PROCEEDINGS OF SCIENCE

Two-loop corrections to $t\bar{t}$ and ZZ/WW production

Andreas von Manteuffel *
PRISMA Cluster of Excellence, Institute of Physics, Johannes Gutenberg University, 55099 Mainz, Germany
E-mail: manteuffel@uni-mainz.de

We report on recent results for two-loop corrections to the pair production of top quarks, Z bosons and W bosons, respectively. In particular, we describe how the master integrals are calculated analytically in terms of multiple polylogarithms with the method of differential equations. For $t\bar{t}$ production, we discuss the light-quark two-loop corrections in the gluon channel. For ZZ and WW production, the solutions for all two-loop master integrals were completed. The results are optimised for numerical evaluation by employing a set of logarithms, classical polylogarithms Li_n ($n = 2,3,4$) and multiple polylogarithms of type $\mathrm{Li}_{2,2}$, where the Li functions are real valued and allow for an immediate power series expansion in the physical region. This method reduces the numerical evaluation time by orders of magnitude with respect to the traditional representation and paved the way to the first NNLO prediction for the total cross section for ZZ production at the LHC.

Loops and Legs in Quantum Field Theory - LL 2014,
27 April - 2 May 2014
Weimar, Germany

*I would like to thank Roberto Bonciani, Andrea Ferroglia, Thomas Gehrmann, Cedric Studerus, Lorenzo Tancredi and Erich Weihs for the pleasant collaboration on the projects presented in this contribution. My special thanks go to Andrea Ferroglia for interesting discussions and kind hospitality during the preparation of this talk. This work was supported in part by the Research Center *Elementary Forces and Mathematical Foundations (EMG)* of the Johannes Gutenberg University of Mainz and by the German Research Foundation (DFG).

PoS(LL2014)003

1. Introduction

Precision measurements at the LHC experiments motivate theoretical predictions at NNLO for top quark pair production and pair production of electroweak gauge bosons. Complete NNLO results have been calculated in the case of $t\bar{t}$ production for the total cross section [1, 2], and in the case of $\gamma\gamma$ [3, 4, 5] and $Z\gamma$ [6, 7, 8] production also for distributions. At this conference [9], the total cross section for ZZ production [10] has been presented for the first time.

Typically, the two-loop amplitudes are one of the bottlenecks of a NNLO calculation. Publicly available programs [11, 12, 13, 14, 15, 16, 17, 18] for integration-by-parts (IBP) reduction [19, 20, 21] are often powerful enough to perform the reduction to master integrals, and new approaches [22, 23] might help to overcome their limitations for more involved processes. A second problem is the calculation of master integrals. Here, considerable improvements are due to new techniques for the treatment of multiple polylogarithms, such as the coproduct formalism [24, 25, 26, 27]. Most calculations of multiscale Feynman integrals are performed with the method of differential equations [28, 29, 30, 31], which has been refined recently [32].

For $t\bar{t}$ production we are interested in a mostly analytical approach towards a fully differential NNLO code. Partial results are available for the subtraction terms [33, 34, 35, 36, 37, 38, 39], one-loop squared [40, 41, 42] and two-loop corrections [43, 44, 45]. Here, we discuss the recently completed light-quark two-loop corrections in the gluon channel [46].

For WW production, the high energy limit for the virtual NNLO corrections has been calculated some time ago [47]. Recently, all planar and non-planar two-loop master integrals for VV' production became available for the equal-mass case [48, 49] and also for the unequal-mass case [50, 51, 52, 53]. Here, we report on the calculation of the master integrals for ZZ and WW production [49].

2. Light fermionic two-loop corrections to $gg \to t\bar{t}$

We consider the two-loop corrections to top quark pair production in the gluon channel,

$$g(p_1) + g(p_2) \to t(p_3) + \bar{t}(p_4), \tag{2.1}$$

with $p_1^2 = p_2^2 = 0$ and $p_3^2 = p_4^2 = m^2$. The Mandelstam invariants $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_2 - p_3)^2$ fulfil the mass-shell condition $s + t + u = 2m^2$. In the following, we report on the calculation [46] of the gauge invariant subset which contains at least one closed massless quark loop.

We generate Feynman diagrams with Qgraf [54] and employ Reduze 2 [15, 55, 56] to match the diagrams to integral families, calculate the interference of the two-loop with the tree amplitude and reduce the resulting Feynman integrals to a set of master integrals. We use Form [57] at different stages of the calculation. Some example Feynman diagrams are shown in Fig. 1.

Most of the master integrals were available already, but some needed to be calculated for this project. The most complicated integrals encountered were the three master integrals occurring in the non-planar double box topology of the last diagram in Fig. 1. In [58] these integrals were computed in terms of multiple polylogarithms, representing the first analytical calculation of non-planar double box integrals with more than one massive leg.

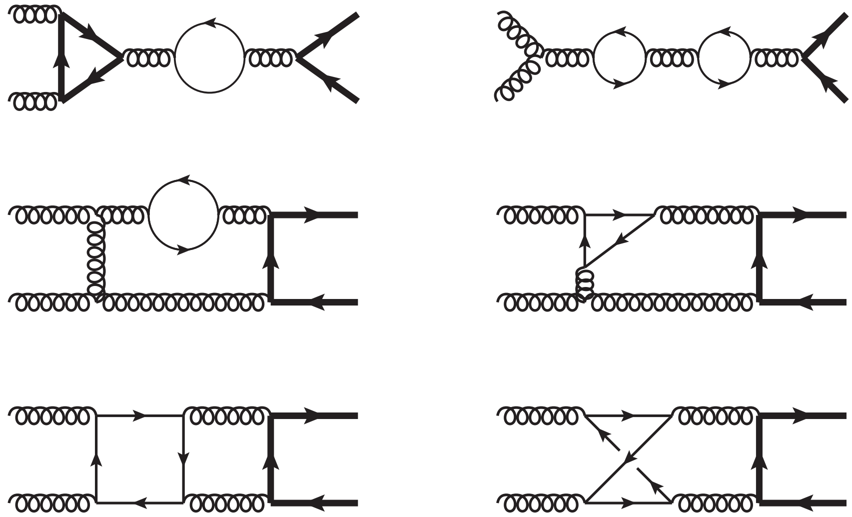

Figure 1: Two-loop corrections to $gg \to t\bar{t}$ with a closed massless quark loop, taken from ref. [46]. Thick fermion lines are used for the massive top quark, thin fermion lines represent massless quarks.

The calculation employs the method of differential equations. Here, we define the dimensionless variables x, y and z by

$$s = -m^2(1-x)^2/x, \quad t = -m^2 y, \quad u = -m^2 z. \tag{2.2}$$

such that the Landau variable x absorbs a root $\sqrt{-s(4m^2-s)}$ appearing in the differential equation due to a two-massive-particle threshold. The derivatives of the master integrals with respect to the external invariants x and y are obtained via IBP reduction in terms of linear combinations of master integrals, where the coefficients are rational functions of x and y. It is not difficult to choose the master integrals such that after an expansion in $\varepsilon = (4-d)/2$, where d is the space-time dimension, the system of differential equations decouples partially such that it can be integrated in a bottom-up approach.

Integrating the differential equations with respect to x and y leads to iterated integrals of the form

$$G(w_1, w_2, \cdots, w_n; z) \equiv \int_0^z \mathrm{d}t \, \frac{1}{t-w_1} G(w_2, \cdots, w_n; t)\,, \tag{2.3}$$

$$G(\underbrace{0, \cdots, 0}_{n}; z) \equiv \frac{1}{n!} \ln^n z\,, \tag{2.4}$$

which are known as multiple polylogarithms. A non-linearity introduced with the variable x leads to non-linear denominators in the integration variable, for which we employ generalised weights [59] $[f(o)]$ defined by

$$G([f(o)], w_2, \cdots, w_n; z) = \int_0^z dt \, \frac{f'(t)}{f(t)} G(w_2, \cdots, w_n; t)\,, \tag{2.5}$$

where f is an irreducible rational polynomial. While it is completely straight-forward to eliminate the generalised weights by complex factorisation, e.g.

$$G([o^2+1];x) = \int_0^x \mathrm{d}t \frac{2t}{t^2+1} = \int_0^x \mathrm{d}t \frac{1}{t-i} + \int_0^x \mathrm{d}t \frac{1}{t+i} = G(i;x) + G(-i;x), \tag{2.6}$$

the bracket notation is more compact and preserves the structure of the $d\ln(f(t))$ integration. In order to fix the integration constants, we use a combination of regularity conditions, symmetry constraints and Mellin-Barnes evaluations in asymptotic limits. For the latter, we employed the packages Ambre [60] (for planar topologies) and MB [61]. The sector decomposition program SecDec 2 [62, 63] was a valuable tool to perform numerical checks of our results.

We insert the master integrals in the interference terms and choose arguments y and x for all multiple polylogarithms including those originating from u-channel contributions. In this way, we obtain a result in terms of multiple polylogarithms of the form

$$G(w_1,\ldots,w_n;y) \quad \text{with } w_i \in \left\{-1,0,-1/x,-x,-(1+x^2)/x,-(1-x+x^2)/x\right\}, \tag{2.7}$$

$$G(w_1,\ldots,w_n;x) \quad \text{with } w_i \in \left\{-1,0,1,[1+o^2],[1-o+o^2]\right\}, \tag{2.8}$$

some of which are multivalued. In order to employ the expressions for phenomenological applications, the performance of the numerical evaluations is not satisfactory yet. The main reason is that known strategies [64] for the evaluation of a single function of the above type requires a series of non-trivial mappings to many power series objects, which are ultimately used for the numerical approximation.

Therefore, we express our results in terms of an alternative functional basis, which we construct to allow for fast and direct numerical evaluations. We choose logarithms, classical polylogarithms

$$\mathrm{Li}_n(x_1) = -G(\underbrace{0,\cdots,0,1}_{n};x_1), \tag{2.9}$$

with $n = 2,3,4$ and genuine multiple polylogarithms of the type

$$\mathrm{Li}_{2,2}(x_1,x_2) = G\left(0,\frac{1}{x_1},0,\frac{1}{x_1x_2};1\right), \tag{2.10}$$

where the arguments are (complicated) rational functions of x and y, such that the resulting functions are real valued. We can actually go one step further and require in addition

$$|x_1| < 1, \qquad |x_1x_2| < 1, \tag{2.11}$$

such that the Li functions allow for an immediate convergent power series representation

$$\mathrm{Li}_n(x_1) = -\sum_{j_1=1}^{\infty} \frac{x_1^{j_1}}{j_1^n}, \tag{2.12}$$

$$\mathrm{Li}_{2,2}(x_1,x_2) = \sum_{j_1=1}^{\infty}\sum_{j_2=1}^{\infty} \frac{x_1^{j_1}}{(j_1+j_2)^2}\frac{(x_1x_2)^{j_2}}{j_2^2}, \tag{2.13}$$

which renders the numerical evaluation straight-forward and very stable. While it is not obvious at all that such a restricted set of functions is sufficient to express our results, we find that it is

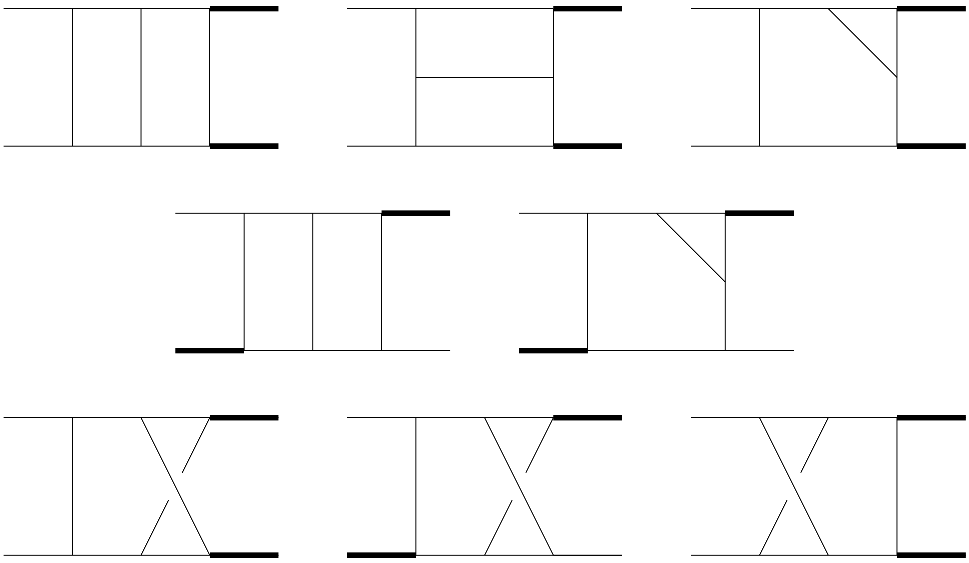

Figure 2: Top level topologies for planar [48] and non-planar [49] master integrals in $q\bar{q} \to WW$ and $q\bar{q} \to ZZ$. Thick lines denote the massive electroweak vector bosons.

indeed the case. In the final expression, all generalised weights are eliminated by multivariate recombinations such as

$$G(-(1+x^2)/x; y) + G([1+o^2]; x) - G(0; x) + i\pi = \ln(-(1+x^2)/x - y) = \ln(z) \tag{2.14}$$

The new functional basis reduces the numerical evaluation time by orders of magnitude. Our methods for the multiple polylogarithms employ and extend symbol and coproduct based algorithms presented in [26, 27].

3. The two-loop master integrals for $q\bar{q} \to ZZ/WW$

We consider the two-loop corrections for the partonic scattering processes

$$q(p_1) + \bar{q}(p_2) \to Z(p_3) + Z(p_4), \tag{3.1}$$

$$q(p_1) + \bar{q}(p_2) \to W^+(p_3) + W^-(p_4), \tag{3.2}$$

with $p_1^2 = p_2^2 = 0$ and $p_3^2 = p_4^2 = m^2$, where m is either the Z or the W mass, respectively. Similarly to the previous section, we choose to work with dimensionless variables x and z, where here

$$s = m^2(1+x)^2/x, \quad t = -m^2 y, \quad u = -m^2 z. \tag{3.3}$$

In the following, we report on the recent analytical calculation [49] of the two-loop master integrals required for two-loop corrections [65] to these processes. Fig. 2 shows the planar and non-planar top-level topologies for the master integrals. A significant part of the technology discussed in the previous section could be employed also for this calculation.

We choose our master integrals such that the differential equations take the normal form

$$d\vec{m}(\varepsilon;x,z) = \varepsilon\, dA(x,z)\,\vec{m}(\varepsilon;x,z) \tag{3.4}$$

where $\vec{m}(\varepsilon;x,z)$ is a vector of master integrals and $A(x,z)$ is a matrix of rational functions of x and z. Normal forms of this type have been proposed in [32]. Finding such a basis is not necessary to solve the integrals, a partial decoupling in ε would be enough for that purpose. However, it simplifies the book-keeping and is considerably more transparent from a conceptional point of view. In our case, the matrix A can be decomposed according to

$$A(x,z) = \sum_{k=1}^{10} A_k \ln(r_k)\,, \tag{3.5}$$

where A_k is a matrix of rational numbers and the letters r_k are rational functions of the external invariants,

$$r_k = \left\{x,\, 1-x,\, 1+x,\, z,\, 1+z,\, x-z,\, 1-xz,\, 1+x^2-xz,\, 1+x+x^2-xz,\, z(1+x+x^2)-x\right\}. \tag{3.6}$$

Except for specific cases it is not known whether for some given set of Feynman integrals a basis with differential equations of the form (3.4) exists at all (see also [66, 67, 68]). Provided such a basis exists, no general algorithm to finding it is available. An approach which works for the master integrals discussed in this section and other cases of practical relevance was formulated in [49]. Starting from a rough first guess which partially decouples the top level topologies, the recipe describes how to systematically clean up unwanted terms.

We integrate the differential equations and use a couple of simple integrals available in the literature as independent input. We impose regularity conditions in some of the following collinear and threshold limits

$$z \to x, \qquad z \to 1/x, \qquad z \to -1, \qquad z \to (1+x+x^2)/x, \qquad x \to 1\,, \tag{3.7}$$

which fixes all of the remaining integration constants.

Similarly as for the $t\bar{t}$ corrections, we convert our results to a functional basis optimised for numerical evaluations. Also here, it turns out to be sufficient to employ Li functions where the arguments fulfil (2.11). In this way, the complete two-loop corrections to $q\bar{q} \to ZZ$ production can be evaluated for some generic phase space point in about 30ms with double precision or in about 0.3s with 30 digits precision on one CPU core.

4. Conclusions

Considerable progress has been made towards fully differential NNLO predictions for the pair production of massive particles at the LHC. In this talk, we reported on recent analytic results for two-loop corrections to the pair production of top quarks, Z bosons and W bosons, respectively. A major ingredient is the analytical calculation of the master integrals, where we put special emphasis on providing them in a form which allows for fast and stable numerical evaluations.

For $t\bar{t}$ production, we presented the light-quark two-loop corrections in the gluon channel [46]. The calculation of the remaining master integrals required for the subleading colour corrections in the quark channel is work in progress [70]. We expect the complete two-loop corrections [71] to be available in the near future.

For ZZ and WW production, we presented the calculation of all two-loop master integrals [49]. These results enabled the calculation of the total ZZ production cross section [10], which was presented at this conference for the first time [9]. For WW production, the calculation of the NNLO cross section is work in progress [69].

References

[1] M. Czakon, P. Fiedler and A. Mitov, *Total Top-Quark Pair-Production Cross Section at Hadron Colliders Through $O(\alpha_S^4)$*, Phys. Rev. Lett. **110** (2013) 25, 252004 [arXiv:1303.6254].

[2] P. Bärnreuther, M. Czakon and P. Fiedler, *Virtual amplitudes and threshold behaviour of hadronic top-quark pair-production cross sections*, JHEP **1402**, 078 (2014) [arXiv:1312.6279].

[3] S. Catani, L. Cieri, D. de Florian, G. Ferrera and M. Grazzini, *Diphoton production at hadron colliders: a fully-differential QCD calculation at NNLO*, Phys. Rev. Lett. **108** (2012) 072001 [arXiv:1110.2375].

[4] Z. Bern, A. De Freitas and L. J. Dixon, *Two loop amplitudes for gluon fusion into two photons*, *JHEP* **0109** (2001) 037, [hep-ph/0109078].

[5] C. Anastasiou, E. N. Glover and M. Tejeda-Yeomans, *Two loop QED and QCD corrections to massless fermion boson scattering*, *Nucl.Phys.* **B629** (2002) 255–289, [hep-ph/0201274].

[6] M. Grazzini, S. Kallweit, D. Rathlev and A. Torre, *$Z\gamma$ production at hadron colliders in NNLO QCD*, *Phys.Lett.* **B731** (2014) 204, [arXiv:1309.7000].

[7] T. Gehrmann and L. Tancredi, *Two-loop QCD helicity amplitudes for $q\bar{q} \to W^{\pm}\gamma$ and $q\bar{q} \to Z^0\gamma$*, *JHEP* **1202** (2012) 004, [arXiv:1112.1531].

[8] T. Gehrmann, L. Tancredi and E. Weihs, *Two-loop QCD helicity amplitudes for $gg \to Zg$ and $gg \to Z\gamma$*, *JHEP* **1304** (2013) 101, [arXiv:1302.2630].

[9] M. Grazzini, these proceedings.

[10] F. Cascioli, T. Gehrmann, M. Grazzini, S. Kallweit, P. Maierhöfer, A. von Manteuffel, S. Pozzorini, D. Rathlev, L. Tancredi and E. Weihs, *ZZ production at hadron colliders in NNLO QCD*, arXiv:1405.2219.

[11] C. Anastasiou and A. Lazopoulos, *Automatic integral reduction for higher order perturbative calculations*, JHEP **0407** (2004) 046 [hep-ph/0404258].

[12] A. V. Smirnov, *Algorithm FIRE – Feynman Integral REduction*, JHEP **0810** (2008) 107 [arXiv:0807.3243].

[13] A. V. Smirnov and V. A. Smirnov, *FIRE4, LiteRed and accompanying tools to solve integration by parts relations*, Comput. Phys. Commun. **184** (2013) 2820 [arXiv:1302.5885].

[14] C. Studerus, *Reduze-Feynman Integral Reduction in C++*, *Comput. Phys. Commun.* **181** (2010) 1293–1300, [arXiv:0912.2546].

[15] A. von Manteuffel and C. Studerus, *Reduze 2 - Distributed Feynman Integral Reduction*, arXiv:1201.4330.

[16] R. N. Lee, *Presenting LiteRed: a tool for the Loop InTEgrals REDuction*, arXiv:1212.2685.

[17] R. N. Lee, *LiteRed 1.4: a powerful tool for reduction of multiloop integrals*, J. Phys. Conf. Ser. **523** (2014) 012059 [arXiv:1310.1145].

[18] R. N. Lee and A. A. Pomeransky, *Critical points and number of master integrals*, JHEP **1311** (2013) 165 [arXiv:1308.6676].

[19] F. Tkachov, *A Theorem on Analytical Calculability of Four Loop Renormalization Group Functions*, *Phys.Lett.* **B100** (1981) 65–68.

[20] K. Chetyrkin and F. Tkachov, *Integration by Parts: The Algorithm to Calculate beta Functions in 4 Loops*, *Nucl. Phys.* **B192** (1981) 159–204.

[21] S. Laporta, *High precision calculation of multiloop Feynman integrals by difference equations*, *Int. J. Mod. Phys.* **A15** (2000) 5087–5159, [hep-ph/0102033].

[22] A. von Manteuffel and R. M. Schabinger, *A novel approach to integration by parts reduction*, arXiv:1406.4513.

[23] S. Badger, these proceedings.

[24] F. Brown, *The Massless higher-loop two-point function*, *Commun. Math. Phys.* **287** (2009) 925–958, [arXiv:0804.1660].

[25] A. B. Goncharov, M. Spradlin, C. Vergu and A. Volovich, *Classical Polylogarithms for Amplitudes and Wilson Loops*, *Phys. Rev. Lett.* **105** (2010) 151605, [arXiv:1006.5703].

[26] C. Duhr, H. Gangl and J. R. Rhodes, *From polygons and symbols to polylogarithmic functions*, *JHEP* **1210** (2012) 075, [arXiv:1110.0458].

[27] C. Duhr, *Hopf algebras, coproducts and symbols: an application to Higgs boson amplitudes*, *JHEP* **1208** (2012) 043, [arXiv:1203.0454].

[28] A. Kotikov, *Differential equations method: New technique for massive Feynman diagrams calculation*, *Phys. Lett.* **B254** (1991) 158–164.

[29] E. Remiddi, *Differential equations for Feynman graph amplitudes*, *Nuovo Cim.* **A110** (1997) 1435–1452, [hep-th/9711188].

[30] M. Caffo, H. Czyz, S. Laporta and E. Remiddi, *The Master differential equations for the two loop sunrise selfmass amplitudes*, *Nuovo Cim.* **A111** (1998) 365–389, [hep-th/9805118].

[31] T. Gehrmann and E. Remiddi, *Differential equations for two loop four point functions*, *Nucl. Phys.* **B580** (2000) 485–518, [hep-ph/9912329].

[32] J. M. Henn, *Multiloop integrals in dimensional regularization made simple*, *Phys. Rev. Lett.* **110** (2013) 251601, [arXiv:1304.1806].

[33] G. Abelof and A. Gehrmann-De Ridder, *Antenna subtraction for the production of heavy particles at hadron colliders*, JHEP **1104** (2011) 063 [arXiv:1102.2443].

[34] G. Abelof and A. Gehrmann-De Ridder, *Double real radiation corrections to $t\bar{t}$ production at the LHC: the all-fermion processes*, JHEP **1204** (2012) 076 [arXiv:1112.4736].

[35] G. Abelof and A. Gehrmann-De Ridder, *Double real radiation corrections to $t\bar{t}$ production at the LHC: the $gg \to t\bar{t}q\bar{q}$ channel* JHEP **1211** (2012) 074 [arXiv:1207.6546].

[36] G. Abelof, O. Dekkers and A. Gehrmann-De Ridder, *Antenna subtraction with massive fermions at NNLO: Double real initial-final configurations*, JHEP **1212** (2012) 107 [arXiv:1210.5059].

[37] G. Abelof, A. Gehrmann-De Ridder, P. Maierhofer and S. Pozzorini, *NNLO QCD corrections to top-antitop production in the $q\bar{q}$ channel*, arXiv:1404.6493.

[38] I. Bierenbaum, M. Czakon and A. Mitov, *The singular behavior of one-loop massive QCD amplitudes with one external soft gluon*, Nucl. Phys. B **856** (2012) 228 [arXiv:1107.4384].

[39] A. Gehrmann-De Ridder, these proceedings.

[40] J. G. Körner, Z. Merebashvili and M. Rogal, *NNLO $O(\alpha_s^4)$ results for heavy quark pair production in quark-antiquark collisions: The One-loop squared contributions*, Phys. Rev. D **77** (2008) 094011 [Erratum-ibid. D **85** (2012) 119904] [arXiv:0802.0106].

[41] B. Kniehl, Z. Merebashvili, J. G. Körner and M. Rogal, *Heavy quark pair production in gluon fusion at next-to-next-to-leading $O(\alpha_s^4)$ order: One-loop squared contributions*, Phys. Rev. D **78** (2008) 094013 [arXiv:0809.3980].

[42] C. Anastasiou and S. M. Aybat, *The One-loop gluon amplitude for heavy-quark production at NNLO*, Phys. Rev. D **78** (2008) 114006 [arXiv:0809.1355].

[43] R. Bonciani, A. Ferroglia, T. Gehrmann, D. Maitre and C. Studerus, *Two-Loop Fermionic Corrections to Heavy-Quark Pair Production: The Quark-Antiquark Channel*, JHEP **0807** (2008) 129 [arXiv:0806.2301].

[44] R. Bonciani, A. Ferroglia, T. Gehrmann and C. Studerus, *Two-Loop Planar Corrections to Heavy-Quark Pair Production in the Quark-Antiquark Channel*, JHEP **0908** (2009) 067 [arXiv:0906.3671].

[45] R. Bonciani, A. Ferroglia, T. Gehrmann, A. von Manteuffel and C. Studerus, *Two-Loop Leading Color Corrections to Heavy-Quark Pair Production in the Gluon Fusion Channel*, JHEP **1101** (2011) 102 [arXiv:1011.6661].

[46] R. Bonciani, A. Ferroglia, T. Gehrmann, A. von Manteuffel and C. Studerus, *Light-quark two-loop corrections to heavy-quark pair production in the gluon fusion channel*, *JHEP* **1312** (2013) 038, [arXiv:1309.4450].

[47] G. Chachamis, M. Czakon and D. Eiras, *W Pair Production at the LHC. I. Two-loop Corrections in the High Energy Limit*, *JHEP* **0812** (2008) 003, [arXiv:0802.4028].

[48] T. Gehrmann, L. Tancredi and E. Weihs, *Two-loop master integrals for $q\bar{q} \to VV$: the planar topologies*, JHEP **1308** (2013) 070 [arXiv:1306.6344].

[49] T. Gehrmann, A. von Manteuffel, L. Tancredi and E. Weihs, *The two-loop master integrals for $q\bar{q} \to VV$*, JHEP **1406** (2014) 032 [arXiv:1404.4853].

[50] J. M. Henn, K. Melnikov and V. A. Smirnov, *Two-loop planar master integrals for the production of off-shell vector bosons in hadron collisions*, JHEP **1405**, 090 (2014) [arXiv:1402.7078].

[51] F. Caola, J. M. Henn, K. Melnikov and V. A. Smirnov, *Non-planar master integrals for the production of two off-shell vector bosons in collisions of massless partons*, arXiv:1404.5590.

[52] V. A. Smirnov, these proceedings.

[53] C. Wever, these proceedings.

[54] P. Nogueira, *Automatic Feynman graph generation*, J. Comput. Phys. **105** (1993) 279.

[55] C. W. Bauer, A. Frink and R. Kreckel, *Introduction to the GiNaC framework for symbolic computation within the C++ programming language*, *J. Symb. Comput.* **33** (2002) 1–12, [cs/0004015].

[56] R. Lewis, *Computer Algebra System Fermat*. http://www.bway.net/~lewis.

[57] J. Vermaseren, *New features of FORM*, math-ph/0010025.

[58] A. von Manteuffel and C. Studerus, *Massive planar and non-planar double box integrals for light N_f contributions to $gg \to t\bar{t}$*, *JHEP* **1310** (2013) 037, [arXiv:1306.3504].

[59] A. von Manteuffel, R. M. Schabinger and H. X. Zhu, *The Complete Two-Loop Integrated Jet Thrust Distribution In Soft-Collinear Effective Theory*, *JHEP* **1403** (2014) 139, [arXiv:1309.3560].

[60] J. Gluza, K. Kajda and T. Riemann, *AMBRE: A Mathematica package for the construction of Mellin-Barnes representations for Feynman integrals*, Comput. Phys. Commun. **177** (2007) 879 [arXiv:0704.2423].

[61] M. Czakon, *Automatized analytic continuation of Mellin-Barnes integrals*, Comput. Phys. Commun. **175** (2006) 559 [hep-ph/0511200].

[62] S. Borowka, J. Carter and G. Heinrich, *Numerical Evaluation of Multi-Loop Integrals for Arbitrary Kinematics with SecDec 2.0*, *Comput. Phys. Commun.* **184** (2013) 396–408, [arXiv:1204.4152].

[63] S. Borowka and G. Heinrich, *Massive non-planar two-loop four-point integrals with SecDec 2.1*, *Comput. Phys. Commun.* **184** (2013) 2552–2561, [arXiv:1303.1157].

[64] J. Vollinga and S. Weinzierl, *Numerical evaluation of multiple polylogarithms*, *Comput. Phys. Commun.* **167** (2005) 177, [hep-ph/0410259].

[65] T. Gehrmann, A. von Manteuffel, L. Tancredi and E. Weihs, in preparation.

[66] E. Remiddi and L. Tancredi, *Schouten identities for Feynman graph amplitudes; The Master Integrals for the two-loop massive sunrise graph*, *Nucl. Phys.* **B880** (2014) 343–377, [arXiv:1311.3342].

[67] L. Tancredi, these proceedings.

[68] P. Mastrolia, these proceedings.

[69] F. Cascioli, T. Gehrmann, M. Grazzini, S. Kallweit, P. Maierhöfer, A. von Manteuffel, S. Pozzorini, D. Rathlev, L. Tancredi and E. Weihs, in preparation.

[70] J. M. Henn, A. von Manteuffel and V. A. Smirnov, in preparation.

[71] R. Bonciani, A. Ferroglia, T. Gehrmann and A. von Manteuffel, in preparation.

PoS(LL2014)003