Spectroscopy and Decay properties of D and D_s mesons with Martin-like confinement potential in Dirac formalism

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We construct quark - antiquark bound states of open charm mesons by assuming that the constituent quark and antiquark are confined by an average Martin-like potential within the Dirac formalism. On fixing the model parameters using the ground state masses, other excited states of the D and D_s meson spectra are computed. The corresponding wave functions are then employed to compute the decay constants of D and D_s meson within the relativistic frame work. These decay constants play an important role in the leptonic and nonleptonic weak decays of the open flavour mesons. The present calculation yields the decay constant of D and D_s meson as 216.02 MeV and 235.18 MeV respectively as against the experimental values of 206.7 ± 8.9 MeV and 260.0 ± 5.4 MeV respectively. The present results are also in accordance with the QCD sum rule predictions (204 ± 6 MeV and 246 ± 6 MeV) and Lattice QCD (218.9 ± 11.3 MeV and 260.1 ± 10.8 MeV) for D and D_s mesons respectively. The branching ratio of the leptonic decays of D and D_s mesons in all the three leptonic channels are also computed and the results are in good agreement with the experimental values particularly in the case of D_s meson.

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1. Introduction

The recent experimental observations of open flavour mesonic states at charm sector have provided a boost to the theoretical attempts towards the understanding of the dynamics of light quarks in the company of heavy flavour quarks. The colour confinement of quarks is understood in terms of multi-gluon exchange processes at the non-perturbative regime of the hadronic size and it is not feasible to compute theoretically from the QCD first principles. The lattice QCD methods, QCD sum rules and potential models etc. are thus being employed to study the hadronic properties. Though lattice calculations are based on ab-initio method at some excited at some extent, it requires huge computing power and thus limited to the study of hadrons properties mainly to the ground states and few cases where excited states are being studied. In this context, other theoretical models particularly to potential models become a tool to study the properties of hadrons. The masses of heavy-light system for low-lying 1S and 1P_J states of D and D_s mesons are known from experiment [1]. Recently, many new resonances of D and D_s systems such as D_0^* (2400), D (2540), D^* (2610), D_s (2638), D_s (2710), D_J(2750), D_J^* (2760), D_s(2860), D_s (3040) are also reported by different experimental groups [2, 3, 4, 5]. But very few of them are comparable with the existing theoretical predictions [6, 7, 8, 9, 10, 11]. In this context, these newly discovered states are very important for theorists to fine tune their models for better understanding of the $Q\bar{q}$ dynamics.

2. Methodology

In the present study, we assume that the constituent quarks in a meson core is independently confined by an average potential of the form

$$V(r) = \frac{1}{2}(1+\gamma_0)(\lambda r^{0.1} + V_0) \tag{2.1}$$

To a first approximation, the confining part of the interaction is believed to provide the zeroth-order quark dynamics inside the meson through the quark Lagrangian density

$$\mathcal{L}_q^0(x) = \bar{\psi}_q(x)\left[\frac{i}{2}\gamma^\mu \overleftrightarrow{\partial_\mu} - V(r) - m_q\right]\psi_q(x). \tag{2.2}$$

The solution of Dirac equation resulting from the Lagrangian can be written as two component (positive and negative energies in the zeroth order) form as

$$\psi_{nlj}^{(+)}(\vec{r}) = N_{nlj}\begin{pmatrix} ig(r)/r \\ (\sigma.\hat{r})f(r)/r \end{pmatrix}\mathcal{Y}_{ljm}(\hat{r}) \tag{2.3}$$

$$\psi_{nlj}^{(-)}(\vec{r}) = N_{nlj}\begin{pmatrix} i(\sigma.\hat{r})f(r)/r \\ g(r)/r \end{pmatrix}(-1)^{j+m_j-l}\mathcal{Y}_{ljm}(\hat{r}) \tag{2.4}$$

Here, N_{nlj} is the overall normalization constant, and $\mathcal{Y}_{ljm}(\hat{r})$ are the solid spherical harmonics and is expressed in terms of the spinor wave functions $\chi_{\frac{1}{2}m_s}$. The reduced radial part $g(r)$ of the upper component and $f(r)$ of the lower component of Dirac spinor $\psi_{nlj}(r)$ are numerically solved and normalized. The optimized model parameters employed for the present study are given below. The quark masses, m_c = 1.27 GeV, m_s =0.1 GeV and $m_{u,d}$ = 0.003 GeV are taken from PDG

[1] while the potential strength (λ) for D and D_s meson is taken as (2.2903 + B) and (2.2655 + B) $GeV^{\nu+1}$ respectively where the centrifugal parameter (B) is (n + 0.153) GeV^{-1} for $l = 0$ and $((n+1)*0.1267)$ for $l \neq 0$. The constant potential parameter (V_0) for D and D_s is taken as -2.6706 GeV and -2.6160 GeV respectively. The computed mass spectra of D and D_s mesons are listed in Table 1 below.

Table 1: Mass spectra of D ($c\bar{q}$, q = d,u) and D_s mesons (in MeV).

State	D Meson				D_s Meson				
	Present	Experiment [1]	[8]	[9]	Present		Experiment [1]	[8]	[9]
1^3S_1	2010.4	$D^*(2010)$ 2010.28 ± 0.13	-	2010	2112.3	D_s^*	2112.3 ± 0.5	-	2111
1^1S_0	1869.6	1869.62 ± 0.15	-	1871	1968.6	D_s	1968.49 ± 0.32	-	1969
1^3P_2	2462.9	$D_2^*(2460)$ 2462.6 ± 0.7	-	2460	2571.0	$D_{s2}(2573)$	2571.9 ± 0.8	-	2571
1^3P_1	2390.6			2469	2498.8	$D_{s1}(2460)$	2459.6 ± 0.6	-	2574
1^3P_0	2386.4	$D_0^*(2400)$ 2318 ± 29	-	2406	2494.5	$D_{s0}^*(2317)$	2317.8 ± 0.6	-	2509
1^1P_1	2322.9	$D_1(2420)$ 2421.3 ± 0.6	-	2426	2430.20	$D_{s1}(2536)$	2535.12 ± 0.13	-	2536
2^3S_1	2606.2	$D^*(2610)$ 2608.7 [4]	2639	2632	2716.4	$D_s^*(2710)$	2710^{+12}_{-7} [2]	2728	2731
2^1S_0	2523.5	$D(2540)$ 2539.4 [4]	2567	2581	2633.3	$D_s(2632)$	2632.5 ± 1.7 [12]	2656	2688
1^3D_3	2835.1	$D_J^*(2760)$ 2763.3 [4]	2760	2863	2947.4	$D_{sJ}^*(2860)$	2862^{+6}_{-3}	2840	2971
1^3D_2	2817.9		2810	2850	2929.8			2885	2961
1^3D_1	2802.9		2790	2788	2914.5			2870	2913
1^1D_2	2745.3	$D_J(2750)$ 2752.4 [4]	2746	2806	2857.2			2828	2931
2^3P_2	2937.9		2965	3012	3051.2			3045	3142
2^3P_1	2891.1		2960	3021	3003.2	$D_{sJ}(3040)$	3044^{+30}_{-9} [3]	3020	3154
2^3P_0	2888.4		2880	2919	3000.4			2970	3054
2^1P_1	2841.9		2940	2932	2954.4			3040	3067
3^3S_1	3148.4		3125	3096	3262.6			3200	3242
3^1S_0	3087.8		3065	3062	3202.0			3140	3219
2^3D_3	3304.6		3212	3335	3419.5			3285	3469
2^3D_2	3292.4		-	3307	3406.9			-	3456
2^3D_1	3281.8		3215	3228	3395.8			3290	3383
2^1D_2	3236.6		-	3259	3351.3			-	3403

3. The Decay constant of the charm flavored mesons

Following the procedure adopted by [13] the decay constant can be expressed through the meson wave function $\psi(p)$ in the momentum space as

$$f_P^2 = \frac{3|I_M|^2}{2\,\pi^2\,M_P\,J_M} \tag{3.1}$$

where

$$I_M = \int_0^\infty dp\ p^2 A(p)\sqrt{g_q(p)\,g_q^*(-p)}, \quad J_M = \int_0^\infty dp\ p^2 g_q(p)\,g_q^*(-p)$$

Table 2: The decay constant f_P of D systems (in MeV).

	f_P			
	1S	2S	3S	4S
Present	**216.02**	185.99	168.14	155.39
Expt./PDG [1]	206.7 ± 8.9			
QCDSR [14]	204 ± 6			
LQCD [15]	218.9 ± 11.3			
LFQM [16]	205.8 ± 8.9			
BSM [17]	230 ± 25			

Table 3: The decay constant f_P of D_s systems (in MeV).

	f_P			
	1S	2S	3S	4S
Present	**235.18**	203.34	184.40	170.82
Expt./PDG [1]	260.0 ± 5.4			
QCDSR [14]	246 ± 6			
LQCD [15]	260.1 ± 10.8			
LFQM [16]	264.5 ± 17.5			
BSM [17]	248 ± 27			

$$A(p) = \frac{(E_{p_1} + m_{q_1})(E_{p_2} + m_{q_2}) - p^2}{[E_{p_1} E_{p_2}(E_{p_1} + m_{q_1})(E_{p_2} + m_{q_2})]^{\frac{1}{2}}} \quad and \quad E_{p_i} = \sqrt{p^2 + m^2}$$

The computed values are listed in Table 2 and 3.

4. The branching ratio of the leptonic decays of D and D_s mesons

Charged mesons produced from a quark and anti-quark can decay to charged lepton pair via a virtual $W^{\pm}$ boson. The theoretical predications are very clean due to the absence of hadrons in the final state. The total leptonic width of D and D_s mesons are given by [18, 10, 11]

$$\Gamma(D_q^+ \to l^+ \nu_l) = \frac{G_F^2}{8\pi} f_{D_q}^2 |V_{cq}|^2 m_l^2 \left(1 - \frac{m_l^2}{M_{D_q}^2}\right)^2 M_{D_q} \quad where \quad q = s, d, u. \tag{4.1}$$

where Fermi coupling constant (G_f) is $1.664 * 10^{-5}$ and the relevant CKM parameters are taken from PDG [1] as $V_{cs} = 1.006$ and $V_{cd} = 0.230$. The branching ratios of the total leptonic widths are obtained as $BR = \Gamma(D_q^+ \to \ell^+ \nu_\ell) \times \tau$, where the lifetime τ of the D and D_s mesons are taken from PDG [1]. The results are shown in Table 4.

Table 4: The leptonic Branching Ratio (BR) of D and D_s mesons.

Process	$\Gamma(M \to \ell \bar{\nu}_\ell)$		$B(M \to \ell \bar{\nu}_\ell)$		
	Present (keV)	[19] (keV)	Present (keV)	[19] (keV)	Expt.[1]
$D_s \to \tau \bar{\nu}_\tau$	13.258×10^{-8}	6.090×10^{-8}	**5.844** $\times 10^{-2}$	4.3×10^{-2}	5.43×10^{-2}
$D_s \to \mu \bar{\nu}_\mu$	13.469×10^{-9}	6.240×10^{-9}	**5.937** $\times 10^{-3}$	4.41×10^{-3}	5.90×10^{-3}
$D_s \to e \bar{\nu}_e$	3.157×10^{-13}	-	1.391×10^{-7}	-	$< 1.2 \times 10^{-4}$
$D^+ \to \tau \bar{\nu}_\tau$	15.288×10^{-10}	4.720×10^{-10}	2.433×10^{-3}	7.54×10^{-4}	$< 1.2 \times 10^{-3}$
$D^+ \to \mu \bar{\nu}_\mu$	5.641×10^{-10}	1.795×10^{-10}	8.977×10^{-4}	2.87×10^{-4}	3.82×10^{-4}
$D^+ \to e \bar{\nu}_e$	1.323×10^{-14}	-	2.105×10^{-8}	-	$< 8.8 \times 10^{-6}$

5. Results and Discussions

The predicted masses of S-wave of D_s meson state $2\,^3S_1$ (2716 MeV) and $2\,^1S_0$ (2633 MeV) are in very good agreement with experimental results 2710^{+12}_{-7} MeV [1] and 2638 MeV [12] respectively. 2^3S_1 (2606 MeV) state and 2^1S_0 (2523 MeV) of D meson are in good agreement with experimental [1] values of 2608 MeV and 2539 MeV reported by BABAR [4]. The expected results of other S-wave excited states of D and D_s meson are in accordance with the other results [8, 9]. The decay constant of D meson 216 MeV and D_s meson 235 MeV are also in agreement with the LQCD [15] result and with the QCD sum rule result [14] respectively. The predicted leptonic branching ratios of the D_s meson are in excellent agreement with the experimental results over other theoretical predictions [19].

6. Acknowledgements

References

[1] J. Beringer et al. (Particle Data Group), Phys. Rev. D 86 (2012) 010001.

[2] J. Brodzicka et al. (Belle Collaboration), Phys. Rev. Lett. 100 (2008) 092001.

[3] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 80 (2009) 092003.

[4] P. Del A. Sanchez et al. (BABAR Collaboration), Phys. Rev. D 82 (2010) 111101.

[5] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 79 (2009) 112004.

[6] S Godfrey and R Kokoski, Phys. Rev. D 43 (1991) 1679.

[7] M. Di Pierro et al., Phys. Rev. D 64, (2001) 114004.

[8] A. M. Badalian, B. L. G. Bakker, Phys. Rev. D 84 (2011) 034006.

[9] D. Ebert, R. N. Faustov and V. O. Galkin, Eur. Phys. J. C 66 (2010) 197.

[10] Bhavin Patel and P C Vinodkumar, Chinese Phys. C 34 (2010) 1497 and arXiv:0908.2212v1 [hep-ph].

[11] N. Devlani and A. K. Rai, Phys. Rev. D 84 (2011) 074030; Eur. Phys. J. A 48 (2012) 104; Int. Theo. Phys. 52 (2013) 2196.

[12] A.V.Evdokimov et al. (SELEX Collaboration), Phys. Rev. Lett. 93 (2004) 242001.

[13] D. Ebert, R N Faustov and V O Galkin, Phys. Rev. D 67 (2003) 014027.

[14] S. Narison, Phys. Lett. B 718 (2013) 1321.

[15] A. Bazavov et al., Phys. Rev. D 85 (2012) 114506.

[16] C. W. Hwang, Phys. Rev. D 81 (2010) 114024.

[17] G. L. Wang, Phys. Lett. B 633 (2006) 492.

[18] S Villa, arXiv:hep-ex/0707.0263v1.

[19] HAKAN IFTCI and *HÜ*SEYIN KORU, Int. J. Mod. Phys. E 9 (2000) 407.