

## Hyperbolic PDEs with non-commutative time

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In this talk, I will report on joint work with Rainer Verch [1] on hyperbolic PDEs with non-commutative time, i.e. linear integro-differential equations of the form  $(D + \lambda W)f = 0$ , where  $D$  is a (pre-)normal hyperbolic differential operator on  $\mathbb{R}^n$ ,  $\lambda \in \mathbb{C}$  is a coupling constant, and  $W$  a regular integral operator which is non-local in time, so that a Hamiltonian formulation is not possible. Such equations appear in the context of wave or Dirac equations on non-commutative deformations of Minkowski space. It will be discussed that at small coupling, the hyperbolic character of  $D$  is essentially preserved, unique advanced/retarded fundamental solutions can be constructed, and the acausal behavior of the solutions is well-controlled. Although the Cauchy problem is ill-posed in general, a scattering operator can be calculated which describes the effect of  $W$  on the space of solutions of  $D$ .

It is also described how these results can be used for the analysis of classical and quantum field theories on non-commutative spaces.

### *References*

- [1] G. Lechner and R. Verch, *Linear hyperbolic PDEs with non-commutative time*, Preprint, arXiv:1307.1780

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