

## Resummed Higgs $p_T$ distribution at NNLO+NNLL in bottom-quark annihilation

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The resummed transverse-momentum distribution for Higgs bosons produced via bottom-quark annihilation at the LHC is presented. Our results are obtained in the five-flavor scheme to NNLO+NNLL accuracy. We present a theoretical prediction which consistently matches the cross section at small and large transverse momenta. Theoretical uncertainties are derived from a variation of the unphysical scales entering the calculation. Their size is significantly reduced with respect to lower orders.

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## 1. Introduction

After the discovery of a Higgs boson [1, 2], differential quantities to determine its precise properties have become crucial. While in the Standard Model (SM), Higgs boson production proceeds predominantly through gluon fusion, in theories with extended Higgs sector, such as the Two-Higgs-Doublet Model (2HDM) or the Minimal Supersymmetric SM (MSSM), the associated production of a Higgs with bottom quarks ( $b\bar{b}H$ ) can become the dominant Higgs production mechanism due to an enhancement of the bottom Yukawa coupling (see Refs. [3, 4, 5] and references therein). Two complementary approaches have been pursued in the literature to obtain a theoretical prediction for this process. In the so-called *four-flavor scheme* (4FS), which is most suitable when final-state bottom quarks are considered as part of the signature, the partonic processes at leading order (LO) are  $q\bar{q} \rightarrow b\bar{b}H$  and  $gg \rightarrow b\bar{b}H$  ( $q \in \{u, d, c, s\}$ ). The cross section is known up to next-to-LO (NLO) QCD [6, 7, 8]. In the *five-flavor scheme* (5FS) the LO partonic process is  $b\bar{b} \rightarrow H$ . The final state bottom quarks are implicitly integrated out in the parton model, which, therefore, are considered as being remnants of the proton. This approach is most suitable for the calculation of the  $b\bar{b}H$  component to inclusive Higgs production, since it implicitly resums collinear logarithms through DGLAP evolution. The inclusive total cross section in the 5FS is known at next-to-NLO (NNLO) [9].

Kinematical distributions of the Higgs boson will become more and more important to decrease the uncertainties on the measured Higgs properties and the search for possible deviations from the SM predictions. The transverse momentum ( $p_T$ ) spectrum of the Higgs is one of the first differential quantities where the experimental analysis will receive sensitivity, once sufficient luminosity is available. In gluon fusion, the  $p_T$  distribution has been studied in great detail (see Ref. [4]). For  $b\bar{b}H$ , the spectrum of the Higgs for  $p_T > 0$  is known at NNLO in the 5FS [10, 11, 12, 13].<sup>1</sup> Until recently, transverse momentum resummation of the Higgs produced in bottom-quark annihilation has been considered only at NLO+NNLL [14]. We report in this article on the first calculation of the resummed NNLO+NNLL transverse momentum distribution in the 5FS [15].

The remainder of the paper is organized as follows: In Section 2, we briefly outline the  $p_T$  resummation formalism for the production of a colorless final state and discuss the required resummation coefficients for  $b\bar{b}H$  production. This discussion includes our result for the second order *hard coefficient* which was the last missing resummation coefficient at NNLO+NNLL. In Section 3, we sketch the calculation of matched-resummed cross section and describe the checks that have been performed on the implementation. We present  $p_T$ -distributions for the LHC at a center-of-mass energy of 8 TeV.

## 2. Transverse momentum resummation

In the calculation of the transverse momentum distribution of a colorless particle of mass  $M$ , large logarithms  $p_T/M$  arise at every order in  $\alpha_s$  for  $p_T \ll M$ . They emerge from an incomplete cancellation of soft and collinear divergences. Therefore the naïve perturbative expansion in  $\alpha_s$  is no longer valid as  $p_T \rightarrow 0$ . However, their all-order resummation is feasible due to the factorization

<sup>1</sup>Throughout this paper we consistently refer to  $b\bar{b} \rightarrow H$  as the LO process, even though its contribution is  $\sim \delta(p_T)$ .

of soft and collinear radiation from the hard process. The resummation formula for the cross section in the low- $p_T$  region is well known [16, 17]<sup>2</sup>

$$\frac{d\sigma^{F,(\text{res})}}{dp_T^2} = \tau \int_0^\infty db \frac{b}{2} J_0(bp_T) W^F(b, M, \tau). \quad (2.1)$$

The Mellin transform<sup>3</sup> of  $W^F$  with respect to the variable  $\tau = M^2/S$  is given by

$$W_N^F(b, M) = \sum_c \hat{\sigma}_{c\bar{c}}^{F,(0)} H_c^F(\alpha_s) \exp \left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dk^2}{k^2} \left[ A_c(\alpha_s(k)) \ln \frac{M^2}{k^2} + B_c(\alpha_s(k)) \right] \right\} \times \sum_{i,j} C_{ci,N}(\alpha_s(b_0/b)) C_{\bar{c}j,N}(\alpha_s(b_0/b)) f_{i,N}(b_0/b) f_{j,N}(b_0/b), \quad (2.2)$$

with  $b_0 = 2 \exp(-\gamma_E)$ , Euler's constant  $\gamma_E = 0.5772\dots$ , the Bessel function  $J_0(x)$ , the hadronic center-of-mass energy  $S$ , and  $\hat{\sigma}_{c\bar{c}}^{F,(0)}$  the Born-level cross section. The superscript  $F$  identifies process specific quantities; e.g.  $F = b\bar{b}H$  for the  $b\bar{b}H$  process and  $F = \text{DY}$  for Drell-Yan. Unless indicated otherwise, the renormalization and factorization scales have been set to  $\mu_F = \mu_R = M$ . The index  $c$  runs over all parton flavors  $c \in \{u, d, s, c, b, g\}$  and their charge conjugates. The functions  $f_{i,N}(q)$  denote the Mellin transform of the parton density functions  $f_i(x, q)$ .

The *resummation coefficients* can be expanded in  $\alpha_s$ :

$$C_{ci,N} = \delta_{ci} + \sum_{n=1}^{\infty} \left( \frac{\alpha_s}{\pi} \right)^n C_{ci,N}^{(n)}, \quad X = \sum_{n=1}^{\infty} \left( \frac{\alpha_s}{\pi} \right)^n X^{(n)}, \quad H_c^F = 1 + \sum_{n=1}^{\infty} \left( \frac{\alpha_s}{\pi} \right)^n H_c^{F,(n)}, \quad (2.3)$$

where  $X \in \{A_c, B_c\}$ . At *leading logarithmic* (LL) accuracy only  $A_c^{(1)}$  is relevant. At *next-to-LL* (NLL) also  $A_c^{(2)}$ ,  $B_c^{(1)}$ ,  $C_{ci}^{(1)}$ , and  $H_c^{F,(1)}$  are included and  $A_c^{(3)}$ ,  $B_c^{(2)}$ ,  $C_{ci}^{(2)}$ , and  $H_c^{F,(2)}$  enter at next-to-NLL (NNLL) [18, 19, 20, 21]. For a fixed *resummation scheme* [22], the coefficients  $A_c$ ,  $B_c$ , and  $C_{ci}$  are process independent, while the entire process dependence is embodied in the *hard coefficient*  $H_c^F$  and the Born factor  $\hat{\sigma}_{c\bar{c}}^{F,(0)}$ . We will work in the DY resummation scheme, where  $H_q^{\text{DY}} \equiv 1$  to all orders (the final results are independent of this choice, of course).

For quark-initiated processes, all resummation coefficients are known through NNLL in this scheme; the process specific hard coefficient  $H_b^{b\bar{b}H} \equiv H_b^H$  was evaluated recently up to second order for the  $b\bar{b}H$  process [15]. Its evaluation requires the knowledge of the purely virtual amplitude for the process  $b\bar{b}H$  which was calculated through NNLO in Refs. [9, 23]. Following Ref. [24], we derive  $H_b^{H,(1)} = 3C_F$  and

$$H_b^{H,(2)} = C_F \left[ \left( \frac{321}{64} - \frac{13}{48} \pi^2 \right) C_F + \left( -\frac{365}{288} + \frac{\pi^2}{12} \right) N_f + \left( \frac{5269}{576} - \frac{5}{12} \pi^2 - \frac{9}{4} \zeta_3 \right) C_A \right]. \quad (2.4)$$

This yields a numerical value of  $H_b^{H,(2)} = 10.52\dots$ . We also calculated  $H_b^{H,(2)}$  using an independent numerical approach. The two results are in perfect agreement with each other [15]. This serves as an important check of our calculation.

<sup>2</sup>Note that resummation occurs in the impact parameter ( $b$ ) space.

<sup>3</sup> $W_N^F(b, M) = \int_0^1 d\tau \tau^{N-1} W^F(b, M, \tau)$

### 3. Outline of the calculation and results

In order to combine the resummed cross section in the low- $p_T$  region and the fixed order cross section at high  $p_T$  we follow the additive matching procedure of Ref. [22] and define the resummed-matched result as follows:

$$\left[ \frac{d\sigma^F}{dp_T^2} \right]_{\text{f.o.}+\text{l.a.}} = \left[ \frac{d\sigma^F}{dp_T^2} \right]_{\text{f.o.}} - \left[ \frac{d\sigma^{F,(\text{res})}}{dp_T^2} \right]_{\text{f.o.}} + \left[ \frac{d\sigma^{F,(\text{res})}}{dp_T^2} \right]_{\text{l.a.}}. \quad (3.1)$$

The fixed order  $p_T$  distribution  $[d\sigma]_{\text{f.o.}}$  is known analytically up to NNLO [11], which has been checked numerically against the partonic Monte Carlo program for  $H$ +jet production of Refs. [10, 12].<sup>4</sup> The logarithmic terms  $[d\sigma^{(\text{res})}]_{\text{f.o.}}$  are obtained from the perturbative expansion of Eq. (2.1) up to NNLO.<sup>5</sup> We verified numerically that  $[d\sigma^{(\text{res})}]_{\text{f.o.}}$  and  $[d\sigma]_{\text{f.o.}}$  are identical in the limit  $p_T \rightarrow 0$ . The calculation of the resummed cross section  $[d\sigma^{(\text{res})}]_{\text{l.a.}}$  at low  $p_T$  was carried out using a modified version of the program HQT [26, 22, 27], which we extended to quark-induced processes, and implement the resummation coefficients of the  $b\bar{b}H$  process.

In addition, the pursued  $p_T$  resummation formalism [22] implies the following *unitarity constraint*:

$$\int dp_T^2 \left[ \frac{d\sigma^F}{dp_T^2} \right]_{\text{f.o.}+\text{l.a.}} \equiv [\sigma_{\text{tot}}^F]_{\text{f.o.}}. \quad (3.2)$$

This relation serves as an important cross-check on our implementation of the matched-resummed distribution. We verified it at the per-mille level for various values of the resummation, factorization, and renormalization scales, separately at order  $\alpha_s$  and  $\alpha_s^2$ , and for the individual partonic sub-channels.

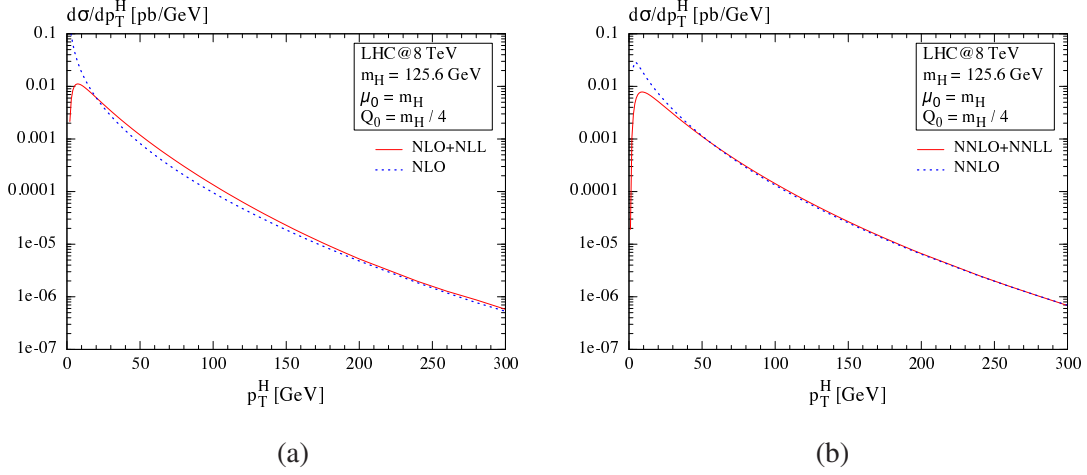
We present results for the LHC at 8 TeV center-of-mass energy in the SM. These results can also be used for neutral (CP even and odd) Higgs production within the 2HDM through rescaling of the bottom Yukawa coupling, and, according to the studies of Refs. [28, 29], also within the MSSM. Our choice for the central factorization and renormalization scale is  $\mu_F = \mu_R = \mu_0 \equiv M$ ; our default value for the resummation scale is  $Q = Q_0 \equiv M/4$ . If not stated otherwise, all numbers are obtained with the MSTW2008 [30] PDF set. The bottom mass is set to zero throughout the calculation, except for the bottom-Higgs Yukawa coupling which we insert in the  $\overline{\text{MS}}$ -scheme at the scale  $\mu_R$ , derived from the input value  $m_b(m_b) = 4.16$  GeV.

Fig. 1 (a) shows the NLO+NLL together with the fixed order NLO distribution. Differences between the fixed-order and the resummed-matched curve at  $p_T \sim M$  are higher order effects [31] which can be quite sizable [14], and were also observed for gluon fusion [32, 31]. However, for our choice of the resummation scale  $Q = M/4$  we observe a good high- $p_T$  matching already at NLO+NLL. The agreement between fixed order and resummed-matched curve is further improved at NNLO+NNLL, see Fig. 1 (b). The two curves are indistinguishable for  $p_T \gtrsim 50$  GeV.

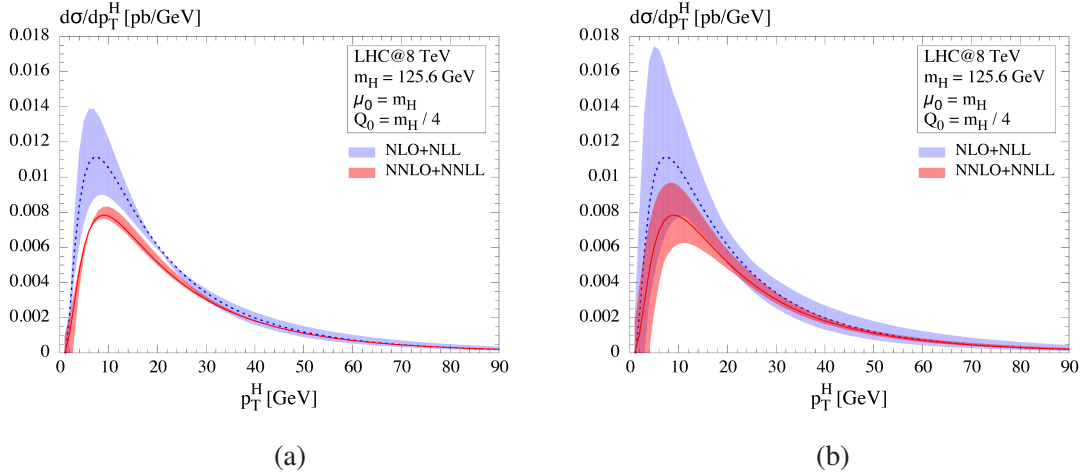
Let us now consider the effect of inclusion of the higher orders in the theoretical uncertainty. At NNLO+NNLL, a reduced sensitivity of the cross section to the resummation scale is expected

<sup>4</sup>For more details see also Ref. [25].

<sup>5</sup>See in Eqs. (72) and (73) of Ref. [22].



**Figure 1:** Transverse momentum spectrum at (a) NLO (blue, dashed) and NLO+NLL (red, solid) and (b) NNLO (blue, dashed) and NNLO+NNLL (red, solid). (Here and in the following plots,  $m_H = M$  is the Higgs mass.)



**Figure 2:** Resummed-matched  $p_T$ -distribution at NLO+NLL (blue, dashed) and NNLO+NNLL (red, solid); lines: central scale choices; bands: (a) uncertainty due to  $Q$ -variation. (b) uncertainty due to variation all scales.

compared to NLO+NLL. This expectation is confirmed in Fig. 2 (a). The bands indicate the uncertainties with respect to  $Q$ , which are obtained by varying  $Q$  between  $Q_0/2$  and  $2Q_0$ , while fixing  $\mu_F$  and  $\mu_R$  at their default values; the lines correspond to  $Q = Q_0$ . In fact, the extremely mild  $Q$ -dependence of the NNLO+NNLL cross section is remarkable.

Finally, Fig. 2 (b) shows the result for an independent variation of all three unphysical scales within  $Q \in [Q_0/2, 2Q_0]$  and  $\mu_F, \mu_R \in [\mu_0/2, 2\mu_0]$ , while excluding the regions  $\mu_F/\mu_R > 2$  and  $\mu_F/\mu_R < 1/2$ . Again, we observe a significant reduction of the scale uncertainties on the resummed-matched cross section at NNLO+NNLL with respect to NLO+NLL. At the maximum, the relative size of the uncertainty band is  $+23\% / -23\%$  for the NNLL curve and  $+48\% / -41\%$  at NLL. The corre-

sponding plots for 13 TeV and PDF uncertainties are presented in Ref. [15].

#### 4. Conclusions

We reported on the calculation of the missing second order hard coefficient  $H_b^{H,(2)}$  and the resummed-matched  $p_T$  distribution of Higgs bosons produced via bottom-quark annihilation at NNLO+NNLL. Our resummed low  $p_T$  distribution matches well to the fixed order distribution already around 50 GeV. Furthermore, the variation of the cross section with the unphysical scales is significantly reduced compared to NLO+NLL. In fact, the extremely weak dependence of the NNLO+NNLL result on the resummation scale is remarkable. Our results provide a precise prediction that consistently combines the cross section in the low- and high- $p_T$  region, which should prove useful particularly in models with enhanced bottom Yukawa coupling.

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