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Freeze-out Condition from Lattice QCD and the Role of Additional Strange Hadrons

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This contribution focuses on the determination of freeze-out conditions heavy-ion collisions through comparisons between lattice QCD calculations and experimental measurements. First, we discuss how the freeze-out condition can be determined by comparing lattice QCD calculated and experimentally measured electric charge fluctuations. Next, we present thermodynamic signatures of additional, yet unobserved strange hadrons and discuss their influence on the freeze-out temperature.

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1. Introduction

Proximity of a second order criticality, such as the $O(4)$ chiral phase transition or the QCD critical point, is universally manifested through long-range correlations at all length scales, resulting in increased fluctuations of the order parameter. These fluctuations can be quantified through the Gaussian (variance) as well as non-Gaussian (skewness, kurtosis *etc.*) cumulants of the distribution of the order parameter. The higher non-Gaussian cumulants grows with higher powers of the correlation length [1] and become increasingly sensitive to the proximity of a criticality. Moreover, even qualitative features, such as the sign change and the associated non-monotonicity, of these non-Gaussian cumulants can encode the presence of a nearby critical region [2, 3, 4]. The non-Gaussian cumulants of the order parameter can be accessed in heavy-ion experiments via the event-by-event fluctuations of various conserved charges and particle multiplicities [5, 6, 7, 8, 9]. In this vein, a major focus of the Beam Energy Scan program at the Relativistic Heavy-Ion Collider is measurements of the event-by-event fluctuations of particle multiplicities and conserved charges [10, 11, 12, 13, 14, 15].

2. Charge fluctuations and freeze-out conditions

Although a direct lattice QCD computation at non-zero baryon (μ_B), electric charge (μ_Q) or strangeness (μ_S) chemical potentials remains difficult due to the infamous sign problem, higher cumulants of fluctuations of these conserved charges can be computed on the lattice using the well established method of Taylor expansion. In this method one expands the logarithm of the QCD partition function, $\ln Z$, or the pressure, $P = -T\ln(Z)/V$, in a power series of the chemical potentials around vanishing values of the chemical potentials. For the electric charge chemical potential

$$\frac{P(T,\mu_Q)}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!}\chi_n^Q(T)\left(\frac{\mu_Q}{T}\right)^n, \text{ where } \quad \chi_n^Q(T) = \frac{1}{VT^3}\left.\frac{\partial^n \ln \mathscr{Z}}{\partial(\mu_Q/T)^n}\right|_{\mu_Q=0}. \tag{2.1}$$

Here, V and T denote the volume and the temperature respectively. Since the generalized charge susceptibilities, χ_n^X, are defined at vanishing chemical potentials, standard lattice QCD techniques can be used to compute them. Among several conserved charges, the net electric charge is of special interest as its fluctuations can be measured both in experiments [12, 15] as well as in lattice QCD through the calculations of its generalized susceptibilities [16, 17]. Further, Taylor expansion in a powers of μ_B, around $\mu_B = 0$, can be employed to obtain these susceptibilities for $\mu_B > 0$,

$$\chi_n^Q(T,\mu_B) = \sum_{k=0}^{\infty} \frac{1}{k!}\chi_{kn}^{BQ}(T)\left(\frac{\mu_B}{T}\right)^k, \quad \text{where} \quad \chi_{kn}^{BQ}(T) = \left.\frac{\partial^k \chi_n^Q}{\partial(\mu_B/T)^k}\right|_{\mu_B=0}. \tag{2.2}$$

These susceptibilities are the measures for the fluctuations of the net electric charge

$$\chi_1^Q(T,\mu_B) = \frac{1}{VT^3}\langle N_Q\rangle, \qquad \chi_2^Q(T,\mu_B) = \frac{1}{VT^3}\left\langle(\delta N_Q)^2\right\rangle,$$
$$\chi_3^Q(T,\mu_B) = \frac{1}{VT^3}\left\langle(\delta N_Q)^3\right\rangle, \qquad \chi_4^Q(T,\mu_B) = \frac{1}{VT^3}\left[\left\langle(\delta N_Q)^4\right\rangle - 3\left\langle(\delta N_Q)^2\right\rangle^2\right], \tag{2.3}$$

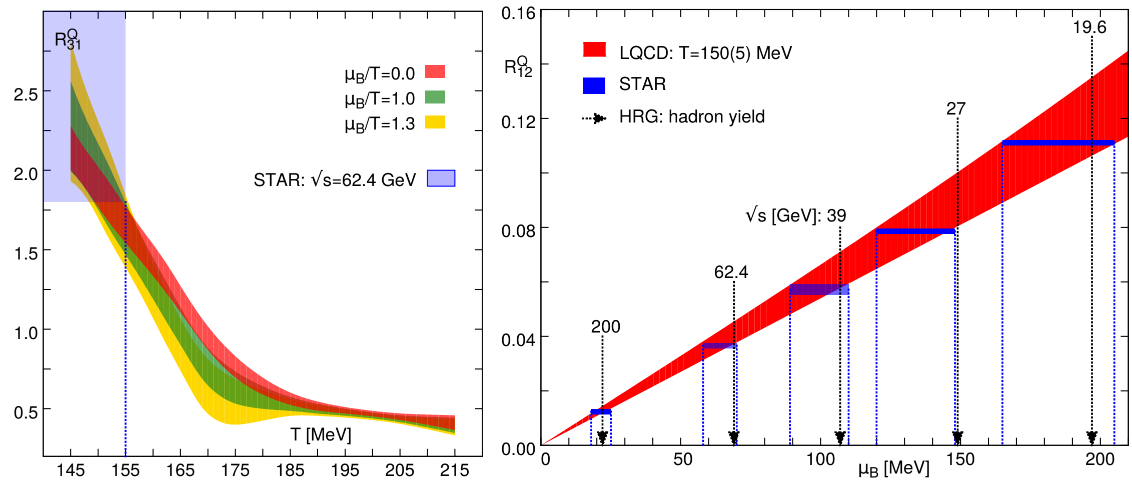

Figure 1: (Left) A comparison between the lattice QCD results for R^Q_{31} and the STAR data [12] for $(S_Q\sigma^3_Q)/M_Q$ at $\sqrt{s} = 62.4$ GeV. The overlap of the experimental results with the lattice QCD calculations provides an upper bound on the freeze-out temperature $T^f \leq 155$ MeV. (Right) Lattice QCD results for R^Q_{12} as a function of μ_B compared with the STAR data [12] for M_Q/σ^2_Q in the temperature range $T^f = 150(5)$ MeV. The overlap regions of the experimentally measured results with the lattice QCD calculations provide estimates for the freeze-out chemical potential μ^f_B for a given $\sqrt{s}$. The arrows indicate the corresponding values of μ^f_B obtained from the statistical hadronization model fits to the experimentally measured hadrons yields [18].

where N_Q is the net (positive minus negative) charge and $\delta N_Q = N_Q - \langle N_Q\rangle$.

On the other hand, through the measurements of the event-by-event distributions of the net electric charge, heavy-ion experiments provide various cumulants, mean (M_Q), variance (σ_Q), skewness (S_Q), and kurtosis (κ_Q), of the electric charge fluctuations for given beam energy ($\sqrt{s}$) [12]

$$M_Q\left(\sqrt{s}\right) = \langle N_Q\rangle \,, \qquad \sigma^2_Q\left(\sqrt{s}\right) = \left\langle (\delta N_Q)^2\right\rangle \,,$$
$$S_Q\left(\sqrt{s}\right) = \frac{\left\langle (\delta N_Q)^3\right\rangle}{\sigma^3_Q} \,, \qquad \kappa_Q\left(\sqrt{s}\right) = \frac{\left\langle (\delta N_Q)^4\right\rangle}{\sigma^4_Q} - 3 \,. \tag{2.4}$$

Thus, the charge susceptibilities obtained from the lattice QCD calculations and the cumulants measured in the heavy-ion experiments are directly related to each other through the appropriate volume-independent ratios [19]

$$\frac{M_Q\left(\sqrt{s}\right)}{\sigma^2_Q\left(\sqrt{s}\right)} = \frac{\chi^Q_1\left(T,\mu_B\right)}{\chi^Q_2\left(T,\mu_B\right)} \equiv R^Q_{12} \,, \tag{2.5a}$$
$$\frac{S_Q\left(\sqrt{s}\right)\sigma^3_Q\left(\sqrt{s}\right)}{M_Q\left(\sqrt{s}\right)} = \frac{\chi^Q_3\left(T,\mu_B\right)}{\chi^Q_1\left(T,\mu_B\right)} \equiv R^Q_{31} \,. \tag{2.5b}$$

In heavy-ion collision experiments the only tunable parameter is the beam energy, $\sqrt{s}$. However, to gain access to the information regrading the QCD phase diagram this tunable parameter needs to be related to the thermodynamic variables, temperature and baryon chemical potential. Traditionally, this $\sqrt{s} \leftrightarrow (T, \mu_B)$ mapping has been done by relying on the statistical hadronization model based analysis [18]. Recent advances in heavy-ion experiments as well as in lattice QCD calculations have placed us placed us in a unique situation where, for the first time, this mapping can now be obtained through direct comparisons between the experimental results and rigorous (lattice) QCD calculations. Recently, it has been shown [19] that by directly comparing lattice QCD calculations for R^Q_{31} [Eq. 2.5b] and R^Q_{12} [Eq. 2.5a] with their corresponding cumulant ratios measured in heavy-ion experiments it is possible to extract the thermal parameters, namely the freeze-out temperature, T^f, and the freeze-out baryon chemical potential μ_B^f. The feasibility of such a procedure has been demonstrated in Refs. [20, 21, 22]. Fig. 1 illustrates a recent example of such a comparison and the subsequent determination of the freeze-out parameters.

3. Influence of additional hadrons on the freeze-out temperature

As can be seen from the Fig. 1, due to large errors on the experiment results for $(S_Q\sigma_Q^3)/M_Q$, at present, only an upper limit on freeze-out temperature can be determined using the method described above. Thus, a complementary procedure for determination of T^f, relying on a separate observable that can be extracted both from heavy-ion experiments and lattice QCD calculations, is certainly welcome. Recently, such a complementary procedure for determination of T^f has been proposed in Ref. [23]. This procedure takes advantage of the fact that the initially colliding nuclei in heavy-ion collisions are free of net strangeness. Thus, the conservation of strangeness under strong interaction ensures that the QGP medium created during the collisions of these heavy-ions is also strangeness neutral.

By Taylor expanding the net strangeness density, $\langle n_S\rangle(\mu_B, \mu_S)$, in μ_B and subsequently imposing the strangeness neutrality condition, $\langle n_S\rangle(\mu_B, \mu_S) = 0$, for a homogeneous thermal medium the strangeness chemical potential, μ_S, can be obtained as [23]

$$\frac{\mu_S}{\mu_B} = s_1(T) + s_3(T)\left(\frac{\mu_B}{T}\right)^2 + \mathscr{O}\left[\left(\frac{\mu_B}{T}\right)^4\right] . \tag{3.1}$$

The coefficients s_1, s_3, *etc.* consist of various generalized baryon, charge and strangeness susceptibilities defined at vanishing chemical potentials and are accessible to standard lattice QCD computations at zero chemical potentials. Fig. 2 (left) shows the leading order contribution to μ_S/μ_B, *i.e.* $s_1(T)$. A comparison of the lattice result with the predictions from the hadron resonance gas model reveal that the inclusion of only experimentally observed, as listed by the Particle Data Group [24], hadrons fails to reproduce the lattice data around the crossover region. Note that, while $\mu_S(T, \mu_B)$ is unique in QCD, for a hadron gas it depends on the relative abundances of the open strange baryons and mesons. For fixed T and μ_B, a strangeness neutral hadron gas having a larger relative abundance of strange baryons over open strange mesons naturally leads to a larger value of μ_S. Astonishingly, the inclusion of additional but yet unobserved strange hadrons predicted within the quark model [25, 26] provides a much better agreement with lattice results, hinting that these additional hadrons become thermodynamically relevant close to the crossover temperature [23]. In fact, other lattice thermodynamics studies also indicate that additional, unobserved charm hadrons also become thermodynamically relevant close to the QCD crossover [27].

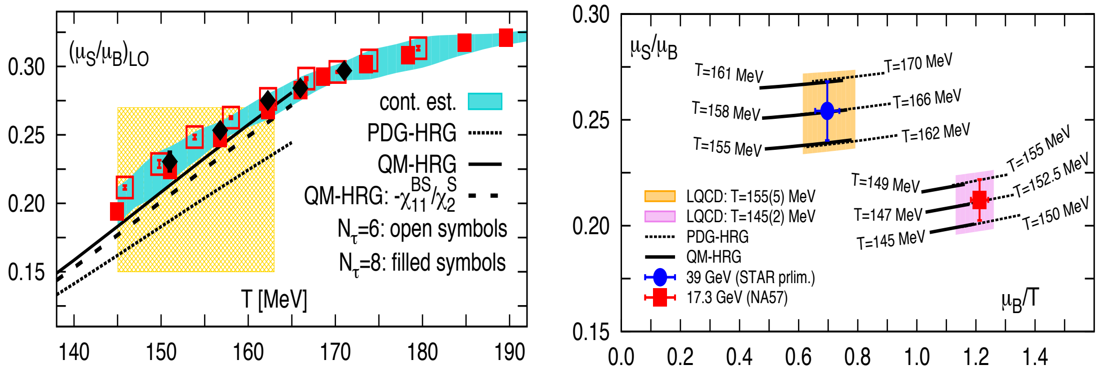

Figure 2: (Left) Lattice QCD results [23] for μ_S/μ_B at the leading order, *i.e.* $s_1(T)$. The dotted line (PDG-HRG) shows the results of hadron resonance gas model containing only hadrons listed by the Particle Data Group [24]. The solid line (QM-HRG) depicts the result for a hadron gas when additional, yet unobserved, quark model predicted strange hadrons [25, 26] are included. The shaded region indicate the chiral crossover region $T_c = 154(9)$ MeV [28]. (Right) A comparison between the experimentally extracted values of $(\mu_S^f/\mu_B^f, \mu_B^f/T^f)$ (filled points) with the lattice QCD results for μ_S/μ_B (shaded bands) [23]. The lattice QCD results are shown for $\mu_B/T = \mu_B^f/T^f$. The temperature range where lattice QCD results match with μ_S^f/μ_B^f provide the values of T^f, *i.e.* $T^f = 155(5)$ MeV and $145(2)$ MeV for $\sqrt{s} = 39$ GeV and 17.3 GeV, respectively.

On the other hand, the experimentally measured yields of the strangeness, S, anti-baryons to baryons at the freeze-out are determined by the thermal freeze-out parameters (T^f, μ_B^f, μ_S^f) [18]

$$R_H(\sqrt{s}) = \exp\left[-\frac{2\mu_B^f}{T^f}\left(1-\frac{\mu_S^f}{\mu_B^f}|S|\right)\right]. \tag{3.2}$$

By fitting the experimentally measured values of R_Λ, R_Ξ and R_Ω, corresponding to $|S| = 1, 2$ and 3, the values of μ_S^f/μ_B^f and μ_B^f/T^f, as 'observed' in a heavy-ion experiment at a given $\sqrt{s}$, can easily be extracted. Matching these experimentally extracted values of μ_S^f/μ_B^f with the lattice QCD results for μ_S/μ_B as a function of temperature, one can determine the freeze-out temperature T^f. Fig. 2 (right) illustrate this procedure. Once again, the inclusion of additional, yet unobserved strange hadrons in the hadron gas model leads to very similar values of the freeze-out temperatures as obtained using the lattice data. However, including only the hadrons listed by the Particle Data Group [24] yields freeze-out temperatures that are $5-8$ MeV smaller.

Acknowledgments

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