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Search for $Z_c(3900)$ on the lattice with twisted mass fermions

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We study the low-energy scattering of $D\bar{D}^*$ using lattice QCD with $N_f = 2$ twisted mass fermion configurations with three pion mass values. The threshold scattering parameters, namely the scattering length a_0 and effective range r_0, for the s-wave scattering in $J^P = 1^+$ channel are extracted. Our results indicate that the interaction of this channel is weakly repulsive. Therefore our results do not support the $D\bar{D}^*$ bound state interpretation of the state $Z_c(3900)$. To further investigate the properties of $Z_c(3900)$, we redo the calculation with some improvements. We employ the stochastic LapH smearing method, which greatly improves the precision of our results. We also enlarge the operator basis and study the coupled channel effects.

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1. Introduction

Recently a charged resonance-like structure $Z_c^{\pm}(3900)$ has been observed at BESIII in the $\pi^{\pm}J/\psi$ invariant mass spectrum from the $Y(4260)$ decays [1]. The same structure was confirmed shortly by the Belle [2] and CLEO collaborations [3]. The mass of this structure is close to the DD^* threshold. One possible interpretation is a molecular bound state formed by the $\bar{D}^*$ and D mesons. Studying the interaction between $\bar{D}^*$ and D mesons is important to investigate the properties of $Z_c^{\pm}(3900)$.

This proceedings is organized as follows. In Sec. 2 we briefly describe some computational strategies. In Sec. 3 we present the simulation details and the results. More details about this work can be found in Ref. [4]. We started a new study to address some unsolved issues in Ref. [4]. The methodology and very preliminary results of this new study are presented in Sec. 4.

2. Strategies for the computation

2.1 Twisted boundary condition

Under the ordinary periodic boundary condition, the three-momentum is quantized as $\mathbf{k} = \frac{2\pi}{L}\mathbf{n}$ with $\mathbf{n}$ being a vector of integers. For a typical lattice size the smallest nonzero momentum is still too large to study low-energy scattering. In order to increase the resolution in momentum space, we have adopted the so-called twisted boundary conditions (TBC) [5, 6] for the valence quark fields. The strategy follows that in Ref. [7]. Basically, the quark field $\psi_{\theta}(\mathbf{x},t)$, when transported by an amount of L along the spatial direction i (designated by unit vector $\mathbf{e}_i$), $i = 1,2,3$, will change a phase $e^{i\theta_i}$:

$$\psi_{\theta}(\mathbf{x}+L\mathbf{e}_i,t) = e^{i\theta_i}\psi_{\theta}(\mathbf{x},t)\ , \tag{2.1}$$

where $\theta = (\theta_1,\theta_2,\theta_3)$ is the twisted angle (vector) for the quark field in spatial directions.

Introduce the new quark fields$\psi'(\mathbf{x},t) = e^{-i\boldsymbol{\theta}\cdot\mathbf{x}/L}\psi_{\theta}(\mathbf{x},t)$, it is easy to verify that $\psi'(\mathbf{x},t)$ satisfy the conventional periodic boundary conditions if the un-primed field $\psi_{\theta}(\mathbf{x},t)$ satisfies the twisted boundary conditions (2.1). For Wilson-type fermions, this transformation is equivalent to the replacement of the gauge link: $U_\mu(x) \Rightarrow U'_\mu(x) = e^{i\theta_\mu a/L}U_\mu(x)$, for $\mu = 0,1,2,3$ and $\theta_\mu = (0,\theta)$.

Normal hadronic operators are constructed using the primed fields. For example, a quark bilinear operator $\mathscr{O}_\Gamma(\mathbf{x},t) = \bar{\psi}'_f\Gamma\psi'_{f'}(\mathbf{x},t)$, after summing over the spatial index $\mathbf{x}$, will carry a non-vanishing momenta: $\mathbf{p} = (\theta_f - \theta_{f'})/L$. The allowed momenta on the lattice are thus modified to:

$$\mathbf{k} = \frac{2\pi}{L}\left(\mathbf{n}+\frac{\theta}{2\pi}\right). \tag{2.2}$$

In principle, we can have any value of momentum on lattice by the variation of the twist angle. Note that, For a generic value of the twist angle θ, the parity is broken. Parity is a good symmetry only for special values $\theta_i = 0$ or π. In this work we apply four different twist angles $\theta = (0,0,\pi/8)$, $\theta = (0,0,\pi/4)$, $\theta = (0,0,\pi)$ and $\theta = (\pi,\pi,0)$. Parity breaking at $\theta = (0,0,\pi/8)$ and $\theta = (0,0,\pi/4)$ has to be taken into account when extracting the scattering information.

2.2 Lüscher's finite volume method

We use Lüscher's finite volume method to extract the scattering parameters. Lüscher has shown that the energy eigenvalue of a two-particle system in a finite box is related to the elastic scattering phase of the two particles in the infinite volume [8, 9]. In the center of mass frame, we write the energy of the two-particle system $E_{1,2}$ as

$$E_{1,2} = \sqrt{m_1^2 + \mathbf{k}^2} + \sqrt{m_2^2 + \mathbf{k}^2} \, . \tag{2.3}$$

It is convenient to define a variable $q^2 = \mathbf{k}^2 L^2/(2\pi)^2$. What Lüscher's formula tells us is a direct relation of q^2 and the elastic scattering phase shift $\delta(q)$ in the infinite volume. In the simplest case of s-wave elastic scattering, it reads [9] :

$$q \cot \delta_0(q) = \frac{1}{\pi^{3/2}} \mathscr{Z}_{00}(1; q^2) \, , \tag{2.4}$$

where $\mathscr{Z}_{00}(1; q^2)$ is the zeta-function which can be evaluated numerically with given q^2 value. Once the two-particle energy $E_{1,2}$ is obtained from lattice simulations, one can infer the elastic energy shift by applying Lüscher's formula given above.

In the case of parity breaking, if we ignore higher partial waves and consider the s-wave and p-wave, Lüscher's formula becomes

$$[q \cot \delta_0(q^2) - m_{00}][q^3 \cot \delta_1(q^2) - m_{11}] = m_{01}^2 \, , \tag{2.5}$$

where m_{00}, m_{11} and m_{01} are known functions of q^2.

3. Simulation details and Results

In this study, we have utilized $N_f = 2$ twisted mass gauge field configurations generated by European Twisted Mass Collaboration (ETMC) at $\beta = 4.05$ for three different pion mass values 300 MeV, 420 MeV and 485 MeV. All lattices used are of the size $32^3 \times 64$ with lattice spacing $a \simeq 0.067$fm. For the valence charm quark, we have used the Osterwalder-Seiler action [10].

3.1 Extraction of two-particle energy levels

Two-particle energies are measured in Monte Carlo simulations by measuring corresponding correlation functions, which are constructed from appropriate interpolating operators with definite symmetries. Since $Z_c^{\pm}(3900)$ was observed in $J/\psi\pi^{\pm}$ final states, the preferable quantum numbers of this state are $I^G(J^P) = 1^+(1^+)$. Expressing in terms of particle contents explicitly, the operator should be $D^{*+}\bar{D}^0 + \bar{D}^{*0}D^+$.

On the lattice, the rotational symmetry group $SO(3)$ is broken down to the corresponding point group. The two-particle system with $J^P = 1^+$ transforms according to T_1 irreducible representation of the cubic group. Thus, we use the following operator to create the two charmed meson state from the vacuum,

$$\mathscr{O}^i_\alpha(t) = \sum_{R \in G} \left[D^{*+}(R \circ \mathbf{k}_\alpha, t+1)\bar{D}^0(-R \circ \mathbf{k}_\alpha, t) + \bar{D}^{*0}(R \circ \mathbf{k}_\alpha, t+1)D^+(-R \circ \mathbf{k}_\alpha, t) \right] \, , \tag{3.1}$$

θ	Irrep	$\Delta E[t_{\min},t_{\max}](\mu=0.003)$		$\Delta E[t_{\min},t_{\max}](\mu=0.006)$		$\Delta E[t_{\min},t_{\max}](\mu=0.008)$	
$\mathbf{0}$	T_1	0.001(1)[8,13]	0.054(2)[7,11]	-0.000(1)[10,14]	0.059(2)[7,11]	0.005(2)[13,17]	0.046(1)[7,11]
$(0,0,\frac{\pi}{8})$	A_1	-0.006(2)[9,16]	0.046(5)[10,15]	-0.005(2)[11,16]	0.051(2)[9,14]	0.005(4)[17,23]	0.056(4)[12,18]
	E	0.005(2)[10,15]	0.061(2)[6,11]	0.016(5)[18,23]	0.064(2)[9,14]	-0.002(1)[4,10]	0.061(4)[14,20]
$(0,0,\frac{\pi}{4})$	A_1	-0.005(2)[9,13]	0.051(5)[10,14]	-0.004(2)[11,16]	0.052(2)[9,14]	0.006(2)[13,20]	0.056(4)[12,18]
	E	0.005(2)[10,15]	0.061(2)[7,11]	0.022(8)[20,25]	0.065(2)[9,14]	-0.001(1)[4,12]	0.065(5)[14,20]
$(0,0,\pi)$	A_2	-0.015(5)[14,19]		0.014(7)[19,24]		0.021(5)[18,24]	
	E	-0.003(10)[17,25]		0.043(9)[20,27]		0.028(6)[19,26]	
$(\pi,\pi,0)$	B_1	0.003(10)[17,22]		0.026(6)[18,26]		0.059(8)[19,26]	
	B_2	0.025(5)[12,17]		0.031(1)[6,12]		0.026(5)[16,22]	
	B_3	0.029(1)[5,10]		0.020(4)[14,21]		0.029(1)[6,12]	

Table 1: Results for the energy shifts ΔE obtained in our calculations for various cases. The time interval $[t_{\min},t_{\max}]$ from which we extract the values of ΔE are also listed. These ranges are relevant for the estimation of the error for the zeta functions as described in the text.

where $\mathbf{k}_\alpha$ is a chosen three-momentum mode. In this study we applied three different modes corresponding to $\mathbf{k}_\alpha^2 = 0$, $(\frac{2\pi}{L})^2$, and $2(\frac{2\pi}{L})^2$ respectively. $G = O(\mathbb{Z})$ designates the cubic group and $R \in G$ is an element of the group. We have used the notation $R \circ \mathbf{k}_\alpha$ to represent the momentum obtained from $\mathbf{k}_\alpha$ by applying the operation R on $\mathbf{k}_\alpha$. To avoid complicated Fierz rearrangement terms, we have put the two mesons on two neighboring time-slices.The single particle operators for the pseudoscalar and vector charmed mesons are local quark bilinears.

In the case of twisted boundary conditions, the operators are constructed similarly with the primed fields for the up/down quark fields. The only difference is the discrete version of the rotational symmetry. It has been reduced from O_h to one of its subgroups: D_{4h}, D_{2h} or C_{4v}, depending on the particular choice of θ. For more details, see Ref. [4].

To obtain the two-particle energies, we need to calculate the correlation matrix $C_{\alpha\beta} = \langle \mathscr{O}_\alpha(t)\mathscr{O}^\dagger_\beta(0)\rangle$ and solve the generalized eigenvalue problem:

$$C(t)\cdot v_\alpha(t,t_0) = \lambda_\alpha(t,t_0)C(t_0)\cdot v_\alpha(t,t_0)\ . \tag{3.2}$$

The eigenvalues $\lambda_\alpha(t,t_0)$ can be shown to behave like [11] $\lambda_\alpha(t,t_0) \simeq e^{-E_\alpha(t-t_0)} + \cdots$, where E_α is the eigenvalue of the Hamiltonian.

The real signal for the eigenvalues in our simulation turns out to be somewhat noisy. To enhance the signal, the following ratio was attempted:

$$\mathscr{R}_\alpha(t,t_0) = \frac{\lambda_\alpha(t,t_0)}{C^{\mathscr{V}}(t-t_0,\mathbf{0})C^{\mathscr{P}}(t-t_0,\mathbf{0})} \propto e^{-\Delta E_\alpha\cdot(t-t_0)}\ , \tag{3.3}$$

where $C^{\mathscr{V}}(t-t_0,\mathbf{0})$ and $C^{\mathscr{P}}(t-t_0,\mathbf{0})$ are one-particle correlation functions with zero momentum for the corresponding mesons. Therefore, ΔE_α is the difference of the two-particle energy measured from the threshold of the two mesons. The energy difference ΔE_α can be extracted by fitting $\mathscr{R}_\alpha(t,t_0)$ to an exponential. The final results for ΔE_α, together with the corresponding ranges from which the ΔE_α's are obtained, are summarized in Table 1. We only list the lowest two energy levels since we are not going to use those higher energy levels to extract the scattering parameters.

3.2 Extraction of scattering information

Close to the scattering threshold, the quantity $q\cot\delta(q^2)$ has the following effective range

	B_0	R_0	B_1	R_1	χ^2/dof
$\mu = 0.003$	-0.513(0.008)	-2.3(0.1)	-0.047(0.006)	-0.1(0.2)	47.0/11
$\mu = 0.006$	-0.16(0.01)	-0.8(0.2)	0.29(0.05)	-2.6(0.3)	28.1/ 11
$\mu = 0.008$	-0.67(0.09)	2.4(0.8)	-0.037(0.008)	-0.1(0.2)	17.0/11

Table 2: Fit results with parity-conserving and parity-mixing points.

expansion:

$$q^{2l+1}\cot\delta_l(q^2) = B_l + \frac{1}{2}R_l q^2 + \cdots \ , \tag{3.4}$$

where B_l and R_l are related to the scattering length a_l and the effective range r_l by $B_l = [L/(2\pi)]^{2l+1}a_l^{-1}$ and $R_l = [L/(2\pi)]^{2l-1}r_l$.

Using Eq. (2.4), (2.5) and (3.4), we fit our parity-conserving and parity-mixing data simultaneously to get the s-wave and p-wave scattering parameters. The fitting results for three different pion masses are collected in Table 2. To get a feeling of the quality of the fits, we plot the quantity $q\cot\delta_0(q^2)$ vs. q^2 in Fig. 1. This figure illustrates the situation for all three pion masses in our simulation.

It is straightforward to convert the fitted values of B_0, R_0, B_1 and R_1 into physical units. The s-wave scattering length and effective range are summarized in Table. 3. It is observed that the values we get for a_0 do not seem to follow a simple regular chiral extrapolation within the range that we have studied. We therefore kept the individual values for a_0 and r_0 for each case. This irregularity might be caused by the smallness of the value $m_\pi L \sim 3.3$ for $\mu = 0.003$. To circumvent this, one has to study a larger lattice. The negative values of a_0 indicate that the two constituent mesons have weak repulsive interactions at low energies. Therefore, our result does not support the bound state scenario for these two mesons. This conclusion is consistent with a recent lattice study using Wilson fermions [12].

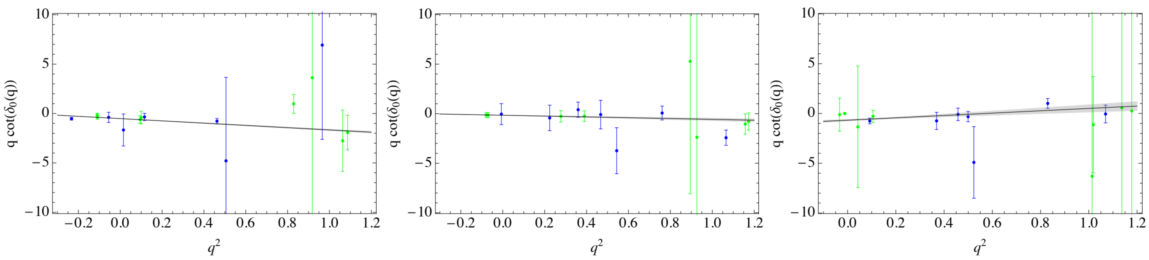


Figure 1: Results for the correlated fits as described in the text. Each panel, from left to right, corresponds to $\mu = 0.003, 0.006$ and 0.008, respectively. The quantity $q\cot\delta_0(q^2)$ is plotted versus q^2 for all our data points, both parity-conserving (blue) case and parity-mixing case (green). The straight lines and the bands indicate the fitted result for $F_0(q^2) = B_0 + (R_0/2)q^2$ and the corresponding uncertainties in B_0 and R_0.

4. A new study

In the previous section we concluded that our results do not support the $D\bar{D}^*$ bound state interpretation for $Z_c(3900)$. However, we are not able to rule out the possible appearance of a bound

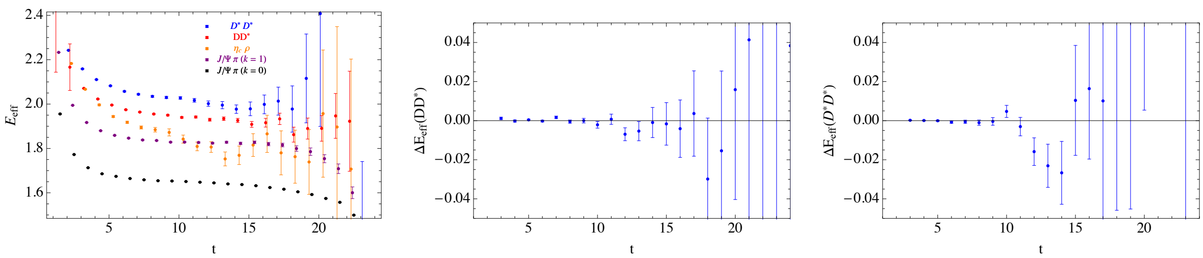
	$\mu = 0.003$	$\mu = 0.006$	$\mu = 0.008$
a_0[fm]	-0.67(1)	-2.1(1)	-0.51(7)
r_0[fm]	-0.78(3)	-0.27(7)	0.82(27)

Table 3: The values for a_0 and r_0 in physical units.

state if the pion mass is lowered and the volume is increased accordingly. We only considered the $D\bar{D}^*$ (and its conjugation under G-parity) interpolating operators. In principle, these operators have overlap with $J/\Psi\pi$, $\eta_c\rho$ and $D^*\bar{D}^*$ states. A more complete set of operators and a coupled-channel study is required. Ideally the operator basis should include all possible two-meson operators and diquark-antidiquark operators with the same quantum numbers in the targeted energy range. We started a new study in order to address these issues.

We use $N_f = 2+1+1$ twisted mass gauge field configurations generated by ETMC. Configurations with various pion masses, volumes and lattice spacings are available for this study. The details about the configurations can be found in Ref. [13]. The action for valence quarks is Osterwalder-Seiler action [10]. Stochastic Laplacian Heavyside quark smearing method [14] is applied for the computation of the quark propagators. With this method we can have all-to-all propagators and it is efficient to compute the correlation functions of a large basis of operators.

As a start point, we calculated the spectrum using five operators which feature the two-meson systems with particle contents $D\bar{D}^*$, $J/\Psi\pi$, $\eta_c\rho$ and $D^*\bar{D}^*$. The operators are built in a similar way as in Eq. 3.1. For $D\bar{D}^*$, $\eta_c\rho$ and $D^*\bar{D}^*$ we only used the zero momentum mode. For $J/\Psi\pi$ we used two momentum modes. Five energy levels are obtained from the variational method as described in Sec. 3.1. The effective masses are shown in Fig. 2(a). The effective mass plots of the energy shift for $D\bar{D}^*$ and $D^*\bar{D}^*$ are also shown Fig. 2(b) and 2(c) respectively. From the plots we can see that the interactions in these two channels are very weak. The analysis to extract the scattering parameters using coupled-channel Lüscher's fomula is undergoing. We are also going to include more operators, such as the diquark-antidiquark operators and the non-local operators.



(a) (b) (c)

Figure 2: Effective mass plots for the five energy levels obtained from the five interpolating operators described in the text (a), the energy shift of $D\bar{D}^*$ (b) and the energy shift of $D^*\bar{D}^*$ (c).

5. Conclusions

In this proceedings, we present an exploratory lattice study for the low-energy scattering of $(D\bar{D}^*)^{\pm}$ two meson system near the threshold using single-channel Lüscher's finite-size technique.

We obtained the s-wave scattering length a_0 and effective range r_0 in the $J^P = 1^+$ channel. Our results indicate that the interaction in this channel is weakly repulsive.

We started a new study in the effort of investigating the lattice artifacts and coupled channel effects. We present some very preliminary results from this study. More results will be available in the near future and provide more solid information about the properties of the charged charmonium-like state $Z_c(3900)$.

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