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Parity violation in the CMB bispectrum by a rolling pseudoscalar

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We investigated the parity violating signal of the temperature and polarization bispectra in the cosmic microwave background (CMB) caused by the rolling pseudoscalar inflation. This model is known for producing large non-Gaussianities in the tensor sector. We found that the CMB bispectra generated in the pseudoscalar inflation have non-zero signals in both parity-even ($\ell_1 + \ell_2 + \ell_3 =$ even) and parity-odd ($\ell_1 + \ell_2 + \ell_3 =$ odd) spaces by using the full-sky formalism. These parity violating signals help us to detect non-Gaussianity in the tensor sector. We also discussed the detectability of the CMB bispectra induced by the pseudoscalar inflation. Parity violating signals improve the detectability more and we found that E-mode bispectra improve of 400% the detectability. Furthermore, B-mode bispectrum is able to improve the signal-to-noise ratio about 3 orders of magnitude.

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1. Introduction

In the recent studies, parity violation is one of the key to explore the early Universe models. And it is known that parity violating models leave traces on the temperature fluctuation and polarization of the cosmic microwave background (CMB).

Recent studies show that there is a model, which has strong relation between parity violation and large tensor non-Gaussianity. The non-Gaussianity in tensor sector has raised less interest than that in scalar sector. This is because tensor-to-scalar ratio may be too small to considered tensor non-Gaussianity and inflation induced scalar fields motivate the scalar non-Gaussianity. In Ref. [1], they proposed the model, which can amplify the vacuum fluctuation of a U(1) gauge field by introducing the coupling between a pseudoscalar field and a gauge field. This enhanced gauge mode also generates chiral gravitational waves because gravitational waves are sourced by the energy momentum tensor of chiral gauge field. Because of chiral gravitational waves in this model, the fluctuations of CMB must have parity violating signals as TB and EB correlations. The rolling pseudoscalar inflation creates small scalar and large tensor non-Gaussianities [2]. They studied the effects of parity violation on the CMB temperature auto-bispectrum by using flat-sky formalism. Based on this formalism, it does not work on large scales where the tensor mode dominates. Furthermore, it is difficult to know that the parity violating signals, analytically, i.e., both the parity-even ($\ell_1+\ell_2+\ell_3 = \mathrm{even}$) and parity-odd ($\ell_1+\ell_2+\ell_3 = \mathrm{odd}$), if we use the flat-sky formalism where the ℓ spaces are not discreet.

In this paper, we present the CMB temperature and polarization bispectra through a full-sky formalism [3]. By a concrete computation based on full-sky formalism, we presented the CMB bispectra have non-zero parity violating signals in ℓ space analytically. We estimated the detectability of the tensor non-Gaussianity for cases with the auto- and cross-bispectra of the CMB temperature and E-mode polarization and with the auto-bispectrum of the CMB B-mode polarization.

2. Parity violation by a pseudoscalar

In this section, we will show that the gravitational waves become chiral and large tensor non-Gaussianity is created naturally in the rolling pseudoscalar inflation. In this paper, we focus on a model where a rolling pseudoscalar field χ couples to U(1) gauge field A_μ in the inflationary epoch. We consider the inflation model as

$$\mathscr{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{2}(\partial\chi)^2 - U(\chi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\chi}{4f}\tilde{F}^{\mu\nu}F_{\mu\nu} \; , \tag{2.1}$$

where ϕ and f are an inflaton and a coupling constant between the pseudoscalar and gauge fields, respectively. And the gauge field strength obeys $F_{\mu\nu} = A_{\mu;\nu} - A_{\nu;\mu}$ and $\tilde{F}^{\mu\nu}$ is its dual tensor, which causes parity violation. $V(\phi)$ and $U(\chi)$ are the potential of the inflaton and the pseudoscalar field.

The equation of motion of the gauge field can be written as

$$\mathbf{A}'' - \nabla^2\mathbf{A} - \frac{\chi'}{f}\nabla\times\mathbf{A} = 0 \; , \tag{2.2}$$

where we imposed the Coulomb gauge $A_0 = 0$ on the gauge field and $\nabla \cdot \mathbf{A} = 0$. We denote $' \equiv \partial_\tau$ which means derivative respect to conformal time. We can solve the above equation of motion by moving to a quantizetion process in Fourier space as

$$A_i(\tau,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \sum_{\lambda=\pm 1} \left[a_\lambda(\mathbf{k}) A_\lambda(\tau,\mathbf{k}) + a_\lambda^\dagger(-\mathbf{k}) A_\lambda^*(\tau,-\mathbf{k}) \right] \varepsilon_i^{(\lambda)}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} , \tag{2.3}$$

where $a_\lambda(\mathbf{k})$ and $a_\lambda^\dagger(\mathbf{k})$ are ordinary creation and annihilation operators. And $\varepsilon_i^{(\lambda)}(\mathbf{k})$ is a divergenceless polarization vector. This solution of the mode function A_λ with rolling condition $\dot{\chi} = \mathrm{const}$ obeys the Coulomb wave function and whose growing mode is written as

$$A_+(\tau,\mathbf{k}) \simeq \frac{1}{\sqrt{2k}} \left(-\frac{k\tau}{2\xi} \right)^{1/4} e^{\pi\xi - 2\sqrt{-2\xi k\tau}} , \tag{2.4}$$

where $\xi = \dot{\chi}/(2fH)$ and a dot represents a derivative with respect to the coordinate time $dt = ad\tau$. We can understand the parity violation in this model by considering the solution of A_λ, namely, A_+ is exponentially amplified by ξ while A_- is not amplified by rolling pseudoscalar.

The tensor perturbation $\delta g_{ij} = a^2 h_{ij}$ obeys the Einstein equation:

$$h_{ij}'' + 2\frac{a'}{a} h_{ij}' - \nabla^2 h_{ij} = -\frac{2a^2}{M_P} \left(E_i E_j + B_i B_j \right)^{TT} , \tag{2.5}$$

where E_i and B_i are electric and magnetic part of the gauge field. Moreover TT means the traceless and transverse part of the energy momentum tensor of the gauge field. The gravitational waves have also chiral property, which are induced by the parity violating sources of the gauge field. To solve the above equation in Fourier space, we also express h_{ij} by the polarization tensor $e_{ij}^{\pm 2}(\hat{\mathbf{k}})$, which is the traceless and transverse tensor, as

$$h_{ij}(\tau,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \sum_{\lambda=\pm 2} h_{\mathbf{k}}^{(\lambda)}(\tau) e_{ij}^{(\lambda)}(\hat{\mathbf{k}}) e^{i\mathbf{k}\cdot\mathbf{x}} . \tag{2.6}$$

The solution is given by the Green function as

$$\begin{aligned} h_{\mathbf{k}}^{(\lambda)}(\tau) &= -\frac{2H^2}{M_P^2} \int d\tau' G_k(\tau,\tau') \tau'^2 \int \frac{d^3\mathbf{k}'}{(2\pi)^{3/2}} d^3\mathbf{k}'' \frac{1}{2} e_{ij}^{(-\lambda)}(\hat{\mathbf{k}}) \\ &\quad \times \left[\mathscr{E}_i(\tau',\mathbf{k}') \mathscr{E}_j(\tau',\mathbf{k}'') + \mathscr{B}_i(\tau',\mathbf{k}') \mathscr{B}_j(\tau',\mathbf{k}'') \right] \delta(\mathbf{k}' + \mathbf{k}'' - \mathbf{k}) , \end{aligned} \tag{2.7}$$

where the Green function is given as

$$G_k(\tau,\tau') = \frac{1}{k^3 \tau'^2} \left[(1 + k^2 \tau\tau') \sin k(\tau - \tau') + k(\tau' - \tau) \cos k(\tau - \tau') \right] \Theta(\tau - \tau') . \tag{2.8}$$

$\mathscr{E}_i$ and $\mathscr{B}_i$ correspond to the electric and magnetic part of the gauge field in Fourier space:

$$\mathscr{E}_i(\tau,\mathbf{k}) \equiv v_+'(\tau,\mathbf{k}) \varepsilon_i^{(+)}(\mathbf{k}) , \tag{2.9}$$

$$\mathscr{B}_i(\tau,\mathbf{k}) \equiv k v_+(\tau,\mathbf{k}) \varepsilon_i^{(+)}(\mathbf{k}) . \tag{2.10}$$

The tensor bispectrum is evaluated through the above solution and amplified in the equilateral limit ($k_1 = k_2 = k_3$) [2], namely

$$\left\langle \prod_{n=1}^{3} h^{(+2)}_{\mathbf{k_n}} \right\rangle_{k_n \to k} \simeq 3 \times 10^{-4} \mathscr{P}^3 X^3 \frac{\delta(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3})}{k^6} , \tag{2.11}$$

where we defined $\mathscr{P} \equiv H^2/(8\pi\varepsilon M_P^2)$ and X is the non-gaussian strength parameter, which is defined as

$$X \equiv \varepsilon \frac{e^{2\pi\xi}}{\xi^3} , \tag{2.12}$$

where ε is slow-roll parameter. This is the only parameter in this inflation model. In the detectability analysis, we estimate the 1σ errors of this parameter. From numerical evaluation, we confirmed that the other spin modes are smaller than Eq. (2.11). Consequently, we dropped these ignorable contributions in this paper.

3. Parity violation in CMB bispectra

In this section, we present results of the CMB temperature and polarization bispectra based on the full-sky formalism [3]. The CMB anisotropies are quantified through a multipole expansion:

$$\frac{\Delta \mathscr{X}(\hat{\mathbf{n}})}{\mathscr{X}} = \sum_{\ell,m} a^{\mathscr{X}}_{\ell m} Y_{\ell m}(\hat{\mathbf{n}}) , \tag{3.1}$$

where $\mathscr{X}$ denotes the temperature (I), E-mode (E) and B-mode (B) fields. The coefficients of the multipole expansion are given as

$$a^{\mathscr{X}}_{\ell m} = 4\pi(-i)^\ell \int_0^\infty \frac{k^2 dk}{(2\pi)^{3/2}} \mathscr{T}^{\mathscr{X}}_\ell(k) \sum_{\lambda=\pm 2} \left(\frac{\lambda}{2}\right)^x \int d^2\hat{\mathbf{k}} h^{(\lambda)}_{\mathbf{k}} {}_{-\lambda}Y^*_{\ell m}(\hat{\mathbf{k}}) , \tag{3.2}$$

where x corresponds to parities of three modes, namely, parity even component: $x = 0$ for $\mathscr{X} = I, E$ and parity odd component: $x = 1$ for $\mathscr{X} = B$, and $\mathscr{T}^{\mathscr{X}}_\ell$ is a radiation transfer function, respectively. The CMB bispectra are expressed as

$$\left\langle \prod_{n=1}^{3} a^{\mathscr{X}_n}_{\ell_n m_n} \right\rangle = B^{\mathscr{X}_1\mathscr{X}_2\mathscr{X}_3}_{\ell_1\ell_2\ell_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} , \tag{3.3}$$

and the CMB bispectra induced by chiral gravitational waves are evaluated as

$$\begin{aligned} B^{\mathscr{X}_1\mathscr{X}_2\mathscr{X}_3}_{\ell_1\ell_2\ell_3} &= -\frac{(8\pi)^{3/2}}{10} \sqrt{\frac{7}{3}} N \mathscr{P}^3 X^3 \left[\prod_{n=1}^{3} \sum_{L_n} (-1)^{\frac{L_n}{2}} (-i)^{\ell_n} I^{20-2}_{\ell_n L_n 2} \right] I^{0\,0\,0}_{L_1 L_2 L_3} \begin{Bmatrix} \ell_1 & \ell_2 & \ell_3 \\ L_1 & L_2 & L_3 \\ 2 & 2 & 2 \end{Bmatrix} \\ &\times \int_0^\infty r^2 dr \left[\prod_{n=1}^{3} \frac{2}{\pi} \int_0^\infty k_n^2 dk_n \mathscr{T}^{\mathscr{X}_n}_{\ell_n}(k_n) j_{L_n}(k_n r) \right] B^{\rm eq}_{k_1 k_2 k_3} , \end{aligned} \tag{3.4}$$

where

$$I^{s_1 s_2 s_3}_{l_1 l_2 l_3} \equiv \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ s_1 & s_2 & s_3 \end{pmatrix} . \tag{3.5}$$

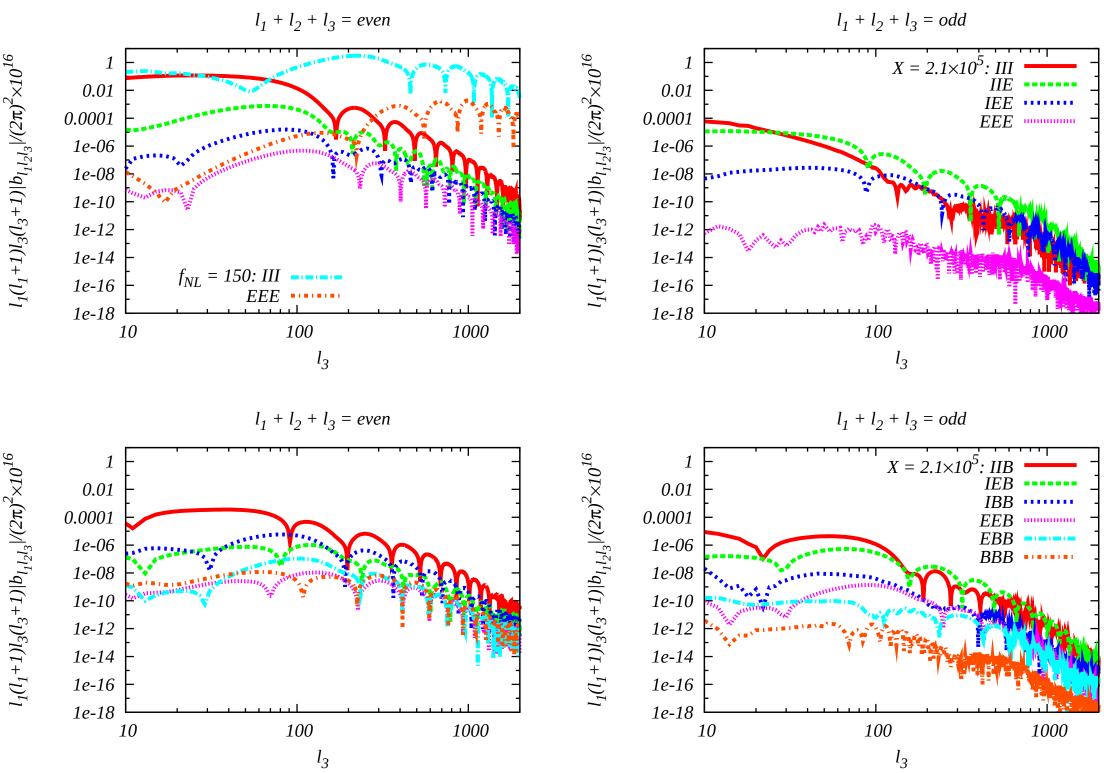

Figure 1: All possible CMB bispectra, i.e., $\langle III\rangle$, $\langle IIE\rangle$, $\langle IEE\rangle$ and $\langle EEE\rangle$ (top two panels), and $\langle IIB\rangle$, $\langle IEB\rangle$, $\langle IBB\rangle$, $\langle EEB\rangle$, $\langle EBB\rangle$ and $\langle BBB\rangle$ (bottom two panels). The model parameter is fixed $X = 2.1\times10^5$ and the primordial power spectrum is $\mathscr{P} = 2.5\times10^{-9}$ for $\ell_1+2 = \ell_2+1 = \ell_3$. Left and right two panels describe the parity-even ($\ell_1+\ell_2+\ell_3 = \text{even}$) and parity-odd ($\ell_1+\ell_2+\ell_3 = \text{odd}$) components, respectively. We also plot $\langle III\rangle$ and $\langle EEE\rangle$ from the equilateral non-Gaussianity with $f_{\rm NL} = 150$.

Note that we confirmed that the primordial tensor-bispectrum correlates closely with the usual equilateral template. Hence the primordial tensor-bispectrum can be replaced the usual equilateral template to compute the CMB bispectra and its normalization factor N is decided numerically without uncertainly.

We can see the selection rules of the Wigner symbols tells us the non-zero values for both $\ell_1+\ell_2+\ell_3 = \text{even}$ and $\ell_1+\ell_2+\ell_3 = \text{odd}$. This selection of ℓ space is induced by parity violating interaction between the pseudoscalar field and the gauge fields. We depict the CMB reduced bispectra for $\ell_1 \approx \ell_2 \approx \ell_3$ induced by pseudoscalar in Fig. 1.

From Fig. 1, the amplitude of parity-odd and cross-bispectra are same as that of parity-even and cross-bispectra. However the amplitude of parity-odd and auto-bispectra are smaller than that of parity-even and auto-bispectra. Total signal coming from parity-odd bispectra would be suppressed slightly because of the decaying nature at $\ell_1 \approx \ell_2 \approx \ell_3$. Note that the peaks at $\ell \sim 100$ in the E- and B-mode polarization are induced by the Thomson scattering.

4. Detectability

In this section, I devote to summarize the detectability analysis, shortly. We evaluate 1σ errors

	III	EEE	all $I+E$	BBB ($r=0.05$)	BBB ($r=5\times10^{-4}$)
Planck	127 (129)	232 (233)	56 (65)	17 (19)	2.1 (2.1)
PRISM	127 (129)	83 (84)	25 (30)	0.87 (1.0)	0.015 (0.017)
ideal	127 (129)	82 (83)	25 (29)	0.12 (0.20)	$1.2\ (2.0)\times10^{-4}$

Table 1: Expected 1σ errors of X^3 normalized by 10^{15} in the III only, EEE only, all $I+E$ cases ($\ell_{\rm max}=1000$) and BBB only case ($\ell_{\rm max}=500$) for each experiment. The errors from parity-even signals alone are written in parentheses.

of the model parameter X^3 using Fisher matrix. When the model parameter X^3 satisfies $X^3 \lesssim O(10^{15})$, the curvature perturbation induced the chiral gravitational waves is negligible. In this parameter region, the usual curvature perturbation would coincide the observed primordial power spectrum. We adopt the *Planck* and the proposed PRISM experiments to consider the instrumental noise. We present the 1σ errors of X^3 in Table 1.

From Table 1, detectable model parameters in PRISM experiments are as follows. First, $X=5\times10^5$ can be detected for cases with I only. Second, when we introduce the additional information as E (parity-odd and parity-even), $X=2.9\times10^5$ is detectable. Furthermore, we use B alone, $X=9.5\times10^4$ for the case with $r=0.05$ and $X=2.5\times10^4$ for the case with $r=5\times10^{-4}$, respectively. These results are comparable to or slightly tighter than the IB and EB correlations.

5. Summary

In this paper, we considered the parity-violating CMB bispectra induced by the rolling pseudoscalar field. The full-sky formalism informs us of the parity violating signal, i.e., existence of the both $\ell_1+\ell_2+\ell_3=$ odd and $\ell_1+\ell_2+\ell_3=$ even in ℓ space, analytically. To compute the CMB bispectra, we analyzed the shape of tensor non-Gaussianity. We depicted the CMB bispectra for $\ell_1\approx\ell_2\approx\ell_3$. We found that the shape of tensor non-Gaussianity induced by the pseudoscalar field correlate to usual equilateral type. From these analyzation, we found that the amplitudes of parity-odd and cross-bispectra are same as that of parity-even and cross-bispectra. However the amplitudes of parity-odd and auto-bispectra are smaller than that of parity-even and auto-bispectra.

We also analyzed the detectability of the model parameter X using Fisher matrix. If we use both temperature and E-mode auto- and cross-bispectra, detectability of the model parameter X is improved of 400% with respect to temperature bispectra alone in the PRISM experiment. Finally, B-mode bispectrum may be more powerful tools than any other information and these are marvelous studies to seek the trace of the early universe.

References

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