Exploring confinement in SU(N) gauge theories with double-trace Polyakov loop deformations

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Recent results applying resurgence theory to finite-temperature field theories yield a detailed analytic structure determined by topological excitations. We examine finite-temperature SU(N) lattice gauge theories in light of these results. Double-trace Polyakov loop deformations move through different regions of the confined phase characterized by continuous change in the adjoint Polyakov loop. Lattice models show how the behavior of monopole constituents of calorons can change in the different confining regions. We conjecture that the pure SU(N) gauge theory is close to a special symmetric point where monopole effects give rise to Casimir string-tension scaling.

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1. Introduction

Topological features of four-dimensional gauge theories on $R^3 \times S^1$, deformed from a pure gauge theory by either a double-trace deformation or by periodic adjoint fermions, have been been shown to lead to confinement in a region where semiclassical methods are valid [1 – 3]. Moreover, this region is smoothly connected to the usual low-temperature confining region [1]. Recents results applying resurgence theory to such models [4, 5] suggest a detailed analytic structure determined by the interplay of perturbative and non-perturbative physics. The behavior of observables are described by a trans-series

$$\langle O \rangle = \sum_{n=0}^{\infty} p_{0,n} \lambda^n + \sum_c e^{-S_c/\lambda} \sum_{n=0}^{\infty} p_{c,n} \lambda^n. \tag{1.1}$$

This sum includes contributions which are not topologically stable, such as an instanton-anti-instanton contributions, and applies even in theories without topologically stable contributions [6]. In light of these results, we want to explore the structure of the confining phase under the action of double-trace Polyakov loop deformations. Such deformations interpolate between different Abelian limiting models and change the interpretation of topological excitations and their role. In lattice field theories, duality can be used in place of semiclassical continuum techniques to obtain the topological content of deformed field theories. What emerges from the interplay between continuum and lattice models is a consistent picture of confinement in which the non-universal weighting of topological objects determines string tension scaling.

2. The $O(3)$ model in $d = 2$

A simple model of the behavior we wish to study in gauge theories is afforded by the $O(3)$ model in $d = 2$ space-time dimensions. The continuum Euclidean action is given by

$$S = \int d^2x \frac{1}{2g^2} (\nabla \vec{\sigma})^2 \tag{2.1}$$

where the fields are constrained to $\vec{\sigma}^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 1$. Like QCD, the $O(3)$ model is asymptotically free and has instantons.

We can deform the $O(3)$ model into an XY-like model by deforming the action [7, 8]

$$S \to S - \int d^2x \frac{1}{2} h\sigma_3^2. \tag{2.2}$$

With $h < 0$, $\sigma_3 = 0$ is preferred; as h becomes increasingly negative, the model approaches the XY, or $O(2)$ limit. The XY model in $d = 2$ has the well-known Kosterlitz-Thouless critical behavior, associated with the transition from a low-temperature (g^2 small) phase dominated by massless spin-waves to a high-temperature phase (g^2 large), a vortex-antivortex plasma with a mass gap. Each Kosterlitz-Thouless vortex-antivortex pair are the constituents of an $O(3)$ instanton, with $\sigma_3 \to \pm 1$ at each vortex or antivortex core. It is also interesting to consider the behavior of the deformed $O(3)$ model for $h > 0$. In this case, the boundary conditions are $\sigma_3 \to \pm 1$ in the limit $|z| \to \infty$, and the system becomes increasingly like the Ising model as $h \to \infty$. The antivortex moves to infinity
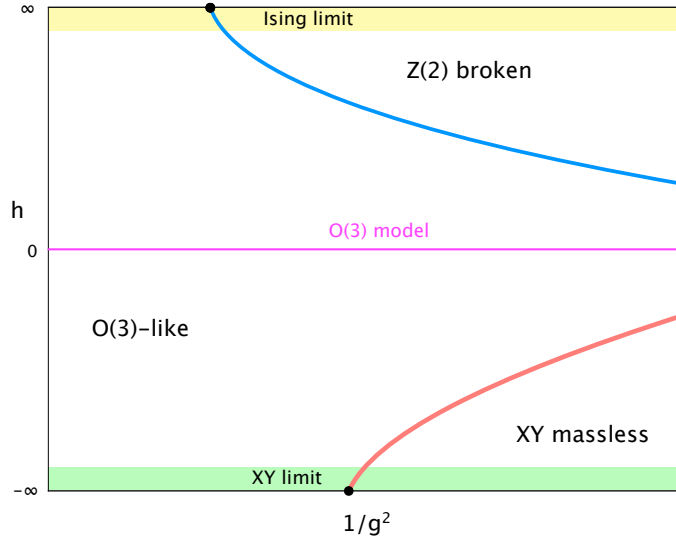
Figure 1: The phase diagram of the deformed $O(3)$ model as a function of $1/g^2$ and h.

and the interpretation of an instanton as a vortex-antivortex pair is lost. For $h > 0$, instantons look like flipped spins in an Ising model low-T expansion: At infinity, $\sigma_3 \to -1$ and $\sigma_3 \to +1$ at the center of the instanton.

The deformed model has three distinct phases, as sketched in Figure 1. There is an Ising-like phase, where the $Z(2)$ symmetry $\sigma_3 \to -\sigma_3$ is spontaneously broken. There is a massless spin-wave phase with the behavior of the low-temperature phase of the XY model. There is a massive, disordered phase with the $O(3)$ model intermediate between the XY and Ising limits. The expectation value of σ_3^2, which is conjugate to h, determines where we are in the phase diagram. The Ising limit corresponds to $\left\langle \sigma_3^2 \right\rangle \to 1$ while the XY limit is obtained when $\left\langle \sigma_3^2 \right\rangle \to 0$. For both $h > 0$ and $h < 0$, instantons disorder the system, but their role appears completely different. If we are deep in the $Z(2)$ broken phase, with $\left\langle \sigma_3 \right\rangle \approx 1$, instantons are highly suppressed and the dilute instanton gas approximation may be used. In order to approach the critical line, instantons must lower $\left\langle \sigma_3 \right\rangle$ towards zero, and the dilute instanton gas approximation must fail. In the massless XY phase, vortices are tightly bound in pairs, and instantons play no role in large-distance behavior. After crossing the $O(2)$ critical line, a Coulomb gas of vortices and antivortices forms, leading to a massive phase. It is only in this region that we have good analytic control of the behavior of topological excitations outside of the dilute instanton gas approximation. If we consider $O(N)$ models with $N > 3$, it is physically obvious that they must have Ising and XY limits obtained using a similar deformation and a similar phase diagram. Although instantons in these higher $O(N)$ models are not topologically stable, the inclusion of topological excitations is absolutely necessary to reproduce the phase structure. This is a confirmation of a key assumption of resurgence theory as applied to quantum field theories: non-topologically stable solutions matter.

3. High-T confinement on $R^3 \times S^1$

Gauge theories on $R^3 \times S^1$ can be treated in a manner similar to the deformed $O(3)$ model discussed above. It is often convenient to identify the circumference L of S^1 with the inverse temperature T^{-1}. In the limit where T is large, two things occur: The coupling $g^2(T) \to 0$ gets weak as $T \to \infty$, and global $Z(N)$ is spontaneously broken. It is possible to modify the action to restore $Z(N)$ symmetry with a so-called double-trace deformation. This is a term added to the action, depending only on Polyakov loops in the adjoint representation, that favors the confined, $Z(N)$-unbroken phase. For the gauge group $SU(2)$, a simple choice for such a deformation is [1]

$$S \to S - \int d^2x H_A \left| Tr_F P \right|^2 \tag{3.1}$$

with $H_A < 0$. For larger gauge groups, a more complicated deformation is required [9 – 11]. It is also possible to restore $Z(N)$ symmetry using fermions in the adjoint representation with periodic boundary conditions [2, 3]. In the high-T limit, A_4 behaves as a three-dimensional scalar with a vacuum expectation value, leading to Euclidean monopole solutions. These in turn give rise to confinement in "spatial" Wilson loops lying in R^3 at constant x_4.

The Euclidean monopoles are constituents of instantons [12 – 15] and confine [2, 9]. Dimensional reduction yields confinement by monopole gas as in the $d = 3$ Georgi-Glashow model [16]. The monopole gas is represented by a sine-Gordon model for SU(2):

$$S_{eff} = \int d^3x \left[\frac{g^2(T)T}{32\pi^2} (\partial_j \sigma)^2 - 4y \cos(\sigma) \right] \tag{3.2}$$

where $y \propto T^3 (\Lambda/T)^{11/3}$. This program works in lattice gauge theories lattice as well [17, 18]. In the lattice case, Abelian lattice duality methods can be used to uncover vortex effects, as opposed to steepest descent methods in continuum models. On the lattice, vortices of higher charge appear naturally. They do not have the instabilities seen in continuum analysis of higher-charge solutions, because on the lattice integrals over translational zero modes are replaced by lattice sums.

The effect of H_A on the gauge theory is similar to that of h on the $O(3)$ model, with some differences. Negative H_A increases the deconfinement temperature, while positive H_A promotes $Z(2)$ breaking and decreases the deconfinement temperature. The observable $Tr_A P$ indicates where we are in the phase diagram of the deformed $SU(2)$ gauge theory. The limit $H_A \to \infty$ is an Ising limit, where $Tr_A P \to 2$ and $Tr_F P \to \pm 1$. When H_A is sufficiently negative, we obtain $Tr_A P \sim -1$ and $Tr_F P \sim 0$, and the gauge theory behaves as a $U(1)$ gauge theory at large distances. The pure gauge theory, with $H_A = 0$, has both $Tr_A P \sim 0$ and $Tr_F P \sim 0$, and sits between the two limiting behaviors. A special feature of the deformed $SU(2)$ model is the existence of a tricritical point where the deconfinement transition changes from 2nd-order to 1st-order. This new critical point is in the same universality class as the tricritical point in the Blume-Emery-Griffiths (BEG) model [19], and has non-Ising critical indices. The observable $Tr_A P$ plays a role in the gauge theory analogous to the role of vacancies in the BEG model.

In the semiclassical region of SU(N), inclusion of only the lightest monopole states gives a generalized sine-Gordon model based on an N-component field ρ with action

$$S_{mag} = \int d^3x \left[\frac{T}{2} (\partial_j \rho)^2 - 2y \sum_{k=1}^N \cos\left(\frac{2\pi}{g} \alpha_k \cdot \rho \right) \right] \tag{3.3}$$
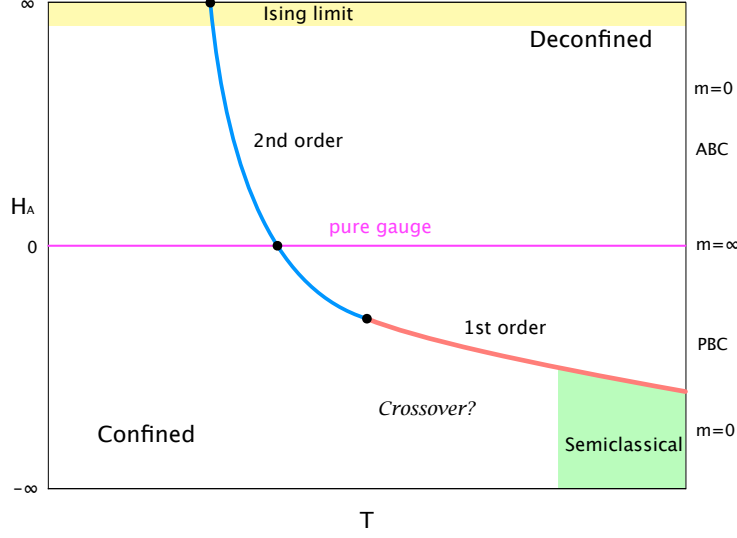
Figure 2: The phase diagram of the deformed $SU(2)$ gauge theory as a function of T and H_A.

where the α_k are the affine roots of SU(N) [9]. This model is one of a whole class of generalized sine-Gordon models that differ in how different weights are included. In general, string tensions for different N-alities may be calculated as the surface tension of kink solutions connecting different vacua. the lowest string tension is known analytically. If all roots are included with equal weighting, an ansatz of straight-line motion in the Lie algebra leads to Casimir scaling [20]. If only positive roots are included, sine-law scaling is obtained [21]. Thus any deformation that changes the weighting of monopoles configurations in the partition function may also change string-tension scaling laws.

4. Lattice models

Using the ideas from previous sections, it is now easy to construct a lattice gauge theory with Casimir scaling. It is an Abelian theory with a $U(1)^{N-1}$ gauge group and a global $Z(N)$ symmetry in an $R^3 \times S^1$ geometry. We begin with a $U(1)^N$ lattice gauge theory with a Villain action [22]:

$$S_1 = \frac{1}{2g^2} \sum_{a=1}^{N} \sum_p Tr \left(\partial_\mu \phi_\nu^a - \partial_\nu \phi_\mu^a - 2\pi n_{\mu\nu}^a \right)^2 \tag{4.1}$$

The integer-valued plaquette variables $n_{\mu\nu}^a$ are summed over all integers to enforce the symmetry. We define a set of monopole currents:

$$m_\mu^a (X) = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} \partial_\nu n_{\rho\sigma}^a (x) \tag{4.2}$$

The remaining degrees of freedom can be integrated out, giving a Coulomb gas representation:

$$S_{dual} = \frac{2\pi^2}{g^2} \sum_{a=1}^{N} \sum_{R,R'} m_\mu^a (R) G \left(R - R' \right) m_\mu^a \left(R' \right) \tag{4.3}$$

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We can restrict $U(1)^N$ to $U(1)^{N-1}$ using a periodic delta function:

$$S_2 = -i \sum_{x,\mu} p_\mu(x) \left[\sum_a \phi_\mu^a(x) \right] \tag{4.4}$$

This gives rise to an electric interaction in a Coulomb gas representation

$$S_{dual} = \frac{2\pi^2}{g^2} \sum_{R,R'} m_\mu^a(R) G(R-R') m_\mu^a(R') + \frac{g^2}{2} \sum_{r,r'} p_\mu(r) G(r-r') p_\mu(r')$$
$$- i \sum_{r,R} \left(\sum_a m_\mu^a(R) \right) \Theta_{\mu\nu}(R-r) p_\nu(r) \tag{4.5}$$

representing the interaction of electric and magnetic charges; see, *e.g.*, [23, 24]. As in continuum field theories, we can add a term to the action that favors or disfavors the $Z(N)$ center subgroup of $SU(N)$. On $R^3 \times S^1$, we can ensure that the dominant terms will be short monopole world lines with $m_4^a = +1$ and $m_4^b = -1$ for pairs (a,b) with $a \neq b$. Unlike the continuum case, an expectation value for A_4, and hence the Polyakov loop, does not play an integral role. All roots are naturally included with equal weight. After dimensional reduction and transformation of the monopole gas to a sine-Gordon form, this leads naturally to Casimir scaling.

5. Conclusions

Resurgence theory suggests the need to include a large class of non-perturbative phenomena in quantum field theories, considerably beyond what has heretofore been included. Deformations allow us to change the non-perturbative content in non-trivial ways. In the case of $SU(N)$ gauge theories on $R^3 \times S^1$, we can interpolate between an $U(1)^{N-1}$ instanton gas picture of confinement and a $Z(N)$ gauge theory, with the pure gauge theory in the middle, while remaining in the confining phase. $Tr_A P$ indicates where we are in the phase diagram, with the system behaving as a generalization of the BEG model. Instantons change their role in moving between regions, but are important throughout. It seems plausible that these deformations may change string tension ratios among different N-alities. Casimir scaling in particular is associated with the inclusion of monopoles on a democratic basis, and appears naturally in a $U(1)^{N-1}$ lattice model.

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