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Rho and A-mesons in external magnetic field in SU(2) lattice gauge theory

Olga Larina*
SSC RF ITEP
E-mail: olarina@itep.ru

Elena Luschevskaya
SSC RF ITEP
E-mail: luschevskaya@itep.ru

We computed correlators of vector, axial and pseudoscalar currents in the external strong magnetic field according to SU(2) lattice gauge theory. Masses of the neutral ρ and A mesons with different spin projections $s = 0, \pm 1$ along the magnetic field direction have been calculated. The masses of the neutral axial and vector mesons with zero spin $s = 0$ decrease under the increasing magnetic field, while the masses with spin $s = \pm 1$ increase with the value of the field. The quark mass extrapolation also were performed on the lattice.

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*Speaker.

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1. Introduction

In physics of strong interactions many interesting effects are caused by strong magnetic fields. It is expected that the magnetic fields ~ 2 GeV existed in the Early Universe during the electroweak phase transition [1]. In non-central heavy-ion collisions the value of magnetic field can reach $15m_\pi^2 \sim 0.29$ (GeV)^2 [2]. Phenomenological models show that the critical temperature of the transition T_c between the phases of confinement and deconfinement varies with a raising of the external magnetic field B, and the phase transition becomes of the first order [3]. An increase of the phase transition temperature T_c is predicted in Nambu-Jona-Lasinio models: NJL, EPNJL, PNJL [4] and PNJL$_8$ [5], the Gross-Neveu model [6, 7]. Calculations on the lattice with $N_f = 2$ flavours [8] and $N_f = 2+1$ flavours revealed the increase of the transition temperature T_c with raising of the magnetic field [9]. The chiral perturbation theory also predicts the decrease of the transition temperature with the magnetic field value [10]. It has been shown in the framework of the Nambu-Jona-Lasinio model that in the presence of sufficiently strong magnetic fields ($B_c = m_\rho^2/e \simeq 10^{16}$ T) QCD vacuum becomes a superconductor [11] along the direction of the magnetic field. The transition to the superconducting phase is accompanied by a condensation of charged ρ mesons. Calculations on the lattice [12] also indicate the existence of the superconducting phase. We have investigated the behavior of the neutral ρ meson masses with different spin projection $s = 0$ and $s = \pm 1$ to the field direction. It can be an evidence of the presence of the superfluidity phase. In [13] the mass of the neutral vector ρ meson was calculated in the relativistic quark- antiquark model, the mass of neutral ρ meson with zero spin does not vanishes with the growth of the magnetic field in the confinement phase in contradiction with the results presented in [11].

2. Details of the calculations

For generation of $SU(2)$ gauge field configurations we use the tadpole improved Symanzik action [14]. The calculations were performed on symmetric lattices with various lattice volumes 14^4, 16^4, 18^4 and lattice spacings $a = 0.0681,\ 0.0998$ and 0.138348 fm. A fermionic spectrum in the background of $SU(2)$ gauge fields were calculated due to Neuberger overlap operator which is chiral invariant Neuberger [15]. We compute the eigenfunction ψ_k and eigenvalues λ_k for a test quark in a gauge field configurations A_μ on the lattice in Euclidean space. A_μ is a sum of $SU(2)$ gauge fields and the external $U(1)$ uniform magnetic field. From the eigenfunctions of the Dirac operator we construct the propagators and correlators. The value of the magnetic field on the lattice is quantized and equal to $qB = 2\pi k/(aL)^2,\ \ k \in \mathbb{Z}$, where $q = -1/3e$ is the charge of a d-quark, we consider one flavour in our theory. The quantization condition imposes a limit on the minimum value of the magnetic field. For our calculations it equals 0.386 GeV^2 for the lattice volume 16^4 and lattice spacing 0.1383 fm. We use statistical independent configurations of the gluon field for each value of the quark mass in the interval $m_q a = 0.01 - 0.8$.

3. Calculation of observables

We calculate the observables

$$\langle \psi^\dagger(x) O_1 \psi(x) \psi^\dagger(y) O_2 \psi(y) \rangle_A \tag{3.1}$$

where $O_1, O_2 = \gamma_5, \gamma_{\mu,\nu}$ are Dirac gamma matrices, $\mu, \nu = 1, .., 4$. In the Euclidean space $\psi^\dagger = \bar{\psi}$ [16]. The correlators (3.1) are defined by Dirac propagators $1/(D+m)$. In our work we used the $M = 50$ lowest eigenstates. On the lattice in the background of a gauge field A_μ the observables (3.1) (for the M lowest eigenstates) have the form

$$\langle \bar{\psi} O_1 \psi \bar{\psi} O_2 \psi \rangle_A = \sum_{k,p<M} \frac{\langle k|O_1|k\rangle \langle p|O_2|p\rangle - \langle p|O_1|k\rangle \langle k|O_2|p\rangle}{(i\lambda_k + m)(i\lambda_p + m)} \tag{3.2}$$

The first term in the numerator of (3.2) represents a disconnected part and the second one with a minus sign - a connected part. We checked the first term is zero within the errors and does not affect the masses, so we use only the connected part of the correlators.

The mass of the neutral ρ meson was obtained from the correlator of vector currents $\langle j^V_\mu(x) j^V_\nu(y) \rangle_A$, where $j^V_\mu(x) = \psi^\dagger(x)\gamma_\mu \psi(x)$. The correlator $\langle j^{PS}(x) j^{PS}(y) \rangle_A$ enables us to compute the mass of π meson, where $j^{PS} = \psi^\dagger(x)\gamma_5 \psi(x)$ is the pseudoscalar current.

For the calculation of meson masses we use spectral expansion of the lattice correlation function

$$C(n_t) = \sum_k \langle 0|O_1|k\rangle \langle k|O_2^\dagger|0\rangle e^{-n_t a E_k} = A_0 e^{-n_t a E_0} + A_1 e^{-n_t a E_1} + ..., \tag{3.3}$$

where A_0, A_1 are some constants, E_0 is the energy of the lowest state. For a particle with average momentum equal to zero $\vec{p} = 0$ its energy equals its mass $E_0 = m_0$, E_1 is the energy of the first excited state, n_t is the number of a lattice site in the line of time direction. From expansion (3.3) one can see that for large values n_t the main contribution comes from the ground energy state.

Due to the periodic boundary conditions the contribution of the ground state into meson propagator has the form

$$f(n_t) = A_0 e^{-n_t a E_0} + A_0 e^{-(N_T - n_t) a E_0} = 2A_0 e^{-N_T a E_0/2} \cosh((N_T - n_t) a E_0) \tag{3.4}$$

The value of the ground state mass can be obtained by fitting the function (3.4) to the lattice correlator (3.2). The second method which we use is the Maximal Entropy Method (MEM) [17]. Euclidean correlator of the imaginary time $G(\tau, \vec{p}) = \int d^3x \langle O(\tau, \vec{x}) O^\dagger(0, \vec{0}) \rangle e^{-i\vec{p}\vec{x}}$ is connected to the spectral function $\rho(\omega, \vec{p})$ as follows:

$$G(\tau, \vec{p}) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega) \rho(\omega, \vec{p}), \quad K(\tau, \omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}, \tag{3.5}$$

where T is the temperature, τ is the Euclidean time, ω is the frequency. The position of the first peak in spectral function $\rho(\omega, \vec{p})$ corresponds to the energy of the ground state. We presume $\langle \vec{p} \rangle = 0$ and do not consider any function from it. The detail of the calculations according with MEM are presented in our previous work [18].

4. Results

At first we calculate the mass of the neutral π meson on the lattice from the correlators of the pseudoscalar currents $C^{PSPS}(n_t) = \langle j^{PS}(\vec{0}, n_t) j^{PS}(\vec{0}, 0) \rangle_A$, where $j^{PS}(\vec{0}, n_t) = \bar{\psi}(\vec{0}, n_t)\gamma_5 \psi(\vec{0}, n_t)$. We found that the squared pion mass is the linear function of the quark mass, this agrees with the predictions of ChPT. In Fig.1 (left) we show the π meson mass versus the value of the squared

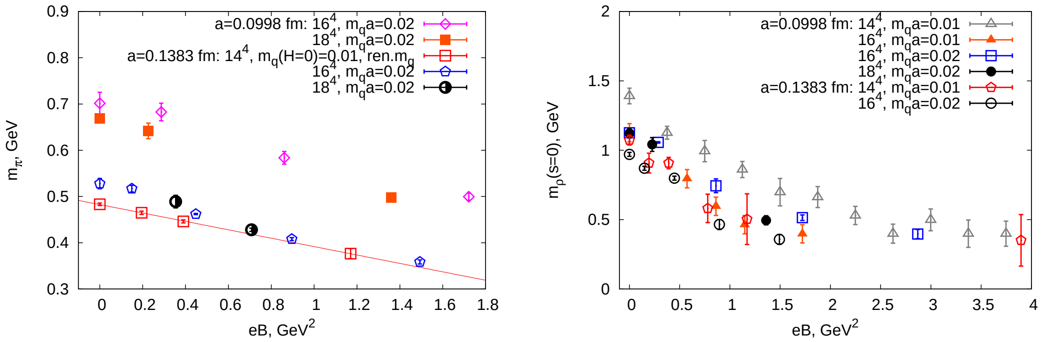

Figure 1: The mass of the neutral π meson obtained via the $C^{PSPS}(nt)$ versus the squared value of the magnetic field for renormalized and nonrenormalized quark masses (left). Dependence of the mass of the neutral vector ρ meson with zero spin $s = 0$ on the value of external magnetic field for the lattice volumes 14^4, 16^4, 18^4 and lattice spacings $a = 0.0998$ fm, 0.1338 fm calculated in accordance with the MEM.

magnetic field. For the renormalized pion mass we get the linear mass dependence from the magnetic field, the slope is negative in accordance with the results of A.Smilga obtained with the Chiral Perturbation Theory [19].

The external magnetic field is directed along the third coordinate axis. We calculate the meson masses with zero spin from the expression (3.2) with $O_1, O_2 = \gamma_3$. Fig.1 (right) shows the mass of the neutral vector meson with zero spin obtained by the Maximal Entropy Method at various lattice volumes, spacings and bare quark masses. The mass decreases under the increasing magnetic field for all the sets of data.

The masses of the vector meson were calculated for various values of the magnetic field. We extrapolate the ρ meson mass to the quark mass m_{q_0} giving the mass of neutral pion equal to 135 Mev. For this purpose we calculate the ρ mass for several values of m_q in the interval $m_q = 0.01 \div 0.8$, perform fitting and find the coefficients a_i and b_i in the equations

$$m_\rho = a_0 + a_1 m_q, \qquad m_A = b_0 + b_1 m_q \tag{4.1}$$

and then extrapolate $m_\rho(m_q)$ to the physical values $m_\rho(m_{q_0})$ at $m_q = m_{q_0}$ using (4.1).

In Fig.2 (left) we depict the mass of the neutral ρ meson with zero spin. The mass decreases under increasing magnetic field for all the lattice volumes and spacings. For the purposes of visualization we connected the points by splines. Fig.2 (right) shows the mass of the ρ meson mass with nonzero spin versus the field value. The masses with spin $s = \pm 1$ increase with the field. The results were obtained after the quark mass extrapolation. Fig. 3 (left) shows the behaviour of the neutral axial meson mass with zero spin versus the external magnetic obtained from the Maximal Entropy Method. In Fig. 3 (right) we see the mass of the neutral axial meson with various spin projections along the field direction. The mass of A with zero spin decreases, while the masses with $s = \pm 1$ increase slowly.

Unfortunately on the lattice in the presence of the magnetic field the quantum numbers of mesons are not precise. The mixing takes place due to the interaction between photons and vector (axial) quark currents and can occur between neutral pion and neutral ρ or A meson with zero spin.

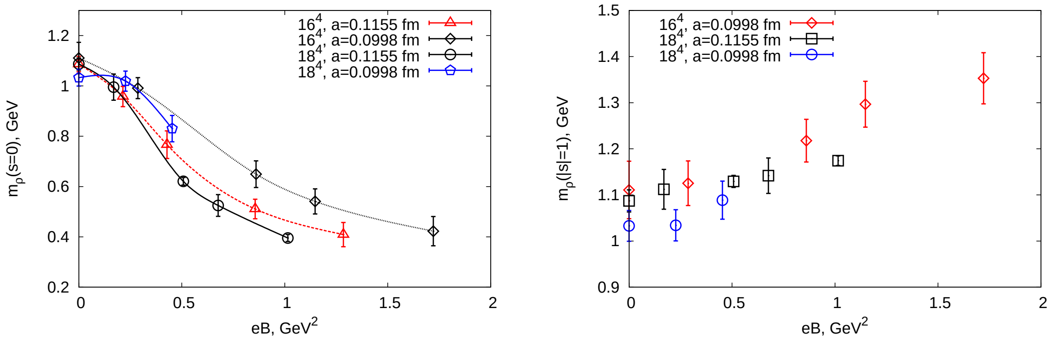

Figure 2: Dependence of the mass of the neutral vector ρ meson with spin $s = 0$ on the value of the external magnetic field for the lattice volumes 16^4, 18^4 and lattice spacings $a = 0.0998,\ 0.1155$ fm (left). The mass of the neutral vector ρ meson with spin $s = \pm 1$ versus the field value for the lattice volumes 16^4, 18^4 and lattice spacings $a = 0.0998,\ 0.1155$ fm (right). The results were obtained after quark mass extrapolation by coshinus function fit.

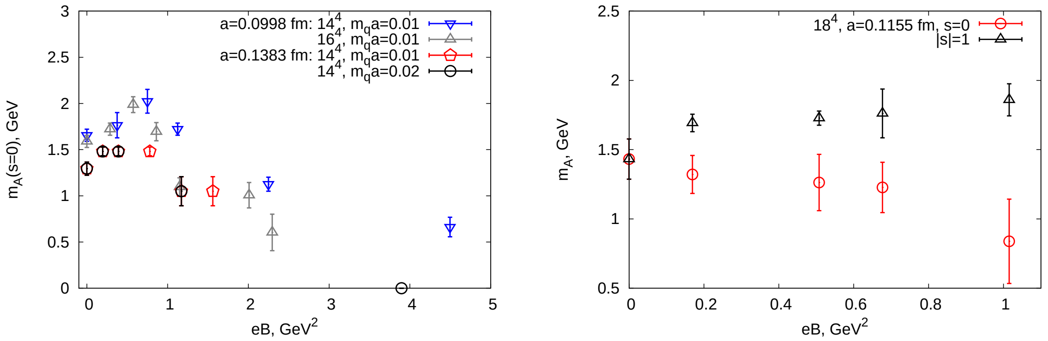

Figure 3: The mass of the neutral axial A meson with zero spin $s = 0$ versus the value of external magnetic field for the lattice volumes 14^4 and 16^4 and lattice spacings $a = 0.0998$ fm, 0.1338 fm in accordance with the Maximal Entropy Method (left). The mass of the neutral axial A meson with various spins versus the value of the magnetic field for the lattice volume 18^4 and lattice spacing $a = 0.1155$ fm (right).

No severe methods occurs to disentangle these two states in the magnetic field. However we have strong indications that the masses of vector and axial mesons with $s = \pm 1$ increase in $SU(2)$ theory.

5. Conclusions

In this work we explore the masses of neutral π, ρ and A mesons in the background of the strong magnetic field of hadronic scale in the confinement phase. The masses with zero spin projection to the magnetic field differ from the masses with spin projection $s = \pm 1$. The masses with $s = 0$ decrease with the magnetic field, while the masses with $s = \pm 1$ increase with the field. We consider this phenomena being the result of an anisotropy created by the strong magnetic field creates. We do not observe any condensation of neutral mesons, so there are no evidences of superfluidity in the confinement phase. However the presence of superconducting phase at high values of the magnetic field B [20] in QCD is a hot topic for discussions. The condensation of charged ρ mesons would be an evidence of the existence of the superconductivity in QCD.

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