PROCEEDINGS OF SCIENCE

Universal three-pion physics with a large scattering length

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We discuss the low-energy properties of three-pion systems when the two-pion scattering length is very large. In the isospin symmetric limit, the three-pion system develops a bound state with isospin one. With the isospin breaking, the three neutral pions exhibit the Efimov effect and infinitely many bound states appear when the scattering length diverges. These phenomena can be realized by the three-pion systems in lattice QCD with an appropriate tuning of the quark mass. The existence of the universal bound state(s) has an implication to the in-medium softening of multi-pion states.

XV International Conference on Hadron Spectroscopy-Hadron 2013
4-8 November 2013
Nara, Japan

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 http://pos.sissa.it/

PoS(Hadron 2013)174

1. Large scattering length of the two-pion system

In recent years, universal physics in few-body systems, such as the Efimov effect, has attracted much attention [1]. The key quantity for the universality is the two-body scattering length a; if the magnitude of the scattering length $|a|$ is much larger than the characteristic length scale of the two-body interaction, the low-energy property of the three-body system is model-independently determined. Thanks to the universality, these phenomena have been investigated in a wide range of physical systems [2, 3].

Here we consider the universal few-body phenomena and their consequences in hadron physics, adopting the three-pion system as an example. In the real world, the $\pi\pi$ scattering length is known to be smaller than the typical QCD scale of 1 fm. There is no shallow bound/virtual state near the $\pi\pi$ threshold, and the σ meson exists in the isospin $I = 0$ channel as a resonance far above the threshold [4]. On the other hand, the lattice QCD simulation shows that the σ meson becomes a bound state for heavy quark masses [5]. This suggests that there is an intermediate region of the quark mass with which the σ meson appears near the $\pi\pi$ threshold and hence the scattering length in the $I = 0$ channel becomes divergent.

In the following, we discuss the universal physics of three pions with a large scattering length [6]. We find the universal low-energy phenomena in the isospin symmetric case (section 2) and in the isospin breaking case (section 3). In principle, these phenomena can be realized by appropriately tuning the quark mass in lattice QCD. Moreover, we show that the universal physics has an implication to the in-medium properties of multi-pion states (section 4).

2. Universal physics with the isospin symmetry

We first discuss the $\pi\pi$ scattering when the isospin symmetry is exact. In our setup, the pion has a nonzero mass m which is chosen such that the $I = 0$ scattering length a is much larger than the range of the interaction and than the $I = 2$ scattering length. In the low-momentum region $p \sim 1/a$, the $I = 0$ amplitude is characterized only by a and the $I = 2$ amplitude can be neglected. In the two-pion sector, a shallow bound state is generated with the binding energy $B_2 = 1/(ma^2)$ for $1/a > 0$.

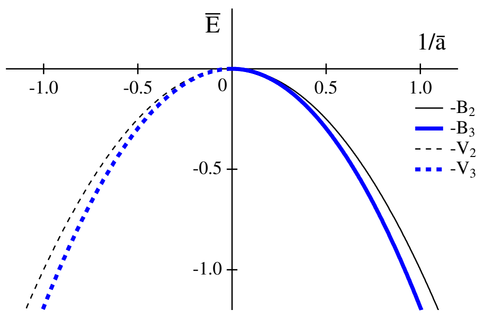

Figure 1: Energies of the bound and virtual states in the isospin symmetric limit with $\bar{E} = \mathrm{sgn}(E)|mE|^4$ and $\bar{a} = \mathrm{sgn}(a)|a|^4$ in arbitrary units [6]. The thick (thin) solid line represents the energy of the three-pion (two-pion) bound state, and the thick (thin) dashed line the energy of the three-pion (two-pion) virtual state.

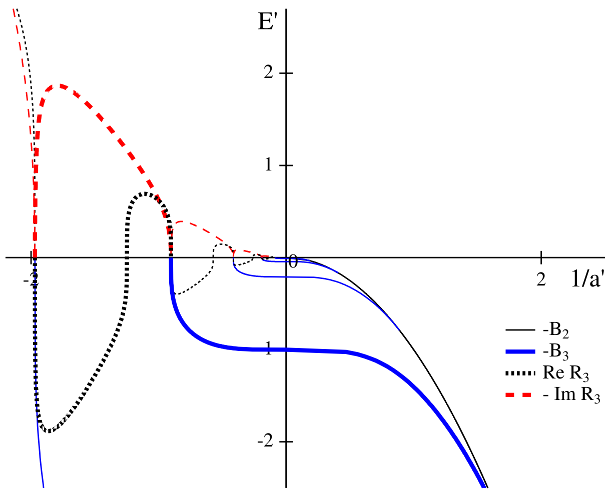

Figure 2: Energies of the bound states of the three π_0 system with the large isospin breaking [6]. The solid lines represent the energies of the bound states, and the dotted (dashed) lines represent the real parts (imaginary parts with the opposite sign) of the resonance poles. Both the axes are normalized to be dimensionless by the Efimov parameter κ_* as $E' = \text{sgn}(E)|mE/\kappa_*^2|^{1/4}$ and $a' = \text{sgn}(a)|a\kappa_*|^{1/4}$.

Next we focus on the low-energy properties of the three-pion system, where the s-wave component dominates the dynamics. To form a three-body bound state, we consider adding another pion to the $I = 0$ pair of two pions. We thus have a three-pion system with $I = 1$ which can also be formed by adding one pion to the $I = 2$ $\pi\pi$ pair. Although the $I = 2$ amplitude is neglected, the existence of this channel reduces the probability of finding $I = 0$ pair after particle exchange in the three-body equation, in comparison with the case of three identical bosons.

Solving the three-body equation, we find a bound state with the binding energy

$$B_3 = \frac{1.04391}{ma^2} \quad \text{for} \quad 1/a > 0. \tag{2.1}$$

This is a universal result, because it depends only on the scattering length. There is no resonance solution at low energy, and one virtual state is found for a negative scattering length $1/a < 0$ at $E = -V_3 = -1.04391/(ma^2)$. In this way, both the two-body and three-body bound states vanish at $1/a = 0$, and turn into the virtual states. The eigenenergies of the two- and three-pion systems are shown in Fig. 1.

3. Universal physics with the isospin breaking

With the isospin symmetry breaking, the neutral π^0 becomes lighter than the charged $\pi^{\pm}$. Around the $\pi^0\pi^0$ threshold, it is always possible to consider the energy region much smaller than the mass difference of the charged and neutral pions. In this case, we can neglect the effect of charged pions, and a large $\pi^0\pi^0$ scattering length leads to the standard Efimov effect of three identical bosons.

According to the universal physics of three identical bosons, the three π^0 system supports infinitely many bound states. The energy levels of the three-body system are shown in Fig. 2 as functions of the inverse scattering length. After the dissociation into the three-pion continuum, the Efimov bound states become resonance, as also discussed in Ref. [7]. The pole trajectory along with the decrease of the inverse scattering length indicates that the resonance pole moves in the complex energy plane and comes back to the threshold when the next Efimov bound state dissociates. This is a peculiar feature of the Efimov physics; the number of the three-body resonance is always one.

4. Implication to in-medium softening of multi-pion states

Universal physics tells us that the three-pion system has at least one bound state when the two-pion system has a shallow bound state. This has an implication to the in-medium properties of hadrons. It has been discussed that the spectrum of the σ meson softens in nuclear medium due to the partial restoration of chiral symmetry [8]. In the presence of the explicit symmetry breaking, the σ meson pole reaches the $\pi\pi$ threshold before the entire restoration of chiral symmetry [9] so the same universal phenomena should occur around the three-pion threshold.

The existence of the universal bound state indicates that the spectrum of the three-pion channel with $I = 1$ and $J = 0$ softens when the softening of the σ channel takes place. Because this corresponds to the quantum number of an excited pion, we may regard it as the softening of π^*. This is another realization of the universal phenomena of three pions in hadron physics.

This work was partially supported by JSPS KAKENHI Grant Numbers 24105702, 24740152, 25887020, by the Yukawa International Program for Quark-Hadron Sciences (YIPQS), and by RIKEN iTHES Project.

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