PROCEEDINGS OF SCIENCE

Friedmann equation and the emergence of cosmic space

Ee Chang-Young∗†
Sejong University, Seoul, Korea, Republic of
E-mail: cylee@sejong.ac.kr

We show that Padmanabhan's conjecture [arXiv:1206.4916] for the emergence of cosmic space holds for the flat Friedmann-Robertson-Walker universe in Einstein gravity, but not for the non-flat case if one uses the proper volume. We check the validity of various works extending Padmanabhan's conjecture to non-Einstein and non-flat cases, and find serious shortfalls in most of them. The analysis is done using the Friedmann equation with the assumptions of the holographic principle and the equipartition rule of energy.

If we accept that Padmanabhan's conjecture is right, then we may interpret our result as follows. The holography of our nature imposes (spatial) flatness on our universe.

Frontiers of Fundamental Physics 14
15-18 July 2014
Aix Marseille University (AMU) Saint-Charles Campus, Marseille, France

∗Speaker.
†This article is based on the work by *Ee Chang-Young and Daeho Lee, JHEP 04 (2014) 125.*

PoS(FFP14)077

1. Introduction

Recently, Padmanabhan proposed a governing equation that would control the expansion of cosmic space [1]. It has been known that the universe of pure de Sitter type satisfies the holographic equipartition, $N_{\mathtt{sur}} = N_{\mathtt{bulk}}$, where $N_{\mathtt{sur}}$ and $N_{\mathtt{bulk}}$ are the degrees of freedom (DOF) of the boundary surface and of the bulk, respectively. Under the assumption that our universe is asymptotically de Sitter, Padmanabhan considered that the expansion of the universe is being driven towards holographic equipartition. Thus, he conjectured that the governing equation for the expansion of cosmic space is given by

$$\frac{dV}{dt} = L_p^2(N_{\mathtt{sur}} - N_{\mathtt{bulk}}), \tag{1.1}$$

where V is the volume of cosmic space enclosed by the apparent horizon $\tilde{r}_A$, and L_p is the Planck length. Using the above relation and with further assumptions of the holographic principle and the equipartition rule of energy, he succeeded in obtaining the Friedmann equation of the (3+1)-dimensional flat Friedmann-Robertson-Walker (FRW) universe in Einstein gravity.

Cai then obtained the Friedmann equations for an (n+1)-dimensional flat FRW universe in the Einstein, Gauss-Bonnet, and Lovelock gravity cases using Padmanabhan's conjecture [2]. However, in the Gauss-Bonnet and Lovelock cases, Cai used an effective volume for the volume change but used the plain ordinary volume for the bulk DOF.

In order to avoid this discrepancy, the plain ordinary volume was used both for the volume change and for the bulk DOF in Ref. [3]. But it was not free. In order to obtain the Friedmann equation, Padmanabhan's relation (1.1) had to be severely modified in both Gauss-Bonnet and Lovelock cases.

Extension to the non-flat case was first done in Ref. [4] with a slight modification of Padmanabhan's conjecture. However, the aerial volume was used instead of the proper invariant volume.

Ref. [5] tried to make up for this shortcoming by using the proper invariant volume in the non-flat Einstein case. They introduced the 'effective Planck length' solely to contain all the complications of the time dependence in 'the proportionality factor', so that the original form of Padmanabhan's conjecture could be maintained. This severe modification could cast strong doubt on the very validity of the conjecture.

In this work, we show that Padmanabhan's original conjecture and its various modified versions can be obtained from the Friedmann equation for the flat FRW universe in the Einstein, Gauss-Bonnet, and Lovelock gravity cases. We extend the same analysis for the non-flat case in (3+1)-dimensions. Throughout the paper, we use the natural units $k_B = c = \hbar = 1$.

2. Emergence of cosmic space for a flat FRW universe

Here, we show that the Friedmann equation of the flat FRW universe yields Padmanabhan's relation when the holographic principle and the equipartition rule of energy are assumed.

For later reference, we consider an (n+1)-dimensional FRW universe with the metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_{n-1}^2\right), \tag{2.1}$$

where $d\Omega_{n-1}^2$ denotes the line element of the $(n-1)$-dimensional unit sphere. Here, the spatial curvature constant k corresponds to a closed, flat and open universe for $k=+1,0,$ and -1, respectively. The metric (2.1) can be rewritten as

$$ds^2 = h_{ab}dx^a dx^b + \tilde{r}^2 d\Omega_{n-1}^2,\ a,b=0,1, \tag{2.2}$$

where $\tilde{r}=a(t)r$, $h_{ab}=\mathtt{diag}(-1,a^2/1-kr^2)$, and $(x^0,x^1)=(t,r)$. The apparent horizon in (2.2) is defined as the marginally trapped surface with vanishing expansion and is determined by the relation $h^{ab}\partial_a\tilde{r}\partial_b\tilde{r}=0$. Thus, the radius of the apparent horizon is given by

$$\tilde{r}_A = \frac{1}{\sqrt{H^2+k/a^2}}, \tag{2.3}$$

where $H\equiv \dot{a}/a$ is the Hubble parameter. The Hawking temperature associated with the apparent horizon is given by

$$T_H = \frac{1}{2\pi\tilde{r}_A}. \tag{2.4}$$

In this work, we will only consider the case in which the distribution of matter and energy takes the form of a perfect fluid. Then the Friedmann equations in the (n+1)-dimensional Einstein gravity are given by [6]:

$$H^2+\frac{k}{a^2} = \frac{16\pi L_p^{n-1}}{n(n-1)}\rho, \tag{2.5}$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi L_p^{n-1}}{n(n-1)}[(n-2)\rho+np]. \tag{2.6}$$

We now restrict ourselves to the flat case ($k=0$). Since the volume enclosed by the apparent horizon $\tilde{r}_A$ in the flat case is given by $V=\Omega_n\tilde{r}_A^n$, where Ω_n is the volume of the unit n-sphere, the rate of volume change is given by

$$\frac{dV}{dt} = n\Omega_n\tilde{r}_A^{n-1}\dot{\tilde{r}}_A. \tag{2.7}$$

Then with the use of the Friedmann equation (2.6), it is given by

$$\frac{dV}{dt} = L_p^{n-1}\left(\frac{A}{L_p^{n-1}}+\frac{8\pi\tilde{r}_A V}{n-1}[(n-2)\rho+np]\right), \tag{2.8}$$

where $A=n\Omega_n\tilde{r}_A^{n-1}$. The bulk Komar energy in an (n+1)-dimensional flat spacetime is given by [7]:

$$E = \frac{[(n-2)\rho+np]}{(n-2)}V. \tag{2.9}$$

With the use of the equipartition rule of energy, the bulk DOF is given by

$$N_{\mathtt{bulk}} = \frac{2|E|}{T_H} = -4\pi\tilde{r}_A V\frac{[(n-2)\rho+np]}{n-2}. \tag{2.10}$$

Note that $(n-2)\rho+np<0$, since $N_{\mathtt{bulk}}>0$.
The surface DOF can be identified as in Ref. [8] by

$$N_{\mathtt{sur}} = \alpha\frac{A}{L_p^{n-1}}, \tag{2.11}$$

PoS(FFP14)077

where $\alpha = (n-1)/2(n-2)$. The inclusion of the coefficient α is necessary to attain the correct identification with the Newton constant in the higher dimensional case [8]. Eq. (2.8) can then be written as

$$\frac{dV}{dt} = \tilde{L}_p^{n-1}(N_{\mathtt{sur}} - N_{\mathtt{bulk}}), \tag{2.12}$$

where $\tilde{L}_p$ is defined by

$$\tilde{L}_p^{n-1} \equiv L_p^{n-1}/\alpha.$$

This is the relation that used in Ref. [2] to derive the Friedmann equation in the (n+1)-dimensional flat Einstein case. Note that in the (3+1)-dimensional case, $\alpha = 1$ and $\tilde{L}_p = L_p$, and thus we recover the relation (1.1) that Padmanabhan conjectured.

Now, we will perform the same analysis in the Gauss-Bonnet and Lovelock gravity cases.

First, we consider the Gauss-Bonnet case. Recall that the apparent horizon and volume are given by the same formulas as in the Einstein case. The Friedmann equations for the Gauss-Bonnet gravity are given by [6]:

$$H^2 + \frac{k}{a^2} + \tilde{\alpha}\left(H^2 + \frac{k}{a^2}\right)^2 = \frac{16\pi L_p^{n-1}}{n(n-1)}\rho, \tag{2.13}$$

$$\left(\dot{H} - \frac{k}{a^2}\right)\left[1 + 2\tilde{\alpha}\left(H^2 + \frac{k}{a^2}\right)\right] + \left(H^2 + \frac{k}{a^2}\right)\left[1 + \tilde{\alpha}\left(H^2 + \frac{k}{a^2}\right)\right] = -\frac{8\pi L_p^{n-1}}{n(n-1)}[(n-2)\rho + np], \tag{2.14}$$

where $\tilde{\alpha} = (n-2)(n-3)\alpha$.

For the flat case ($k = 0$), the rate of the volume change (2.7) is given by

$$\frac{dV}{dt} = \frac{L_p^{n-1}}{(1 + 2\tilde{\alpha}\tilde{r}_A^{-2})}\left(\frac{A(1 + \tilde{\alpha}\tilde{r}_A^{-2})}{L_p^{n-1}} + \frac{8\pi\tilde{r}_A V}{n-1}[(n-2)\rho + np]\right). \tag{2.15}$$

In the above we used the Friedmann equation (2.14) with $k = 0$. The bulk DOF is given by the same formula as in the Einstein case:

$$N_{\mathtt{bulk}}^{GB} = -4\pi\tilde{r}_A V \frac{[(n-2)\rho + np]}{n-2}. \tag{2.16}$$

If we use the same ansatz for the surface DOF as in Ref. [2],

$$N_{\mathtt{sur}}^{GB} = \frac{A(1 + \tilde{\alpha}\tilde{r}_A^{-2})}{\tilde{L}_p^{n-1}}, \tag{2.17}$$

then Eq. (2.15) can be expressed as

$$\frac{dV}{dt} = \frac{\tilde{L}_p^{n-1}}{(1 + 2\tilde{\alpha}\tilde{r}_A^{-2})}(N_{\mathtt{sur}}^{GB} - N_{\mathtt{bulk}}^{GB}). \tag{2.18}$$

PoS(FFP14)077

This is still different from Padmanabhan's relation (1.1) by the factor $(1+2\tilde{\alpha}\tilde{r}_A^{-2})^{-1}$. To deal with this, the effective volume related to the effective area, which has been used to deal with the entropy of black holes in the Gauss-Bonnet gravity case, was introduced in [2]:

$$\tilde{A} = A\left(1+\frac{n-1}{n-3}2\tilde{\alpha}\tilde{r}_A^{-2}\right). \tag{2.19}$$

The effective volume can be obtained using the relation $d\tilde{V}/d\tilde{A} = \tilde{r}_A/(n-1)$. In this way, one can get the relation

$$\frac{d\tilde{V}}{dt} = \tilde{L}_p^{n-1}(N_{\mathtt{sur}}^{GB} - N_{\mathtt{bulk}}^{GB}), \tag{2.20}$$

which was used in Ref. [2] in the Gauss-Bonnet case. However, there is a catch in this derivation; in the above equation, on the right-hand side the plain ordinary volume was used for the bulk DOF (2.16), but on the left-hand side the effective volume was used to calculate the rate of volume change.

In order to avoid the above discrepancy, Ref. [3] used a severely modified version of Padmanabhan conjecture. Now we derive this modified version from the Friedmann equation. Here, the bulk and surface DOFs are the same as in the flat Einstein case which are given by (2.10) and (2.11). Then, it is easy to show that the rate of volume change (2.15) can be written as

$$\frac{dV}{dt} = L_p^{n-1}\frac{(N_{\mathtt{sur}} - N_{\mathtt{bulk}})/\alpha + \tilde{\alpha}K(N_{\mathtt{sur}}/\alpha)^{1+\frac{2}{1-n}}}{1+2\tilde{\alpha}K(N_{\mathtt{sur}}/\alpha)^{\frac{2}{1-n}}}, \tag{2.21}$$

where $K = (n\Omega_n/L_p^{n-1})^{2/(n-1)}$. Indeed Eq. (2.21) is the modified version used in Ref. [3].

Next we consider the Lovelock gravity [9] case. The spacetime can be described by the same metric (2.1) as in the Gauss-Bonnet case. The Friedmann equations in the Lovelock gravity are given by [6]:

$$\sum_{i=1}^{m}\hat{c}_i\left(H^2+\frac{k}{a^2}\right)^i = \frac{16\pi L_p^{n-1}}{n(n-1)}\rho, \tag{2.22}$$

$$\left(\dot{H}-\frac{k}{a^2}\right)\sum_{i=1}^{m}i\hat{c}_i\left(H^2+\frac{k}{a^2}\right)^{i-1} + \sum_{i=1}^{m}\hat{c}_i\left(H^2+\frac{k}{a^2}\right)^i = -\frac{8\pi L_p^{n-1}}{n(n-1)}[(n-2)\rho+np], \tag{2.23}$$

where $m = [n/2]$ and the coefficients are given by

$$\hat{c}_1 = 1,\ \hat{c}_i = c_i\prod_{j=3}^{2m}(n+1-j) \ \mathtt{for}\ \ i>1. \tag{2.24}$$

For the flat case ($k=0$), the rate of volume change (2.7) is given by

$$\frac{dV}{dt} = \frac{L_p^{n-1}}{\sum_{i=1}^{m} i\hat{c}_i\tilde{r}_A^{2(1-i)}}\left(\frac{A\sum_{i=1}^{m}\hat{c}_i\tilde{r}_A^{2(1-i)}}{L_p^{n-1}} + \frac{8\pi\tilde{r}_A V}{(n-1)}[(n-2)\rho+np]\right), \tag{2.25}$$

and the bulk DOF is the same as in the Einstein case,

$$N^L_{\texttt{bulk}} = -4\pi\tilde{r}_A V \frac{[(n-2)\rho + np]}{n-2}. \tag{2.26}$$

If we use the same ansatz for the surface DOF as in Ref. [2],

$$N^L_{\texttt{sur}} = \frac{A}{\tilde{L}_p^{n-1}} \sum_{i=1}^{m} \hat{c}_i \tilde{r}_A^{2(1-i)}, \tag{2.27}$$

then Eq. (2.25) can be written as

$$\frac{dV}{dt} = \frac{\tilde{L}_p^{n-1}}{\sum_{i=1}^m i\hat{c}_i \tilde{r}_A^{2(1-i)}} (N^L_{\texttt{sur}} - N^L_{\texttt{bulk}}). \tag{2.28}$$

Here again, we introduce the effective volume related to the effective area, as in [2],

$$\tilde{A} = A\sum_{i=1}^{m} \frac{i(n-1)}{(n-2i+1)} \hat{c}_i \tilde{r}_A^{2(1-i)}. \tag{2.29}$$

Using the relation $d\tilde{V}/d\tilde{A} = \tilde{r}_A/(n-1)$, Eq. (2.28) can then be written as

$$\frac{d\tilde{V}}{dt} = \tilde{L}_p^{n-1}(N^L_{\texttt{sur}} - N^L_{\texttt{bulk}}). \tag{2.30}$$

This is just the relation used in Ref. [2] in the Lovelock case. However, this derivation has the same problem as in the Gauss-Bonnet case; on the right-hand side the plain ordinary volume was used for the bulk DOF (2.26), but on the left-hand side the effective volume was used to calculate the rate of volume change.

To avoid this discrepancy, the same bulk and surface DOFs as in the flat Einstein case, Eqs. (2.10) and (2.11), were used in Ref. [3]. With these relations one can easily show that the rate of volume change (2.25) can be written as

$$\frac{dV}{dt} = L_p^{n-1} \frac{(N_{\texttt{sur}} - N_{\texttt{bulk}})/\alpha + \sum_{i=2}^m \tilde{c}_i K_i (N_{\texttt{sur}}/\alpha)^{1+\frac{2(i-1)}{1-n}}}{1+\sum_{i=2}^m i\tilde{c}_i K_i (N_{\texttt{sur}}/\alpha)^{\frac{2(i-1)}{1-n}}}, \tag{2.31}$$

where $K_i = (n\Omega_n/L_p^{n-1})^{2(i-1)/(n-1)}$. This is indeed the modified version used in Ref. [3] in the Lovelock case.

3. Emergence of cosmic space for a non-flat FRW universe

Now, we check Padmanabhan's conjecture in the non-flat (3+1)-dimensional case.

The invariant volume of the space enclosed by the apparent horizon $\tilde{r}_A$ for the (3+1)-dimensional non-flat FRW universe is given by

$$V_k = 4\pi a^3 \int_0^{\tilde{r}_A/a} \frac{r^2}{\sqrt{1-kr^2}} dr, \tag{3.1}$$

where $k = \pm 1$. In the limit $k \to 0$, V_k becomes $4\pi\tilde{r}_A^3/3$.

Extension to the non-flat case was first done in [4]. However, the aerial volume $V = \Omega_n \tilde{r}_A^n$ was used there instead of the proper invariant volume given above. Thus in Einstein gravity, with the rate of volume change (2.7), the Friedmann equation (2.6), the relation (2.3), and by adopting the same definitions of the bulk and surface DOFs as in the flat case, and from Eqs. (2.10) and (2.11), one can easily check that the rate of volume change is given by

$$\frac{dV}{dt} = \tilde{L}_p^{n-1} H\tilde{r}_A (N_{\mathtt{sur}} - N_{\mathtt{bulk}}), \tag{3.2}$$

where $V = \Omega_n \tilde{r}_A^n$. This is just the modified version that was used in [4] for the non-flat Einstein case.

For the Gauss-Bonnet and Lovelock theories, the same tactic as in Ref. [2] was adopted. Using the same volume as in the Einstein case and with the aid of the Friedmann equations (2.14) and (2.23), the rates of volume change in the Gauss-Bonnet and Lovelock cases are given by the RHS of Eqs. (2.15) and (2.25), respectively, with the same additional multiplication factor, $H\tilde{r}_A$.

Then using the same definitions of the bulk and surface DOFs for the flat case, one can get the rate of change of the effective volume as

$$\frac{d\tilde{V}}{dt} = \tilde{L}_p^{n-1} H\tilde{r}_A (N_{\mathtt{sur}}^{GB} - N_{\mathtt{bulk}}^{GB}) \tag{3.3}$$

in the Gauss-Bonnet case, and

$$\frac{d\tilde{V}}{dt} = \tilde{L}_p^{n-1} H\tilde{r}_A (N_{\mathtt{sur}}^{L} - N_{\mathtt{bulk}}^{L}) \tag{3.4}$$

in the Lovelock case. These two relations are the ones used in [4] in the Gauss-Bonnet and Lovelock cases. Obviously these results have the same problem that plagued [2]; namely the discordance between the effective volume for the volume change and the ordinary volume for the bulk DOF.

Finally, we will check the conjecture when the proper volume is used, and see how the modified version of [5] emerges. By applying the Friedmann equation (2.6) and with the use of the relation (3.1), the rate of the change of the invariant volume is given by

$$\begin{aligned} \frac{dV_k}{dt} &= 4\pi\tilde{r}_A^2 \left(\frac{\dot{\tilde{r}}_A}{H\tilde{r}_A} - 1 + H\tilde{r}_A \frac{V_k}{\bar{V}_k} \right) \\ &= L_p^2 \Big[\frac{A}{L_p^2} H\tilde{r}_A \frac{V_k}{\bar{V}_k} + \frac{\bar{V}_k}{V_k} 4\pi\tilde{r}_A (\rho + 3p) V_k \Big] \end{aligned} \tag{3.5}$$

where $\bar{V}_k = 4\pi\tilde{r}_A^3/3$ and $A = 4\pi\tilde{r}_A^2$.

Since the bulk Komar energy in the non-flat case is given by [7]

$$E_k = (\rho + 3p)V_k, \tag{3.6}$$

the bulk DOF with the assumption of the equipartition rule of energy is given by

$$N_{\mathtt{bulk}} = \frac{2|E_k|}{T_H} = -4\pi\tilde{r}_A(\rho + 3p)V_k. \tag{3.7}$$

Here $\rho + 3p < 0$, since $N_{\texttt{bulk}} > 0$. Applying the holographic principle, the surface DOF is given by

$$N_{\texttt{sur}} = A/L_p^2. \tag{3.8}$$

Now, the rate of volume change (3.5) can be written as

$$\frac{dV_k}{dt} = L_p^2 \left(H\tilde{r}_A \frac{V_k}{\bar{V}_k} N_{\texttt{sur}} - \frac{\bar{V}_k}{V_k} N_{\texttt{bulk}} \right) \equiv L_p^2 \Delta \mathcal{N}. \tag{3.9}$$

Here, we introduce $\Delta\mathcal{N}$ for later use.

Obviously, the above result indicates that Padmanabhan's conjecture does not hold in the non-flat case if one uses the proper invariant volume. In order to sidestep this stumbling block, new 'effective Planck length' which is a complicated function of time was introduece in [5]. Now we will see how this 'effective Planck length' is obtained.

From Eqs. (3.7) and (3.8) and using the Friedmann equation (2.6), one can write the following relation:

$$N_{\texttt{sur}} - N_{\texttt{bulk}} = \frac{4\pi \tilde{r}_A^2}{L_p^2} \frac{V_k}{\bar{V}_k} \left[\left(\frac{\dot{\tilde{r}}_A}{H\tilde{r}_A} - 1 + \frac{\bar{V}_k}{V_k} \right) \right] \equiv \Delta N. \tag{3.10}$$

Using this, one can now rewrite Eq. (3.9) in the following form:

$$\frac{dV_k}{dt} \equiv L_p^2 f_k(t) \Delta N, \tag{3.11}$$

where

$$f_k(t) \equiv \frac{\Delta\mathcal{N}}{\Delta N} = \frac{L_p^2}{4\pi \tilde{r}_A^2} \frac{\bar{V}_k}{V_k} \frac{\left(H\tilde{r}_A \frac{V_k}{\bar{V}_k} N_{\texttt{sur}} - \frac{\bar{V}_k}{V_k} N_{\texttt{bulk}} \right)}{\left(\frac{\dot{\tilde{r}}_A}{H\tilde{r}_A} - 1 + \frac{\bar{V}_k}{V_k} \right)}. \tag{3.12}$$

Relation (3.11) is what was used in Ref. [5], and $\sqrt{L_p^2 f_k(t)}$ was dubbed there as 'effective Planck length'. Nonetheless, due to the complicated time dependence of the function $f_k(t)$ one cannot say that the rate of volume change is simply proportional to $\Delta N = N_{\texttt{sur}} - N_{\texttt{bulk}}$, the crux of Padmanabhan's conjecture.

4. Conclusion

In this work we have shown that how Padmanabhan's original conjecture on the evolution of cosmic space and its modified versions in various cases can be obtained from the Friedmann equation.

Padmanabhan's original relation emerges without difficulty from the Friedmann equation in the flat Einstein case. However, in the non-flat Einstein case, the Friedmann equation emerges only when one uses the aerial volume instead of the proper volume. Furthermore, in the Gauss-Bonnet and Lovelock cases, Padmanabhan's conjecture has to be severely modified even for the flat case, jeopardizing its original intention.

In the non-flat Einstein case, a simply modified version was used at first in Ref. [4]. However, the aerial volume was used there instead of the proper invariant volume. Tried to make up for this shortcoming, the proper invariant volume was used in Ref. [5]. However, Padmanabhan's conjecture has to be modified so severely to lose its original meaning.

The idea behind Padmanabhan's conjecture that the expansion of the universe is being driven towards holographic equipartition seems quite reasonable and very attractive. Nevertheless, our analysis shows that the conjecture is applicable to the non-flat case only when the aerial volume is used in Einstein gravity.

On the other hand, if we assume that Padmanabhan's conjecture is right, then we may interpret this result as follows. The (spatial) flatness of our universe is dictated by the holography of nature, since Padmanabhan's conjecture whose backbone is holography is only compatible with (spatially) flat geometry.

Acknowledgments

This work was supported by the National Research Foundation (NRF) of Korea Grant funded by the Korean Government, NRF-2011-0025517.

References

[1] T. Padmanabhan, *Emergence and Expansion of Cosmic Space as due to the Quest for Holographic Equipartition,* arXiv:1206.4916.

[2] R.-G. Cai, *Emergence of space and spacetime dynamics of Friedmann-Robertson-Walker universe,* JHEP **11** (2012) 016.

[3] K. Yang, Y. Liu and Y. Wang, *Emergence of cosmic space and the generalized holographic equipartition,* Phys. Rev. D **86**, 104013 (2012).

[4] A. Sheykhi, *Friedmann equations from emergence of cosmic space,* Phys. Rev. D **87**, 061501(R) (2013).

[5] M. Eune and W. Kim, *Emergent Friedmann equation from the evolution of cosmic space revisited,* Phys. Rev. D **88**, 067303 (2013).

[6] R.-G. Cai and S. P. Kim, *First law of thermodynamics and Friedmann equations of Friedmann-Robertson-Walker universe,* JHEP **02** (2005) 050.

[7] R.-G. Cai, L. Cao and N. Ohta, *Friedmann equations from entropic force,* Phys. Rev. D **81**, 061501(R) (2010).

[8] E. P. Verlinde, *On the origin of gravity and the laws of Newton,* JHEP **04** (2011) 029.

[9] D. Lovelock, *The Einstein Tensor and Its Generalizations,* J. Math. Phys. **12**, 498 (1971); *The four-dimensionality of space and the Einstein tensor,* ibid. **13**, 874 (1972).