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Transverse-momentum resummation and the structure of hard factors at the NNLO

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In this proceeding we consider QCD radiative corrections to the production of colourless high-mass systems in hadron collisions. At small transverse momentum the logarithmically-enhanced contributions can be organized to all perturbative orders by a universal resummation formula that depends on a single process-dependent hard factor. We show that the hard factor is directly related to the all-order virtual amplitude of the corresponding partonic process by a universal (process independent) formula, which we explicitly evaluate up to two-loop level. Once the next-to-next-to-leading order (NNLO) scattering amplitude is available, the corresponding hard factor is directly determined. It can be used in fully-exclusive perturbative calculations (*via* q_T subtraction formalism) up to NNLO, in resummed calculations at full next-to-next-to-leading logarithmic (NNLL) accuracy, and also, it's a necessary ingredient to the next subsequent logarithmic order (N^{3}LL).

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1. Introduction

We consider the inclusive hard-scattering reaction

$$h_1(p_1) + h_2(p_2) \rightarrow F(\{q_i\}) + X\,, \tag{1.1}$$

where the collision of the two hadrons h_1 and h_2 with momenta p_1 and p_2 produces the observed final state F, accompanied by an arbitrary and undetected final state X. The triggered final state F is a generic system of one or more *colourless* particles, such as lepton pairs (produced by Drell–Yan (DY) mechanism), photon pairs, vector bosons, Higgs boson(s), and so forth. The momenta of these final state particles are denoted by q_1,q_2...q_n. The system F has *total* invariant mass $M^2 = (q_1 + q_2 + ...q_n)^2$, transverse momentum $\mathbf{q_T}$ and rapidity y. We employ $\sqrt{s}$ to denote the centre-of-mass energy of the colliding hadrons, which are treated in the massless approximation ($s = (p_1 + p_2)^2 = 2p_1 \cdot p_2$).

It is possibile to calculate the transverse-momentum (q_T) cross section for the process in Eq. (1.1) by using perturbative QCD. In the small-q_T region (roughly, in the region where $q_T \ll M$) the convergence of the fixed-order perturbative expansion in powers of the QCD coupling α_S is spoiled by the presence of large logarithmic terms of the type $\ln^n(M^2/q_T^2)$. We can recover the predictivity of perturbative QCD performing the summation of these logarithmically-enhanced contributions to all order in α_S [1, 2, 3].

If the final state F is colourless, the large logarithmic contributions to the q_T cross section can be systematically resummed to all perturbative orders, and the structure of the resummed calculation can be arranged in a *process-independent* form [1, 3, 5, 6]. Starting from the resummation formula for the DY process [2], two additional steps were needed to arrive at the process-independent version of the formalism: the understanding of the all-order process-independent structure of the Sudakov form factor (through the factorization of a single process-dependent hard factor) [5], and the complete generalization to processes that are initiated by the gluon fusion mechanism [6].

The all-order process-independent form of the resummed calculation has a factorized structure, whose resummation factors are (see Sect. 2) the (quark and gluon) Sudakov form factor, process-independent *collinear* factors and a process-dependent *hard* or, more precisely (see Sect. 3), hard-virtual factor. These factors (which are a set of perturbative functions whose perturbative *resummation coefficients* are computable order-by-order in α_S) control the resummation of the logarithmic contributions. The perturbative coefficients of the Sudakov form factor are known, since some time [3, 7, 4, 8], up to the second order in α_S, and the third-order coefficient $A^{(3)}$ (which is necessary to explicitly perform resummation up to the next-to-next-to-leading logarithmic (NNLL) accuracy) is also known [9]. The next-to-next-to-leading order (NNLO) QCD calculation of the q_T cross section (in the small-q_T region) has been done in analytic form for two benchmark processes, namely, SM Higgs boson production [10] and the DY process [11]. The results of Refs. [10, 11] provide us with the complete knowledge of the process-independent *collinear* resummation coefficients up to the second order in α_S, and with the explicit expression of the hard coefficients for these two specific processes. As shown in Ref. [12], the hard factor (which is process dependent) has an universal (process-independent) structure. The universality structure of the factorization formula has a *soft* (and collinear) origin, and it is closely (though indirectly) related to the universal structure of the infrared divergences [13] of the scattering amplitude. This process-independent structure of the hard-virtual term, which generalizes the next-to-leading order (NLO) results of Ref. [8], is valid to all perturbative orders [12].

The NNLO universal formula for the hard-virtual term completes the q_T resummation formalism in explicit form up to full NNLL+NNLO accuracy. This permits direct applications to NNLL+NNLO resummed calculations for any processes of the class in Eq. (1.1) (provided the corresponding NNLO amplitude is known), as already done for the cases of SM Higgs boson [14] and DY [15, 16] production. The NNLO information of the q_T resummation formalism is also relevant in the context of *fixed order* calculations. Indeed, it enables to carry out fully-exclusive NNLO calculations by applying the q_T *subtraction formalism* of Ref. [17] (the subtraction counterterms of the formalism follow [17] from the fixed-order expansion of the q_T resummation formula, as in Sect. 2.4 of Ref. [14]). The q_T subtraction formalism has been applied to the NNLO computation of Higgs boson [17, 18] and vector boson production [19], associated production of the Higgs boson with a W boson [20], diphoton production [21], $Z\gamma$ production [22] and ZZ production [23]. The computations of Refs. [17, 18, 19, 20] were based on the specific calculation of the NNLO hard-virtual coefficients of the corresponding processes [10, 11]. The computations of Refs. [21, 22, 23] used the NNLO hard-virtual coefficients that are determined by applying the universal form of the hard-virtual term that is derived in [12] and illustrated in the present proceeding.

Transverse-momentum resummation can equivalently be reformulated by using q_T-dependent partonic distributions (see, e.g., Refs. [9, 24]). The explicit NNLO results for the process-independent collinear coefficients [17, 19, 10, 11] and for the structure of the hard-virtual coefficients [12] have been confirmed by the fully-independent computation of Ref. [25], which uses the formalism of Ref. [9].

2. Small-q_T resummation

We consider the inclusive-production process in Eq. (1.1), and we introduce the corresponding *fully* differential cross section[1]

$$\frac{d\sigma_F}{d^2\mathbf{q_T}\, dM^2\, dy\, d\Omega}\,(p_1,p_2;\mathbf{q_T},M,y,\Omega) \;, \tag{2.1}$$

which depends on the total momentum of the system F (i.e. on the variables $\mathbf{q_T},M,y$). To evaluate the $\mathbf{q_T}$ dependence of the differential cross section in Eq. (2.1) within QCD perturbation theory, we first propose the following decomposition:

$$d\sigma_F = d\sigma_F^{(\mathrm{sing})} + d\sigma_F^{(\mathrm{reg})} \;. \tag{2.2}$$

The two last terms in the right-hand side already include the convolutions of partonic cross sections and the scale-dependent parton distributions $f_{a/h}(x,\mu^2)$ ($a = q_f, \bar{q}_f, g$ is the parton label) of the colliding hadrons. We use parton densities as defined in the $\overline{\mathrm{MS}}$ factorization scheme, and $\alpha_S(q^2)$ is the QCD running coupling in the $\overline{\mathrm{MS}}$ renormalization scheme. The partonic cross sections that enter the singular component (the first term in the right-hand side of Eq. (2.2)) contain all the contributions that are enhanced (or 'singular') at small q_T. These contributions are proportional

[1] In this section we briefly recall the formalism of transverse-momentum resummation in impact parameter space [1, 3, 4, 5, 6]. We closely follow the notation of Ref. [6] (more details about our notation can be found therein).

to $\delta^{(2)}(\mathbf{q_T})$ or to large logarithms of the type $\frac{1}{q_T^2}\ln^m(M^2/q_T^2)$. The partonic cross sections of the second term in the right-hand side of Eq. (2.2) are regular (i.e. free of logarithmic terms) order-by-order in perturbation theory as $q_T \to 0$. In the following we focus on the singular component, $d\sigma_F^{(\rm sing)}$, which has an universal all-order structure. The corresponding resummation formula is written as [1, 5, 6]

$$\frac{d\sigma_F^{(\rm sing)}(p_1,p_2;\mathbf{q_T},M,y,\Omega)}{d^2\mathbf{q_T}\,dM^2\,dy\,d\Omega} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{c\bar{c},F}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}\,e^{i\mathbf{b}\cdot\mathbf{q_T}}\,S_c(M,b)$$
$$\times\sum_{a_1,a_2}\int_{x_1}^1\frac{dz_1}{z_1}\int_{x_2}^1\frac{dz_2}{z_2}\,\left[H^F C_1 C_2\right]_{c\bar{c};a_1a_2}\,f_{a_1/h_1}\,f_{a_2/h_2}\;, \quad (2.3)$$

where $b_0 = 2e^{-\gamma_E}$ ($\gamma_E = 0.5772\ldots$ is the Euler number) is a numerical coefficient, and the kinematical variables $x_1 = \frac{M}{\sqrt{s}}\,e^{+y}$ and $x_2 = \frac{M}{\sqrt{s}}\,e^{-y}$. The function $S_c(M,b)$ is the Sudakov form factor, which is universal (process independent) [5]: it only depends on the type ($c = q$ or $c = g$) of colliding partons, and it resums the logarithmically-enhanced contributions of the form $\ln M^2b^2$ (the region $q_T \ll M$ corresponds to $Mb \gg 1$ in impact parameter space). The all-order expression of $S_c(M,b)$ is [2]

$$S_c(M,b) = \exp\left\{-\int_{b_0^2/b^2}^{M^2}\frac{dq^2}{q^2}\left[A_c(\alpha_S(q^2))\,\ln\frac{M^2}{q^2}+B_c(\alpha_S(q^2))\right]\right\}\;, \quad (2.4)$$

where $A_c(\alpha_S)$ and $B_c(\alpha_S)$ are perturbative series in α_S. The perturbative coefficients $A_c^{(1)}, B_c^{(1)}, A_c^{(2)}$ [3], $B_c^{(2)}$ [7, 4, 8] and $A_c^{(3)}$ [9] are explicitly known.

The Born level factor[2] $\left[d\sigma^{(0)}_{c\bar{c},F}\right]$ in Eq. (2.3) is obviously process dependent, although its process dependence is elementary (it is simply due to the Born level scattering amplitude of the partonic process $c\bar{c} \to F$). The remaining process dependence of Eq. (2.3) is embodied in the 'hard-collinear' factor $\left[H^F C_1 C_2\right]$. This factor includes a process-independent part and a process-dependent part. The structure of the process-dependent part is the main subject of the present proceeding.

In the case of processes that are initiated at the Born level by the $q\bar{q}$ annihilation channel ($c = q$), the symbolic factor $\left[H^F C_1 C_2\right]$ in Eq. (2.3) has the following explicit form [5]

$$\left[H^F C_1 C_2\right]_{q\bar{q};a_1a_2} = H_q^F(x_1p_1,x_2p_2;\Omega;\alpha_S(M^2))\;C_{qa_1}(z_1;\alpha_S(b_0^2/b^2))\;C_{\bar{q}a_2}(z_2;\alpha_S(b_0^2/b^2))\;, \quad (2.5)$$

and the functions H_q^F and $C_{qa} = C_{\bar{q}\bar{a}}$ have the perturbative expansion

$$H_q^F(x_1p_1,x_2p_2;\Omega;\alpha_S) = 1+\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n H_q^{F\,(n)}(x_1p_1,x_2p_2;\Omega)\;, \quad (2.6)$$

$$C_{qa}(z;\alpha_S) = \delta_{qa}\;\delta(1-z)+\sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n C_{qa}^{(n)}(z)\;. \quad (2.7)$$

The function H_q^F is process dependent, whereas the functions C_{qa} are universal (they only depend on the parton indices). The factorized structure in the right-hand side of Eq. (2.5) is based on the

[2] The cross section at its corresponding *lowest order* in α_S.

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following fact: the scale of α_S is M^2 in the case of H_q^F, whereas the scale is b_0^2/b^2 in the case of C_{qa}. The appearance of these two different scales is essential [5] to disentangle the process dependence of H_q^F from the process-independent Sudakov form factor (S_q) and collinear functions (C_{qa}). In the case of processes that start at Born level by the gluon fusion channel ($c = g$), the physics of the small-q_T cross section has a richer structure, which is the consequence of collinear correlations [6] that are produced by the evolution of the colliding hadrons into gluon partonic states (the interested reader is referred to [6, 12]). Despite its richer structure, it is possible to disentangle [6] the process dependence of H_g^F from the process-independent Sudakov form factor (S_c) and collinear tensor functions ($C_{ga}^{\mu\nu}$) analogously to the case of the $q\bar{q}$ channel.

As a consequence of the renormalization-group symmetry (Eqs.(22)–(25), in Ref. [12]), the resummation factors H^F, S_c and C_{qa} are not *separately* defined (and, thus, computable) in an unambiguous way. Equivalently, each of these separate factors can be precisely defined only by specifying a *resummation scheme* [5]. We choose the *hard scheme*, that is defined as follows. The flavour off-diagonal coefficients $C_{ab}^{(n)}(z)$, with $a \neq b$, are 'regular' functions of z as $z \to 1$. The z dependence of the flavour diagonal coefficients $C_{qq}^{(n)}(z)$ and $C_{gg}^{(n)}(z)$ in Eqs. (2.7) is instead due to both 'regular' functions and 'singular' distributions in the limit $z \to 1$. The 'singular' distributions are $\delta(1-z)$ and the customary plus-distributions of the form $[(\ln^k(1-z))/(1-z)]_+$ $(k = 0,1,2\ldots)$. The *hard scheme* is the scheme in which, order-by-order in perturbation theory, the coefficients $C_{ab}^{(n)}(z)$ with $n \geq 1$ do not contain any $\delta(1-z)$ term. We highlight (see also Sect. 3) that this definition directly implies that all the process-dependent virtual corrections to the Born level subprocesses are embodied in the resummation coefficient H_c^F.

We note that the specification of the hard scheme (or any other scheme) has sole practical purposes of presentation (theoretical results can be equivalently presented, as actually done in Refs. [10] and [11], by explicitly parametrizing the resummation-scheme dependence of the resummation factors). The q_T cross section, its all-order resummation formula (2.3) and any consistent perturbative truncation (either order-by-order in α_S or in classes of logarithmic terms) of the latter [5, 14] are completely independent of the resummation scheme.

The first-order coefficients $C_{ab}^{(1)}(z)$ are explicitly known [4, 7, 8, 26]. The second-order process-independent collinear coefficients $C_{ab}^{(2)}(z)$ have been independently computed in Refs. [17, 19, 10, 11] and in Ref. [25] by using two completely different methods, and the results of the two computations are in agreement.

The universality structure of the process-dependent coefficients H_c^F at NNLO and higher orders (see Sect. 3) is one of the main results that we are discussing in the present proceeding.

3. Hard-virtual coefficients

In the hard scheme that we are using, the hard-virtual coefficient contains all the information on the process-dependent virtual corrections, and, therefore, we can show that H^F can be related in a process-independent (universal) way to the multiloop virtual amplitude $\mathcal{M}_{c\bar{c}\to F}$ of the partonic process $c\bar{c} \to F$.

We consider the partonic *elastic*-production process

$$c(\hat{p}_1) + \bar{c}(\hat{p}_2) \to F(\{q_i\}) , \tag{3.1}$$

where the two colliding partons with momenta $\hat{p}_1$ and $\hat{p}_2$ are either $c\bar{c} = gg$ or $c\bar{c} = q\bar{q}$ and $F(\{q_i\})$ is the triggered final-state system in Eq. (1.1). The loop scattering amplitude of the process in Eq. (3.1) contains ultraviolet (UV) and infrared (IR) singularities, which are regularized in $d = 4-2\varepsilon$ space-time dimensions by using the customary scheme of conventional dimensional regularization. The renormalized all-loop amplitude of the generic process in Eq. (3.1) is denoted by $\mathcal{M}_{c\bar{c}\to F}$ and it has the perturbative (loop) expansion

$$\mathcal{M}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) = \left(\alpha_S(\mu_R^2)\,\mu_R^{2\varepsilon}\right)^k \left\{ \mathcal{M}^{(0)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\}) \right.$$
$$\left. + \left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right) \mathcal{M}^{(1)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\};\mu_R) + \sum_{n=3}^{\infty} \left(\frac{\alpha_S(\mu_R^2)}{2\pi}\right)^n \mathcal{M}^{(n)}_{c\bar{c}\to F}(\hat{p}_1,\hat{p}_2;\{q_i\};\mu_R) \right\}, \quad (3.2)$$

where the value k of the overall power of α_S depends on the specific process. The perturbative terms $\mathcal{M}^{(l)}_{c\bar{c}\to F}$ $(l = 1,2,\ldots)$ are UV finite, but they still depend on ε (although this dependence is not explicitly denoted in Eq. (3.2)) and, in particular, they are IR divergent as $\varepsilon \to 0$. The IR divergent contributions to the scattering amplitude have a universal structure [13], which is explicitly known at the one-loop [27, 13], two-loop [13, 28] and three-loop [29, 30] level for the class of processes in Eq. (3.1).

In Ref. [8] we can find the universal (process-independent) relation between the NLO hard-virtual coefficient $H^{F\,(1)}$ and the leading-order (LO) amplitude $\mathcal{M}^{(0)}_{c\bar{c}\to F}$ and to the IR finite part of the NLO amplitude $\mathcal{M}^{(1)}_{c\bar{c}\to F}$. The relation between H_c^F and $\mathcal{M}_{c\bar{c}\to F}$ can be extended to NNLO and to higher-order levels [12]. This extension can be formulated and expressed in simple and general terms by introducing an auxiliary (hard-virtual) amplitude $\widetilde{\mathcal{M}}_{c\bar{c}\to F}$ that is directly obtained from $\mathcal{M}_{c\bar{c}\to F}$ in a universal (process-independent) way[3]. In practice, $\widetilde{\mathcal{M}}_{c\bar{c}\to F}$ is obtained from $\mathcal{M}_{c\bar{c}\to F}$ by removing its IR divergences and a *definite* amount of IR finite terms. The (IR divergent and finite) terms that are removed from $\mathcal{M}_{c\bar{c}\to F}$ originate from real emission contributions to the cross section and, therefore, these terms and $\widetilde{\mathcal{M}}_{c\bar{c}\to F}$ *specifically* depend on the transverse-momentum cross section of Eq. (2.1). The relation between H_c^F and $\mathcal{M}_{c\bar{c}\to F}$ is based on an universal all-order factorization formula [12] that emerges from the factorization properties of soft (and collinear) parton radiation. We have explicitly determined this relation up to the NNLO [12]. More precisely, we have shown [12] that this relation is fully determined by the structure of IR singularities of the all-order amplitude $\mathcal{M}_{c\bar{c}\to F}$ and by renormalization-group invariance up to a *single* coefficient (of *soft* origin) at each perturbative order.

We can relate the subtracted amplitude $\widetilde{\mathcal{M}}_{c\bar{c}\to F}$ to the process-dependent resummation coefficients H_c^F of Eqs. (2.3) and (2.5). For processes initiated by $q\bar{q}$ annihilation (see Eqs. (2.5) and (2.6)), the *all-order* coefficient H_q^F can be written as

$$\alpha_S^{2k}(M^2)\, H_q^F(x_1p_1, x_2p_2; \Omega; \alpha_S(M^2)) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}(x_1p_1, x_2p_2; \{q_i\})|^2}{|\mathcal{M}^{(0)}_{q\bar{q}\to F}(x_1p_1, x_2p_2; \{q_i\})|^2}, \quad (3.3)$$

where k is the value of the overall power of α_S in the expansion of $\mathcal{M}_{c\bar{c}\to F}$ (see Eq. (3.2)).

The expression (3.3) allows us the explicit computation of the process-dependent resummation coefficients H_c^F for an arbitrary process of the class in Eq. (1.1). The computation of H_c^F up to the

[3] The interested reader is referred to [12], where there are all the formulae to obtain $\widetilde{\mathcal{M}}_{c\bar{c}\to F}$.

NNLO is straightforward, provided the scattering amplitude $\mathcal{M}_{c\bar{c}\to F}$ of the corresponding partonic subprocess is available (known) up to the NNLO (two-loop) level.

Some examples (DY and Higgs boson production) are explicitly reported in Appendix A of Ref. [12]. In particular, in Appendix A of Ref. [12], we used Eq. (3.3), and we presented the explicit expression of the NNLO hard-virtual coefficient $H_q^{\gamma\gamma(2)}$ for the process of diphoton production [21]. Recently, Eq. (3.3) was used to obtain the hard-virtual factor in the case of Higgs production in bottom quark annihilation [31], in order to calculate the transverse momentum distribution at NNLO+NNLL.

The same procedure that was applied to derive the universal formula for the hard-virtual coefficient H_c^F can be used within the related formalism of threshold resummation [32] for the *total* cross section. The process-independent formalism of threshold resummation also involves a corresponding process-dependent hard factor which has a universality structure [12] that is analogous to the case of transverse-momentum resummation. Recently, we also extended the threshold resummation results of Ref. [12] to the next subsequent order (N^{3}LL) [33]. The general (process-independent) N^{3}LL results of Ref. [33] are based on the universality structure of the hard-virtual factor, and they exploit the recent computation of the N^{3}LO Higgs boson cross section [34] within the soft-virtual approximation. For the specific case of DY production we confirm [33] the soft-virtual N^{3}LO results of Ref. [35].

The results enumerated in this proceeding, with the knowledge of the other process-independent resummation coefficients, complete the q_T resummation formalism in explicit form up to full NNLL and NNLO accuracy for all the processes in the class of Eq. (1.1). This allows applications to NNLL+NNLO resummed calculations for *any* processes whose NNLO scattering amplitudes are available. Moreover, we have all the ingredients to implement the q_T subtraction formalism [17] straightforwardly, to perform fully-exclusive NNLO computations for each of these processes.

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