PROCEEDINGS OF SCIENCE

PoS(LL2014)034

GRACE for ILC

J. Fujimoto∗[a], **P.H. Khiem**[a,b] **and Y. Kurihara**[a],

[a]*High Energy Accelerator Research Organization (KEK), 1-1 Oho Tsukuba, Ibaraki 305-0801, Japan*
[b]*SOKENDAI University, Shonan Village, Hayama, Kanagawa 240-0193 Japan.*
E-mail: junpei.fujimoto@kek.jp,
khiemph@post.kek.jp,
yoshimasa.kurihara@kek.jp

The automatic Feynman amplitude calculation system, GRACE is a generator of event generators, which was originally designed to calculate the higher order corrections to processes for e^+e^- collider experiments. The GRACE system has been used to calculate the $\mathcal{O}(\alpha)$ electroweak corrections to $2 \to 3$ processes for the International Linear Collider (ILC), such as $e^+e^- \to ZHH$, $e^+e^- \to t\bar{t}H$, $e^+e^- \to \nu\bar{\nu}H$, and also to the $2 \to 4$ process $e^+e^- \to \nu_\mu\bar{\nu}_\mu HH$. In this paper, we present the calculation of the full $\mathcal{O}(\alpha)$ electroweak corrections to the process $e^+e^- \to e^+e^-\gamma$ towards the full $\mathcal{O}(\alpha^2)$ electroweak corrections to the process $e^+e^- \to e^+e^-$ in the ILC energy region. We find that the size of the corrections ranges from -2% to $\sim -20\%$ in the range of 250 GeV to 1TeV for the center-of-mass energy.

Loops and Legs in Quantum Field Theory - LL 2014,
27 April - 2 May 2014
Weimar, Germany

∗Speaker.

1. Introduction

The automatic Feynman amplitude calculation system, GRACE is a generator of event generators, which was originally designed to calculate the higher order corrections to processes for e^+e^- collider experiments. The system is described in detail in Ref. [1] where a variety of $2 \to 2$ processes is presented and compared with other papers. The GRACE system has also been used to calculate the full $\mathscr{O}(\alpha)$ electroweak corrections to $2 \to 3$ processes at the International Linear Collider (ILC), such as $e^+e^- \to ZHH$ [2], $e^+e^- \to t\bar{t}H$ [3], $e^+e^- \to \nu\bar{\nu}H$ [4], and also to the $2 \to 4$ process $e^+e^- \to \nu_\mu\bar{\nu}_\mu HH$ [5].

The expected measurement errors performed at the ILC experiments will be around 0.1% or less. These measurements will require a very precise luminosity measurement. The luminosity will be measured by counting Bhabha events. Thus a precise evaluation of the cross section of Bhabha scattering is mandatory. The full $\mathscr{O}(\alpha)$ electroweak corrections had been calculated by several authors, Ref. [6]. The size of corrections reach around $\mathscr{O}(10\%)$ in the high energy region. In order to control the luminosity measurement at the 0.1% level, the higher order corrections beyond $\mathscr{O}(\alpha)$ should be taken into account. Such calculations are also performed by many authors, Ref. [7]. However, the full $\mathscr{O}(\alpha^2)$ electroweak corrections are not yet available.

In this paper, we present the calculation of the full $\mathscr{O}(\alpha)$ electroweak corrections to radiative Bhabha scattering based on the calculation in Ref. [8] towards the full $\mathscr{O}(\alpha^2)$ corrections for the Bhabha process. The computation was performed with the GRACE system. The lowest-order calculation of this process was performed in Ref. [9]. The one-loop QED corrections to this process were presented in Ref. [10]. An analytical expression of the one-loop QED corrections is also available from Ref. [11].

2. GRACE

The GRACE system adopts the on-shell renormalization scheme of the Kyoto group described in Ref. [12]. The ultraviolet (UV) divergences are regulated by the dimensional regularization. While the infrared (IR) divergences are regularized by introducing a fictitious photon mass λ.

Ref, [1] describes the reduction method of the one-loop five- and six-point tensor-functions into one-loop four-point functions by GRACE. The one-, two-, three- and four-point tensor-functions are reduced to scalar one-loop integrals which are numerically evaluated by the FF [13] or Loop Tools [14] packages.

The system equips the non-linear gauge fixing terms [15] in the Lagrangian for consistent checks of the numerical results, which are defined as

$$\begin{aligned}\mathscr{L}_{GF} = & -\frac{1}{\xi_W}|(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{\mu+} + \xi_W\frac{g}{2}(v+\tilde{\delta}H+i\tilde{\kappa}\chi_3)\chi^+|^2 \\ & -\frac{1}{2\xi_Z}(\partial\cdot Z+\xi_Z\frac{g}{2c_W}(v+\tilde{\varepsilon}H)\chi_3)^2 - \frac{1}{2\xi_A}(\partial\cdot A)^2 \,. \end{aligned} \tag{2.1}$$

In the latest version, GRACE treats the axial gauge in the projection operator of external photons. It cures a problem with large numerical cancellations. This is very powerful in calculating the cross sections of events with small scattering angle. It also provides a check for the Ward identities.

This check has worked well to calculate the electroweak corrections to the process $e^+e^- \to t\bar{t}\gamma$ in Ref. [16].

In the step of the phase space integration, we use a parallel processing version of the Monte-Carlo integration package `BASES` [17] with MPI [18] (Message Passing Interface) to reduce the calculation time.

3. The process $e^+e^- \to e^+e^-\gamma$

The full set of Feynman diagrams with non-linear gauge fixing consists of 32 tree diagrams and 3456 one-loop diagrams.

The calculation was checked numerically by three tests, the ultraviolet and infrared finiteness, and independence of the gauge parameters. In general the total cross-section with the full one-loop electroweak corrections is written by

$$\begin{aligned}\sigma_{\mathbf{tot}}^{e^-e^+\gamma_H} &= \int d\sigma_{\mathbf{T}}^{e^-e^+\gamma_H} + \int d\sigma_{\mathbf{V}}^{e^-e^+\gamma_H}(C_{UV},\{\tilde{\alpha},\tilde{\beta},\tilde{\delta},\tilde{\varepsilon},\tilde{\kappa}\},\lambda) \\ &+ \int d\sigma_{\mathbf{T}}^{e^-e^+\gamma_H}\delta_{\mathbf{soft}}(\lambda \le E_{\gamma_S} < k_c) + \int d\sigma_{\mathbf{H}}^{e^-e^+\gamma_H\gamma_S}(E_{\gamma_S} \ge k_c),\end{aligned} \tag{3.1}$$

where $\sigma_{\mathbf{T}}^{e^-e^+\gamma_H}$ is the tree-level cross-section, $\sigma_{\mathbf{V}}^{e^-e^+\gamma_H}$ is the cross-section due to the interference of the one-loop amplitudes and the tree ones. The numerical results must be independent of the ultraviolet cutoff parameter (C_{UV}) and the non-linear gauge parameters ($\tilde{\alpha},\tilde{\beta},\tilde{\delta},\tilde{\varepsilon},\tilde{\kappa}$). The function $\sigma_{\mathbf{V}}^{e^-e^+\gamma_H}$ depends on the photon fictitious mass λ. The λ-dependence has to be canceled against the soft photon contribution with a soft photon factor

$$\delta_{\mathbf{soft}}(\lambda \le E_{\gamma_S} < k_c) = -e^2 \int_{\lambda \le q_0 \le k_c} \frac{d^3q}{(2\pi)^3 2q^0}\left|\frac{p^-}{q\cdot p^-} - \frac{p^+}{q\cdot p^+}\right|^2, \tag{3.2}$$

where q and $p^\pm$ are 4-momenta of the photon and the $e^\pm$, respectively.

The ultraviolet finiteness, the gauge invariance and the infrared finiteness were examined at a phase space point chosen randomly. These tests were performed in quadruple precision. We confirmed that the results are stable over a range of 30 digits, 28 digits and 15 digits, respectively.

Finally, we consider the contribution of the hard photon bremsstrahlung, $\sigma_{\mathbf{H}}^{e^-e^+\gamma\gamma}(k_c)$. The relevant diagrams of the process $e^+e^- \to e^-e^+\gamma\gamma$ were generated by the tree level version of `GRACE` [19] and the phase space integration was also done with `BASES` . By adding this contribution to the total cross-section, the final results have to be independent of k_c. By changing the value of k_c from 10^{-3} GeV to 0.1 GeV, we find that the results are stable at the level of 0.05%.

The reduction method for the one-loop five point function in `GRACE` is also cross-checked with the one in Ref. [20] by calculating a typical diagram with 5-point functions. The two methods are both in agreement over a range of 19 digits.

After checking, we proceed to the phase-space integration step to get the final results. Hereafter we set the following parameters: $\lambda = 10^{-17}$ GeV, $C_{UV} = 0$, $k_c = 10^{-3}$ GeV and $\tilde{\alpha} = \tilde{\beta} = \tilde{\delta} = \tilde{\kappa} = \tilde{\varepsilon} = 0$. In order to reduce the execution time of the phase space integration, we neglected the diagrams which contained the coupling of Higgs boson to electron and positron ($\lambda_{He^-e^+}$), because it's contribution is much smaller than the statistical error of the Monte Carlo integration.

PoS(LL2014)034

4. Results

The input parameters for the calculation are set as follows. The fine structure constant in the Thomson limit is $\alpha^{-1} = 137.0359895$. For the boson masses we use $M_H = 126$ GeV, $M_Z = 91.1876$ GeV and $M_W = 80.385$ GeV. For the lepton masses we take $m_e = 0.510998928$ MeV, $m_\tau = 1776.82$ MeV and $m_\mu = 105.6583715$ MeV, for the quark masses we take $m_u = 2.3$ MeV, $m_d = 4.8$ MeV, $m_c = 1.275$ GeV, $m_s = 95$ MeV, $m_t = 173.5$ GeV and $m_b = 4.18$ GeV. From all the decay channels of the Z boson, the decay width of the Z boson has been estimated as 2.3549 GeV. We applied it to the complex propagators of Z boson in order to regulate its resonance.

For the final state particles, we apply an energy cut of $E^{cut} \geq 10$ GeV and an angle cut of $10^\circ \leq \theta^{cut} \leq 170^\circ$ with respect to the beamline. In order to isolate the photon from the electron or positron we apply an opening angle cut with 10°. Moreover, to distinguish $e^-e^+\gamma$ events from $\gamma\gamma$ ones, we apply an angle cut between the final state electron and positron of 10°.

The total electroweak corrections factor is defined as

$$\delta_{EW} = \frac{\sigma(\alpha)}{\sigma_{Tree}} - 1. \tag{4.1}$$

In order to obtain the genuine weak corrections, we first calculate the pure QED correction factor defined by

$$\delta_{QED} = \frac{\sigma^{QED}(\alpha) - \sigma_0^{QED}}{\sigma_{Tree}}. \tag{4.2}$$

Here σ_0^{QED} is the cross-section of the QED tree-level diagrams and $\sigma^{QED}(\alpha)$ is the cross-section due to the interference of the QED one-loop diagrams and the QED tree diagrams.

We also define the genuine weak correction factor δ_W, in the α-scheme, as follows;

$$\delta_W = \delta_{EW} - \delta_{QED}. \tag{4.3}$$

In Fig. 1 the cross-section and the correction factors are shown as a function of $\sqrt{s}$. The center-of-mass energy ranges from 250 GeV which is near the threshold of $M_H + M_Z$ to 1 TeV. We find that the size of the electroweak corrections factor reaches from -2% to $\sim -20\%$ when varying $\sqrt{s}$ from 250 GeV to 1 TeV. The right part of Fig. 1 clearly shows that QED corrections provides the dominant contribution as compared to the weak corrections. The size of the QED corrections becomes -14% at $\sqrt{s} = 1$ TeV, while one of the weak corrections changes from $\sim 2\%$ to $\sim 6\%$. It is clear that the corrections are a sizable contribution to the total cross-section.

5. Conclusions

The QED and full $\mathscr{O}(\alpha)$ electroweak corrections to $e^+e^- \to e^+e^-\gamma$ at the International Linear Collider have been calculated by using the GRACE system.

This system incorporates a generalized non-linear gauge fixing condition which includes five gauge parameters. Together with the UV, IR finiteness, it provides a powerful tool to test the consistency of the results.

As a conclusion, we find that the size of the full electroweak corrections vary from -2% to $\sim -20\%$ in the range of 250 GeV to 1TeV for the center-of-mass energy.

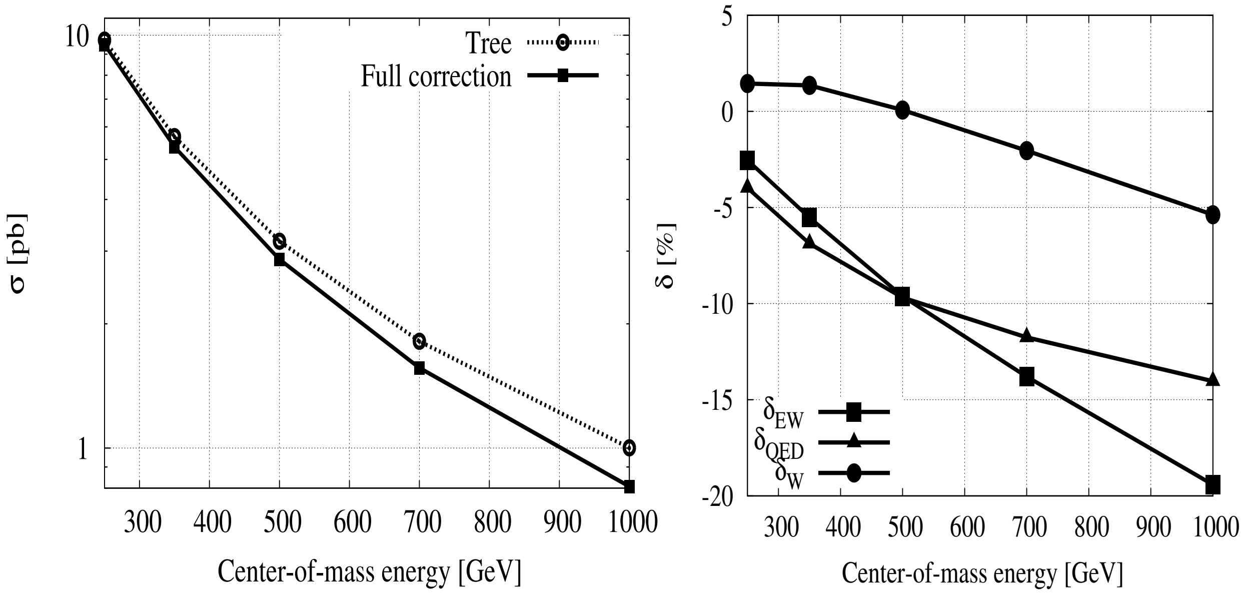

Figure 1: The cross-section (left) and full electroweak corrections (right) as a function of the center-of-mass energy.

Acknowledgments

Authors would like to thank the GRACE group members, Prof. J.A.M. Vermaseren and Dr. T. Ueda for continuous discussion of this subject and their useful suggestions. One author (JF) is grateful to the organizers of the Loops and Legs conference for their creating a very stimulating environment and warm hospitality.

References

[1] G. Belanger, et.al. Phys. Rept. **430** (2006) 117 [hep-ph/0308080].

[2] G. Belanger,et.al. Phys. Lett. B **576** (2003) 152 [hep-ph/0309010].

[3] G. Belanger, et.al. Phys. Lett. B **571** (2003) 163 [hep-ph/0307029].

[4] G. Belanger, et.al. Nucl. Phys. Proc. Suppl. **116** (2003) 353 [hep-ph/0211268].

[5] K. Kato, et.al. PoS HEP **2005** (2006) 312.

[6] K.Tobimatsu and Y.Shimizu, Prog. Theor. Phys. **74** (1985) 567,
K.Tobimatsu and Y.Shimizu, Prog. Theor. Phys. **75** (1986) 905,
M. Böhm, A. Denner, W. Hollik and R. Sommer, Phys. Lett. B **144** (1984) 414,
M. Böhm, A. Denner and W. Hollik, Nucl. Phys. B **304** (1988) 687,
F. A. Berends, R. Kleiss and W. Hollik, Nucl. Phys. B **304** (1988) 712,
J. Fleischer, J. Gluza, A. Lorca and T. Riemann, Eur. J. Phys. **48** (2006) 35 [hep-ph/0606210].

[7] A. A. Penin, Phys. Rev. Lett. **95** (2005) 010408 [hep-ph/0501120],
A. A. Penin, Nucl. Phys. B **734** (2006) 185 [hep-ph/0508127],

R. Bonciani, et.al. Nucl. Phys. B **701** (2004) 121 [hep-ph/0405275],
R. Bonciani, et.al. Nucl. Phys. B **716** (2005) 280 [hep-ph/0411321],
R. Bonciani, et.al. Phys. Rev. Lett. **100** (2008) 131601 [arXiv:0710.4775 [hep-ph]],
R. Bonciani, A. Ferroglia and A. A. Penin, JHEP **0802** (2008) 080 [arXiv:0802.2215 [hep-ph]],
A. A. Penin and G. Ryan, JHEP **1111** (2011) 081 [arXiv:1112.2171 [hep-ph]].

[8] P. H. Khiem, Y. Kurihara, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato, N. Nakazawa, Y. Shimizu, T. Ueda, J.A.M. Vermaseren and Y. Yasui, arXiv:1403.6557 [hep-ph].

[9] K. Tobimatsu and M. Igarashi, Comput. Phys. Commun. **136** (2001) 105.

[10] S. Actis, P. Mastrolia and G. Ossola, Phys. Lett. B **682** (2010) 419 [arXiv:0909.1750 [hep-ph]].

[11] M. Igarashi et al, *in preparation*.

[12] K. Aoki, Z. Hioki, R. Kawabe, M. Konuma and T. Muta, Suppl. Prog. Theor. Phys. **73** (1982) 1.

[13] G. J. van Oldenborgh, *Comput. Phys. Commun.* **58** (1991) 1.

[14] T. Hahn, `LoopTools`, `http://www.feynarts.de/looptools/`.

[15] F. Boudjema and E. Chopin, *Z. Phys.* **C73** (1996) 85 [hep-ph/9507396].

[16] P. H. Khiem, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato, Y. Kurihara, Y. Shimizu and T. Ueda *et al.*, Eur. Phys. J. C **73** (2013) 2400 [arXiv:1211.1112 [hep-ph]].

[17] S. Kawabata, *Comp. Phys. Commun.* **41** (1986) 127; *ibid.,* **88** (1995) 309.

[18] http://www.mcs.anl.gov/research/projects/mpi/

[19] T. Ishikawa, T. Kaneko, K. Kato, S. Kawabata, Y. Shimizu and H. Tanaka, KEK Report 92-19, 1993, `GRACE` manual Ver. 1.0.

[20] A. Denner and S. Dittmaier, Nucl. Phys. B **734** (2006) 62 [hep-ph/0509141].