considered to play an important role to understand origin of the proton spin. Additional measurements of spin asymmetries at other  $\sqrt{s}$  are needed to verify this hypothesis. Based on the functional form (5.12) and on the assumption on energy independence of the spin-dependent fractal dimensions, we have used data on  $\pi^+$  production at  $\sqrt{s} = 27.4$  GeV and  $\theta_{cms} = 90^0$  to estimate longitudinal double spin asymmetry at this energy. Figure 3 demonstrates the scaled spin-dependent functions  $\Psi(z)/\text{fit}$  for the reaction  $\vec{p} + \vec{p} \rightarrow \pi^{+,0} + X$  at both energies  $\sqrt{s} = 27.4$  and 200 GeV. The self-similarity of spin processes expressed by these functions corresponds to (5.12). Accordingly, the correction to the fractal dimension  $\delta$  of unpolarized proton is found to be  $\Delta\delta = 0.01$ .

The STAR collaboration measured longitudinal spin transfer coefficient  $D_{LL}$  for the process  $\vec{p} + p \rightarrow \vec{\Lambda} + X$  at  $\sqrt{s} = 200$  GeV and  $\langle \eta \rangle = 0.5$  [4]. The coefficient can provide sensitivity to spin-dependent hadronization of the  $\Lambda$  hyperon. Unlike the transversal coefficient  $D_{NN}$  which reaches 30% at  $x_F \simeq 0.85$  [24], the coefficient  $D_{LL}$  is rather small ( $\leq 5\%$ ), but non-zero. We have studied corrections to the fractal dimensions  $\delta$  and  $\epsilon_\Lambda$  for the longitudinal spin transfer process as quoted in (5.10) and (5.11) exploiting the self-similarity condition (5.12). Using the value of  $\Delta\delta = 0.01$  obtained from the analysis of the double spin asymmetry mentioned above and approximating the measured  $p_T$ -dependence of the coefficient  $D_{LL}$  by a linear function, the spin-dependent correction to  $\epsilon_\Lambda$  is found to be  $\Delta\epsilon_\Lambda = 0.01$ . Figure 4 demonstrates the corresponding scaled spin-dependent functions  $\Psi(z)/\text{fit}$ , for the reaction  $\vec{p} + p \rightarrow \vec{\Lambda} + X$  at  $\sqrt{s} = 200$  GeV. Based on constancy of  $\epsilon_\Lambda$  with  $\sqrt{s}$  in unpolarized reactions, one can assume that the value of  $\Delta\epsilon_\Lambda$  is independent of the collision energy (similarly as  $\Delta\delta$ ) in the same kinematic range. We consider that the self-similarity of spin-dependent structure of proton and lambda hyperon encoded in the parameters  $\delta, \Delta\delta$  and  $\epsilon_\Lambda, \Delta\epsilon_\Lambda$  and also in the functions  $\Psi_{++}, \Psi_{+-}$  play an important role to understand the mechanism of spin polarization transfer.



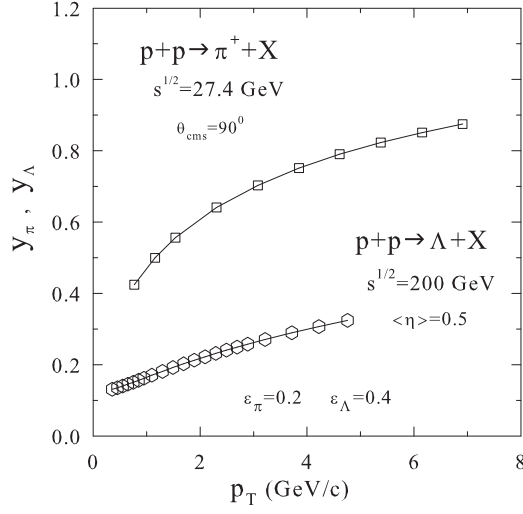
**Figure 4:** The scaled spin-dependent  $\Psi_{++}$ ,  $\Psi_{+-}$  and spin-independent  $\Psi_{00}$  functions for  $\Lambda$  hyperon production in  $p + p$  collisions at  $\sqrt{s} = 200$  GeV and  $\langle \eta \rangle = 0.5$  in  $z$ -presentation.

The suggested procedure of data analysis based on the  $z$ -scaling approach can be applied to a wide class of the polarization processes. Among them there are reactions with production of direct photons,  $J/\psi$ 's, Drell-Yan pairs, pions, kaons, hyperons in  $\vec{p} + \vec{p}$  collisions, as well as production of polarized and unpolarized particles with different flavor content in  $\vec{p} + p$  and  $p + p$  interactions. Systematic experimental investigations of the processes with polarized protons will contribute to further development of theory and understanding of spin as one of the most important and basic property of particles.

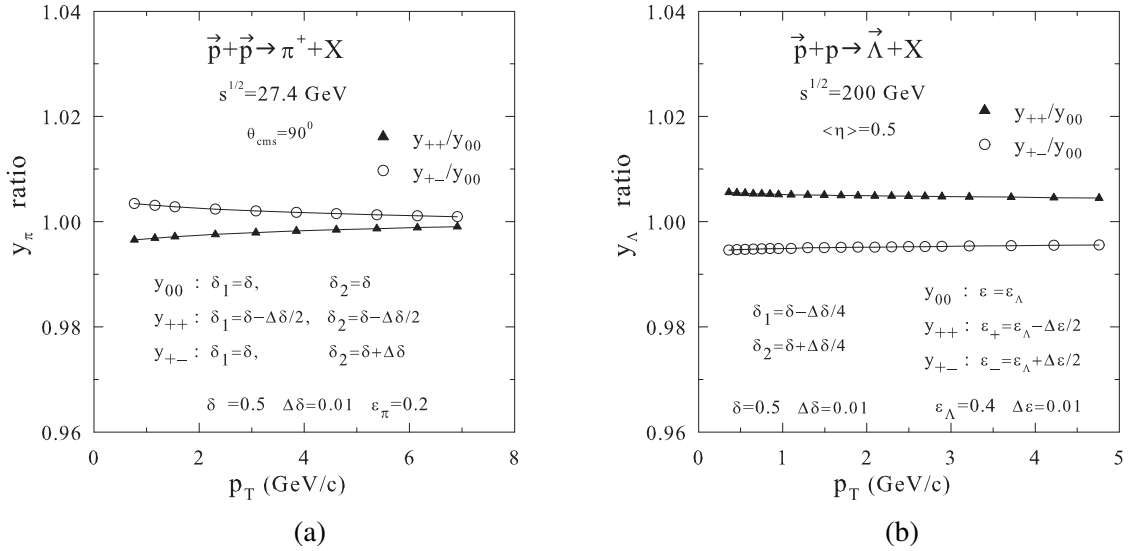
## 6. Spin-dependent energy loss

The concept of  $z$ -scaling allowed us to develop a microscopic scenario of the interaction between hadrons and nuclei at the level of interacting constituents. The microscopic picture of hadron collisions reflects fractal structure of the colliding objects, properties of constituent interactions and hadronization process characterized by the momentum fractions  $x_1, x_2, y_a, y_b$  and fractal dimensions  $\delta_1, \delta_2, \varepsilon_a, \varepsilon_b$ . The scaling observed in many reactions under various (kinematic and multiplicity) conditions predicts values of energy losses during process of inclusive production in dependence on the collision energy, centrality, transverse momentum and type of the inclusive particle [14, 15]. The energy loss is proportional to  $1 - y_a$ .

Figure 5 illustrates the dependence of the momentum fractions  $y_\pi$  and  $y_\Lambda$  of the scattered constituents carried away by the inclusive pion and  $\Lambda$  hyperon, respectively. The symbols correspond to data on production of both particles in the unpolarized  $p + p$  collisions. As seen from Fig.5, the energy losses decrease with increasing  $p_T$ . For pion production at  $\sqrt{s} = 27.4$  GeV and  $p_T = 7$  GeV/c the energy loss is found to be about 10%, whereas for  $\Lambda$  production at  $\sqrt{s} = 200$  GeV and  $p_T = 5$  GeV/c its value is significantly larger, about 70%. As it was mentioned in [14, 15], smaller energy losses are preferable in dealing with interesting physical problems such as localiza-



**Figure 5:** Momentum fractions  $y_\pi$  and  $y_\Lambda$  as a function of the transverse momentum  $p_T$  and collision energy  $\sqrt{s}$  for the processes  $p + p \rightarrow \pi^+ + X$  at  $\sqrt{s} = 27.4$  GeV and  $p + p \rightarrow \Lambda + X$  at  $\sqrt{s} = 200$  GeV, respectively.



**Figure 6:** Spin-dependent momentum fractions  $y_\pi$ ,  $y_\Lambda$  as function of the transverse momentum  $p_T$  and collision energy  $\sqrt{s}$  for processes  $\vec{p} + \vec{p} \rightarrow \pi^+ + X$  and  $\vec{p} + p \rightarrow \vec{\Lambda} + X$ .

tion of critical point and detection of phase transitions in hadron and nuclei collisions. We assume that this applies also to spin structures of hadrons at different scales.

Figure 6(a) shows the ratio of spin-dependent  $y_{++}$ ,  $y_{+-}$  and spin-independent  $y_{00}$  fractions for pion production in the reaction  $\vec{p} + \vec{p} \rightarrow \pi^+ + X$  at  $\sqrt{s} = 27.4$  GeV as a function of the transverse momentum  $p_T$ . The momentum fractions correspond to the same values of fractal dimensions as quoted in Fig.3. One can see that the energy loss is slightly larger for the combination  $(++)$  of proton polarizations than for the combination  $(+-)$ . The first one corresponds to the

collisions with the opposite ( $\rightarrow\leftarrow$ ) and the second one with the same ( $\rightarrow\rightarrow$ ) spin orientations of the incident protons. This feature is in accord with behavior of the ratio of momentum fractions of the polarized particles for the polarization transfer process in the reaction  $\vec{p} + p \rightarrow \vec{\Lambda} + X$  at  $\sqrt{s} = 200$  GeV. As seen from Fig.6(b), the energy loss is smaller for the spin combination ( $++$ ) than for the combination ( $+ -$ ) of the polarized particles in this reaction. The difference between  $y_{++}$  and  $y_{+-}$  represents about 1% and is approximately independent of transverse momentum over the range  $P_T = 0.5 - 4.5$  GeV/c. The momentum fractions correspond to the same values of fractal dimensions as quoted in Fig.4.

The scaling hypothesis (5.12) leads us thus to a conjecture that opposite spin orientation of proton spins (in reactions with longitudinal double spin asymmetries) and spin-flip process (in reactions with longitudinal polarization transfer) result in somewhat larger energy losses relative to the situation where the spins of both polarized particles are aligned in "the same direction". Such natural inference relies on the fractal structure of hadron constituents as implemented in the  $z$ -scaling formalism and, as we consider, reflects the self-similarity of hadron interactions in spin dependent processes at a constituent level.

## 7. Conclusions

Search for new features of spin structure in polarized proton-proton collisions has been discussed. We assume that among basic properties of spin as a quantum characteristic of proton is self-similarity of its internal spin composition and fractality at small scales. The hypothesis of self-similarity of the proton structure, constituent interactions and hadronization process confirmed in unpolarized  $p + p$  collisions over a wide kinematical range is extended for processes with polarized particles. The established properties of inclusive cross sections in  $z$ -presentation, like energy, angular and flavor independence give us basis to study the spin structure of proton in the framework of  $z$ -scaling theory. The requirement of a universal description of hadron spectra in proton collisions at different energies and spin orientations gives restrictions on the values of spin-dependent parameters of the  $z$ -scaling and their dependencies on respective polarizations of particles.

The parameters  $\delta$  and  $\varepsilon_F$  interpreted as fractal dimension of the proton structure and fractal dimension of the fragmentation process are modified for processes with polarized particles. The spin-dependent energy loss was studied as function of  $p_T$  and  $\sqrt{s}$  in the framework of microscopic scenario developed within the  $z$ -scaling approach. The analysis exploits results of measurements for the reactions  $\vec{p} + \vec{p} \rightarrow \pi + X$  and  $\vec{p} + p \rightarrow \vec{\Lambda} + X$ . More accurate estimation of the energy loss needs new precise data with polarized particles and requires further detailed study.

We assume that considered scaling property for polarization processes reflects self-similarity of spin structure of the colliding objects, interaction mechanism of their constituents and process of fragmentation of the polarized particles in the final state. Study of fractality in the reactions with polarized particles can give more deep understanding of the origin of spin. The investigation is motivated by expectations that particle production in  $\vec{p} + \vec{p}$  collisions in the energy range  $\sqrt{s} = 10-30$  GeV is suitable for obtaining new information on fractal properties of proton spin. Such experiments are planned to be carried out at the future SPD NICA facility in Dubna.

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