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Some Field Theoretic Issues Regarding the Chiral Magnetic (Vortical) Effects

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In this contribution, we shall address some field theoretic issues regarding the chiral magnetic (vortical)effects. The general structure of the chiral magnetic current consistent with the electromagnetic gauge invariance is obtained and the impact of the infrared divergence is examined. Some subtleties on the relation between the chiral magnetic effect and the axial anomaly are clarified through a careful examination of the infrared limit of the relevant thermal diagrams. In addition ,the two loop contributions to the chiral vortical conductivity are considered. The Kubo formula together with the anomalous Ward identity of the axial vector current suggest that there may be a nonzero correction to the coefficient of the T^2 term of the conductivity.

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1. Introduction

The chiral magnetic effect (CME) proposed in [1, 2, 3] provides a new probe of the QCD phase transition and the formation of quark-gluon plasma(QGP) via relativistic heavy ion collisions(RHIC). The physical picture of CME relies on the interplay between the helicity of a quark and the external magnetic field. For QGP of a nonzero axial charge density, a net electric current will be generated in (opposite to) the direction of the external magnetic field if the positive (negative) helicity is in excess.

The conditions that support CME are likely implemented in heavy-ion collisions Firstly, for off-central collisions, a strong magnetic field is produced perpendicular to the collision plane[4, 5, 6]; Secondly, because of the high temperature, there may be a sizable probability for the transition to a topologically nontrivial gluon configuration accompanied by a change of the axial charge according to the winding number [2, 7]. Thirdly, the de-confined quarks that carry the chiral magnetic current can travel sufficiently far before hadronization to lead to observable charge asymmetry perpendicular to the collision plane. It has been suggested recently that such a charge asymmetry is correlated with the baryon number asymmetry through a similar mechanism, the chiral vortical effect [8, 9]. So far a lot of efforts have been made to search for such strong field effects in heavy-ion collisions[10, 11, 12, 13, 14].

The chiral magnetic effect for a free quark gas in a static and homogeneous magnetic field $\mathscr{B}$ at thermal equilibrium has been analyzed in great details. With the aid of the grand partition function at a nonzero axial chemical potential μ_5, one obtains the chiral magnetic current $\mathbf{J} = \eta \mathbf{j}$ where $\eta = N_c \sum_f q_f^2$ with q_f the charge number of the flavor f and the current per unit charge given by the classical expression

$$\mathbf{j} = \frac{e^2}{2\pi^2}\mu_5 \mathscr{B}. \tag{1.1}$$

The chiral magnetic effect has also been examined with holographic models [16, 17, 18] and the lattice simulation [19]. A diagrammatic proof of (1.1) to all orders at high density has been attempted in [20].

It was pointed out in [18] that the naive axial charge is not the right object to define the grand canonical ensemble since it is not conserved because of the axial anomaly. Instead one needs to use a conserved modified axial charge. Furthermore, the author of [18] argued that the gauge invariance prevents a nonzero chiral magnetic current to be generated from the grand canonical ensemble defined with an naive axial charge and the chiral magnetic current comes from the anomaly, which is universal to all orders, the classical expression (1.1) is robust against higher order corrections.

In this contribution, we shall analyze the chiral magnetic effect via the current-current correlator in the light of Ref. [18].

2. The General Structure of the Chiral Magnetic Current

The Lagrange density of a quark matter at nonzero baryon number and axial charge densities is given by[27]

$$\mathscr{L} = -\frac{1}{4}F^l_{\mu\nu}F^l_{\mu\nu} - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \bar{\psi}\left(\gamma_\mu \frac{\partial}{\partial x_\mu} - igT^l A^l_\mu - ie\hat{q}A_\mu\right)\psi \tag{2.1}$$

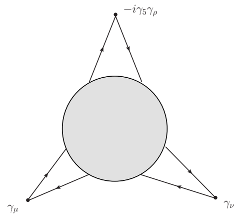

Figure 1: The diagrammatic representation of the contribution to the chiral magnetic current from the photon self-energy, where the contribution of each vertex to the Feynman amplitude is indicated explicitly.

$$+\ \mu\bar{\psi}\gamma_4\psi + \mu_5\left(\bar{\psi}\gamma_4\gamma_5\psi + i\Omega_4\right) + J_\mu^{\text{ext.}} A_\mu$$
$$+\ \text{gauge fixing terms and renormalization counter terms}$$

where $\hat{q}$ is the diagonal matrix of electric charge in flavor space, μ is the quark number chemical potential and μ_5 is the axial charge chemical potential. An external electric current $J_\mu^{\text{ext.}}$ has been added to the Lagrange.

The generating functional of the connected Green function of photons is the logarithm of the partition function $Z[J^{\text{ext.}}]$. The external current $J_\mu^{\text{ext.}}$ generates a nonzero thermal average of the electromagnetic potential, given by $\mathscr{A}_\mu(x) = -i\frac{\delta \ln Z}{\delta J_\mu^{\text{ext.}}(x)}$ The induced current in the medium up to the linear response is

$$J_i(Q) = \mathscr{K}_{ij}(Q)\mathscr{A}_j(Q), \tag{2.2}$$

where

$$\mathscr{K}_{ij}(Q) = -\Pi_{ij}(Q) - i\eta\frac{e^2}{4\pi^2}\mu_5\varepsilon_{ijk}q_k + O(\mathscr{A}^2) \tag{2.3}$$

where $\Pi_{ij}(Q)$ is the usual photon self-energy tensor, subject to higher order corrections.

The antisymmetric part of $\mathscr{K}_{ij}(Q)$,

$$\mathscr{K}_{ij}^A(Q) \equiv \frac{1}{2}[\mathscr{K}_{ij}(Q) - \mathscr{K}_{ji}(Q)] \tag{2.4}$$

which is odd in μ_5, carries odd parity and generates the chiral magnetic current. Expanding the response function $\mathscr{K}_{ij}^A(Q)$ in the powers of μ_5, we have $\mathscr{K}_{ij}^A(Q) = \mu_5\mathscr{K}_{ij}^{(1)}(Q) + O(\mu_5^3)$, where

$$\mathscr{K}_{ij}^{(1)}(Q) = -\frac{\partial}{\partial\mu_5}\Pi_{\mu\nu}(Q)|_{\mu_5=0} - i\eta\frac{e^2}{2\pi^2}\varepsilon_{ijk}q_k \tag{2.5}$$

and underlies the classical form of the chiral magnetic current (1.1).

The first term of (2.5) is represented by the 1PI diagram with two external vector vertices and an external axial vector vertex, shown in Fig.1, at $\mu = i$, $\nu = j$ and $\rho = 4$ denoted by $\Delta_{\mu\nu}(Q_1, Q_2)$ with Q_1, Q_2 the incoming 4-momenta at the photon vertices .In the limit $Q_1 \to -Q_2$ with $Q_1 \equiv Q = (\mathbf{q}, i\omega)$, we find that

$$\frac{\partial}{\partial\mu_5}\Pi_{\mu\nu}(Q)|_{\mu_5=0} = \Delta_{\mu\nu}(Q, -Q). \tag{2.6}$$

The rotation invariance and the Bose symmetry $\Delta_{\mu\nu}(Q_1,Q_2)=\Delta_{\nu\mu}(Q_2,Q_1)$ dictates the following most general tensorial structure

$$\begin{aligned}\Delta_{ij}(Q_1,Q_2) \;=\; & i\eta\frac{e^2}{2\pi^2}[C_0(q_1^2,q_2^2,\mathbf{q}_1\cdot\mathbf{q}_2;\omega)\varepsilon_{ijk}q_{1k}-C_0(q_2^2,q_1^2,\mathbf{q}_1\cdot\mathbf{q}_2;-\omega)\varepsilon_{ijk}q_{2k} \\ & + C_1(q_1^2,q_2^2,\mathbf{q}_1\cdot\mathbf{q}_2;\omega)\varepsilon_{jkl}q_{1k}q_{2l}q_{1i}-C_1(q_2^2,q_1^2,\mathbf{q}_1\cdot\mathbf{q}_2;-\omega)\varepsilon_{ikl}q_{1k}q_{2l}q_{2j}],\end{aligned} \tag{2.7}$$

$$\Delta_{4k}(Q_1,Q_2)=\eta\frac{e^2}{2\pi^2}C_2(q_1^2,q_2^2,\mathbf{q}_1\cdot\mathbf{q}_2;\omega)\varepsilon_{ijk}q_{1i}q_{2j}=\Delta_{k4}(Q_2,Q_1) \tag{2.8}$$

and $\Delta_{44}(Q_1,Q_2)=0$, where C_0, C_1 and C_2 are dynamical form factors.

It follows from (2.5) and (2.7) that

$$\mathscr{K}_{ij}^{A}(Q)=i\eta\frac{e^2}{2\pi^2}\mu_5[F(Q)-1]\varepsilon_{ijk}q_k+O(\mu_5^3) \tag{2.9}$$

with

$$F(Q)=-C_0(q^2,q^2,-q^2;\omega)-C_0(q^2,q^2,-q^2;-\omega). \tag{2.10}$$

The chiral magnetic current in a constant magnetic field corresponds to the limit $F(0)$, which is subtle as we shall see. The electromagnetic gauge invariance,

$$Q_{1\mu}\Delta_{\mu\nu}(Q_1,Q_2)=Q_{2\nu}\Delta_{\mu\nu}(Q_1,Q_2)=0 \tag{2.11}$$

gives rise

$$F(Q)=q^2[C_1(q^2,q^2,-q^2;\omega)+C_1(q^2,q^2,-q^2;-\omega)]+\omega[C_2(q^2,q^2,-q^2;\omega)-C_2(q^2,q^2,-q^2;-\omega)]. \tag{2.12}$$

If the infrared limit of the dynamical form factors C_1 and C_2 exists, then $F(0)$=0 . and there is no chiral magnetic current associated to the *naive* axial charge. This is the case in the static limit $q\to 0$ with $Q=(\mathbf{q},0)$ to one-loop order at nonzero T and/or μ. It remains so if there exists an nonperturbative IR cutoff to remove the $\frac{1}{q^2}$ singularities brought about by QCD corrections[21] (Such kind of singularities is likely to occur for diagrams with more than one quark loops linked by gluon lines). In that case, the chiral magnetic current takes the classical form (1.1) to all orders.

It is a common feature of thermal field theories that the different orders of the double limits $\lim_{q\to 0}\lim_{\omega\to 0}$ and $\lim_{\omega\to 0}\lim_{q\to 0}$ may not agree. While the former order of limits of C_1 and C_2 converges and leads to the classical form of the chiral magnetic current, the latter order of limits leads to IR divergence. The explicit one-loop calculation of the triangle diagram yields[27]

$$C_2(0,0,0;\omega)=\frac{1}{3\omega} \tag{2.13}$$

as $\omega\to 0$ and $\lim_{\omega\to 0}\lim_{q\to 0}F(Q)=\frac{2}{3}$. Consequently, the magnitude of the one-loop chiral magnetic current is reduced to one third of the classical magnitude. This is consistent with the direct one-loop calculation in the literature [15] . Since the form factor $F(Q)$ is not linked to the axial anomaly, the chiral magnetic current in this order of limits is likely to be subject to higher order corrections.

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The IR singularity also shows up via the massless poles if the zero temperature and zero chemical potential limits are taken prior to the limit $Q\to 0$ and $\Delta_{\mu\nu}(Q_1,Q_2)$ becomes fully covariant then. To the one-loop triangle diagram gives rise to

$$C_1(q^2,q^2,-q^2;\omega)=\frac{1}{2(q^2-\omega^2)} \tag{2.14}$$

and

$$C_2(q^2,q^2,-q^2;\omega)=-\frac{\omega}{2(q^2-\omega^2)}. \tag{2.15}$$

Both C_1 and C_2 are infrared divergent and we find $F(0)=1$ and therefore zero chiral magnetic current for $T=\mu=0$ but $\mu_5\neq 0$.

3. The one-loop contribution

The one-loop contribution to the chiral magnetic current has been discussed extensively in the literature. Here we shall supplement this calculation with the Pauli-Villars regularization, since the photon self-energy as a whole suffers from the UV divergence. As the regularization respects the gauge invariance, the result will be consistent with the Ref.[18] and the statement of the previous section. The trivial color-flavor factor η will be suppressed below.

The antisymmetric part of the one-loop self-energy tensor is parametrized as

$$\Pi^A_{ij}(Q)=-i\frac{e^2}{2\pi^2}\mu_5F_1(q,\omega)\varepsilon_{ijk}q_k, \tag{3.1}$$

with $F_1(q,\omega)$ at $\mu_5=0$ corresponds to the one-loop approximation of $F(q,\omega)$ as defined in Eq. (2.10). The dependences on the spatial momentum and the energy are indicated separately here.

The static limit : At zero frequency, $q_0=0$, we find that

$$\lim_{q\to 0}\lim_{\omega\to 0}F_1(q,\omega)=0. \tag{3.2}$$

This result is expected according to the discussion in the last section because the nonzero Matsubara frequency, $(2n+1)\pi T$ regularizes the infrared behavior of the quark propagator even in the massless limit. If, on the other hand, T and μ as well as the quark mass are set to zero first, we find $F_1(q,0)=-1$ at $\mu_5=0$, in agreement with the covariant result reported at the end of the last section.

Massless limit: In the massless limit, $m=0$ we have

$$\lim_{q\to 0}\lim_{\omega\to 0}F_1(q,\omega)=0 \tag{3.3}$$

but

$$\lim_{\omega\to 0}\lim_{q\to 0}F_1(q,\omega)=\frac{2}{3} \tag{3.4}$$

consistent with the result reported in [15]. The nonzero value of the latter limit signals infrared divergence of the form factor $C_2(q^2,q^2,-q^2;\omega)$ defined in the last section under the same orders of limits.

4. The Relation to the Triangle Anomaly

In the section 2, we related the chiral magnetic current to the infrared limit of the three point Green's function in Fig.1 with two electric currents and the fourth component of the axial vector current. We analyzed the general structure of the chiral magnetic current as is required by the electromagnetic Ward identity. For the sake of simplicity, we restricted our attention to zero energy flow . To explore the the impact of the anomalous axial current Ward identity, this restriction can be be relaxed to nonzero energy-momentum fow $K = (\mathbf{k}, k_0)$ at the axial vector vertex.

At a nonzero temperature and/or chemical potential, the limit $K \to 0$ becomes very subtle. Because of the discreteness of the energy in the Matsubara Green's function, one has to switch to the real time formalism for the analysis, of which, the closed time path (CTP) Green's function is most convenient. Explicit calculations of the triangle diagram via the CTP show that

$$\lim_{\mathbf{k}\to 0}\lim_{k_0\to 0}\Lambda_{ij4}(Q_1,Q_2) \neq \lim_{k_0\to 0}\lim_{\mathbf{k}\to 0}\Lambda_{ij4}(Q_1,Q_2). \tag{4.1}$$

The limit order on RHS leads to zero CME current, while the limit order on LHS gives rise to result of the last section, obtained from the Matsubara formulation and its analytic continuation to real energy. Therefore, there is no contradiction between the universality of the anomaly and the statement of [18]. We also explored the subtlety of this infrared limit in general using CTP formalism [27].

5. Chiral vortical effects

The chiral magnetic effect and the chiral vortical effect have been actively investigated for recent years. Because of the triangle anomaly, an external magnetic field and/or an fluid vorticity will induce an electric current, a baryon current and an axial vector current in a relativistic plasma. These currents will lead to separations of electric charges, the baryon numbers and chirality, which may be observed in the quark-gluon plasma created through heavy ion collisions [3, 22]. To the order of the linear response, we have

$$\begin{aligned}\vec{J}_{\rm em} &= \sigma^B_{\rm em}\vec{B} + \sigma^V_{\rm em}\vec{\omega}\\ \vec{J}_{\rm b} &= \sigma^B_{\rm b}\vec{B} + \sigma^V_{\rm b}\vec{\omega}\\ \vec{J}_5 &= \sigma^B_5\vec{B} + \sigma^V_5\vec{\omega}.\end{aligned} \tag{5.1}$$

for the currents driven by the magnetic field and the fluid vorticity. The anomalous transport coefficients σ's above have been exploreded from field theoretic point of view and from the holographic method [3, 23, 9, 24, 25, 26]. An important question along the former approach is if these coefficients are free from the higher order corrections of coupling constants, like their origin, the triangle anomaly. in case of CME, the nonrenormalization of $\sigma^B_{\rm em}$ in the homogeneous limit of a static magnetic field has been established [18, 27] and the classical expression (1.1) holds to all orders of gauge coupling. We shall address the parallel issue for the chiral vortical conductivity σ^V_5 to see whether it is subject to higher order corrections.

The anomalous transport coefficient σ^V_5 was first introduced in [9] where the anomalous Ward identity together with the 2nd law of thermodynamics yields for a relativistic plasma with an axial charge chemical potential μ_5 yields $\sigma_5^V = \frac{\mu_5^2}{2\pi^2}$. It was soon extended to [9, 31]

$$\sigma_5^V = \frac{\mu_5^2}{2\pi^2} + cT^2 \tag{5.2}$$

with $c = \frac{1}{12}$ using Kubo formula [30] at one-loop calculation [32]. This result is also confirmed by kinetic theories[33]. The authors of [32] related the T^2 term to the gravity anomaly and a recent analysis [34] from a geometric point of view within a general hydrodynamical framework suggests the nonrenormalization of the T^2 term. But a field theoretic aspect regarding the higher corrections remains murky.

In a recent work [28], the authors addressed the issue based on diagrammatic analysis. They generalized the Coleman-Hill theorem [35] to the stress tensor insertion and proved the nonrenormalization of σ_5^V for a σ model.We shall study this issue for gauge theories. Upon a close examination at two-loop level, we found a diagram that does have contribution[29]. to σ_5^V below.

For the sake of clarity, we shall consider a QED plasma with the Lagrangian density

$$\mathscr{L} = -\frac{1}{4e_0^2} V^{\mu\nu} V_{\mu\nu} - i\bar{\psi}\gamma^\mu D_\mu \psi + \frac{1}{2} h^{\mu\nu} T_{\mu\nu} + A^\mu J_{5\mu}, \tag{5.3}$$

where $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ is the electromagnetic field tensor with V_μ in gauge potential, the covariant derivative $D_\mu = \partial_\mu - iV_\mu$ and we have added couplings of the axial current $J_{5\mu}$ to an external axial vector field A^μ and the energy-momentum tensor $T_{\mu\nu}$ to a metric perturbation $h^{\mu\nu}$. The axial current $J_{5\mu}$ satisfies its anomalous Ward identity .

According to the Kubo formula [30], the chiral vortical conductivity σ_5^V is given by the correlators between the axial current density and energy flux density as $\mathscr{G}_{ij}(Q) = \sigma_5^V \varepsilon_{ijk} q_k$ in the limit $Q = (0, \vec{q}) \to 0$, where

$$\mathscr{G}_{ij}(Q) = -\int_0^\infty dt \int d\vec{r} e^{-i\vec{q}\cdot\vec{r}} \frac{\mathrm{Tr}\{e^{-\beta H}[J_{5i}(\vec{r},t), T_{0j}(0,0)]\}}{\mathrm{Tr} e^{-\beta H}} \tag{5.4}$$

and can be evaluated perturbatively in terms of thermal diagrams, where H the Hamiltonian corresponding to the Lagrangian density (5.3) at $A_\mu = h_{\mu\nu} = 0$.

All two-loop diagrams are shown in Fig.2. If there were no axial anomaly, the sum of all diagrams (a)-(f) would be of the order $O(q^2)$ in the limit $Q = (0, \vec{q}) \to 0$ according to the Coleman-Hill like argument employed in [28]. As to the contribution from the anomaly, following an elegant argument of [28], sum of that from Fig. 2(b-f) couples only to the trace of the metric perturbation. Therefore the anomaly does not contribute the diagrams Fig.2(b)-(f) with the insertion of an off-diagonal component. The anomaly contribution to diagram Fig.2(a), however, is not covered by the above argument and has to be examined separately. After lengthy calculation,we obtain its contribution to the CVE[29]

$$\sigma_5^{V(2)} = \frac{e_0^2}{48\pi^2} T^2 \tag{5.5}$$

and the coefficient c of (5.2) takes the form

$$c = \frac{1}{12} + \frac{e_0^2}{48\pi^2} \tag{5.6}$$

Because of the universality of the axial anomaly, the second term above are intact if the fermion number and the axial charge chemical potentials are switched on. In another word, the μ_5^2 of (5.2) is not renormalized by higher order terms and our result is not in contradiction with the thermodynamic argument of [9].

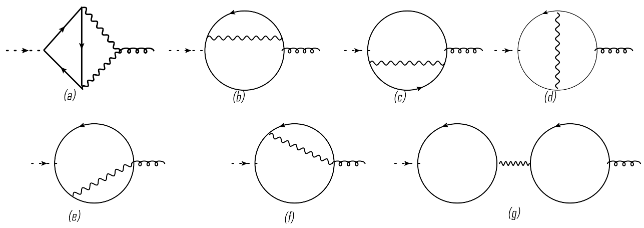

Figure 2: The two-loop diagrams for the chiral vortical conductivity

Our analysis can be trivially generalized a QCD like nonAbelian gauge theory with N_c colors and N_f flavors,

$$\sigma_5^V = N_c N_f \left(\frac{\mu_5^2}{2\pi^2} + cT^2 \right), \tag{5.7}$$

we have

$$c = \frac{1}{12} + \frac{N_c^2 - 1}{2N_c} \frac{g_0^2}{48\pi^2} \tag{5.8}$$

and the 2nd term is not suppressed in the large N_c limit for a fixed 't Hooft coupling $N_c g_0^2$. This makes the strong 't Hooft coupling limit nontrivial, an issue that may be addressed by the holographic principle.

6. Summary Discussions

In this contribution, From quantum field theory view, we addressed the question that if the anomalous transport coefficients(CME,CVE for instance) are free from the higher order corrections of coupling constants and some subtleties in their calculations[27, 29].

we investigated the interplay between the gauge invariance and the infrared limit in the CME. The part of the induced electric current that is linear in the axial chemical potential μ_5 and the magnetic field $\mathscr{B}$ is divided into two terms, i.e.

$$\mathbf{J}(Q) = -\eta \frac{e^2}{2\pi^2} \mu_5 F(Q) \mathscr{B}(Q) + \eta \frac{e^2}{2\pi^2} \mu_5 \mathscr{B}(Q) \tag{6.1}$$

where the first term corresponds to the loop diagrams of the photon self-energy tensor and the second term comes from the Chern-Simons term of the conserved axial charge $\tilde{Q}_5$, which is dictated by the anomaly. The gauge invariance relates the form factor $F(Q)$ to two form factors, C_1 and C_2 underlying a three point diagram of two vector current vertices and an axial current vertex. If the infrared limit of these form factors exists, $F(0) = 0$ to all orders of coupling and the classical form of the chiral magnetic current in a constant magnetic field, eq. (1.1) emerges. Our statements are illustrated with explicit one-loop calculations subject to the Pauli-Villars regularization. At zero temperature, however, both C_1 and C_2 are infrared divergent and $F(0) = 1$. Consequently, the two terms on RHS of (6.1) cancel each other and the chiral magnetic current vanishes. At a nonzero temperature and/or a nonzero chemical potential, $F(0)$ depends on how the limit $Q \to 0$ is approached. The magnitude of the chiral magnetic current is reduced if the zero momentum limit is taken prior to the zero energy limit, as is implied by the infrared divergence of C_2 under the same order of limits. More subtle is the situation with a coordinate dependent μ_5. If the four momentum associated with μ_5, $K = (\mathbf{k}, ik_0)$ is set to zero in the order $\lim_{\mathbf{k}\to 0}\lim_{k_0\to 0}$, the results of sections 2 and 3 are recovered. With the opposite order of the limit, however, $F(0) = 1$ as is dictated by the anomaly and the two terms of (6.1) cancel again.

Unlike what happens with the axial anomaly, the difference between different orders of the infrared limits is unlikely robust against higher order corrections [37]. Since the ambiguity stems from quasi particle poles, it will disappear when the quasi particle weight is diminished by strong coupling. Then the chiral magnetic current will revert to its classical expression with the order $\lim_{\omega\to 0}\lim_{q\to 0}$ of the infrared limit $Q \to 0$. This is consistent with the holographic result reported in [16].

For the CVE transport coefficient, we obtained the next-leading order correction for the first time using the Kubo formula due to dynamical gauge fields' contribution, revealing the normalizibility of the CVE anomalous transport coefficient, unlike the axial anomaly itself [29].

It would be interesting to study the (Non)renormalization of Anomalous Conductivities systematically in strong coupling with Holographic approach[36] as well as lattice method .

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