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Radial and orbital excited states of Λ_c^+ and Σ_c^+ in a Hypercentral Quark Model

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We present excited states of Λ_c^+ and Σ_c^+ using the hypercentral description of the three-body system. The confinement potential is assumed as hyper coulomb plus power potential ($hCPP_\nu$) with power index ν. The 2S, 3S and 4S radial excited states and first negative parity orbital excited states are computed for different power indices (ν), starting from 0.5 to 2.0. We also incorporate spin dependent contribution perturbatively. Our calculated results for excited states of Λ_c^+ and Σ_c^+ are in good agreement with known experimental results as well as other theoretical predictions.

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1. Introduction

In beginning of the 21^{st} century significant progress has been achieved in the experimental facilities at Belle, BaBar, CLEO, CDF, SELEX, ALICE, LHC *etc.,* to study the properties and production of heavy flavour hadrons [1]. Out of the huge data bank related to the heavy flavour hadrons, majority of them are in the meson sector. However considerable number of them are also recorded in the baryon sector. So many surprises are also expected in the hadronic sector containing new states coming from the above mentioned world wide experimental setups. Therefore the theoretical calculation of the properties of hadrons containing heavy flavour quarks is desired to conform the experimental observations. At the theoretical front, there exist vast literature on the ground state properties of heavy flavour baryons and few cases of excited states. There are less number of excited heavy baryons available from experimental side due to its fast decay properties. But the recent experimental facilities has recorded and has shown progress in the baryon spectroscopy. The quark composition of Λ_c^+ and Σ_c^+ is udc and Λ_c^+ was the first known charm baryon shortly after the discovery of J/ψ meson. Excited charm baryons were discovered long after the initial observation of the Λ_c^+ ground state. Recently, the CDF collaboration [1] observed a number of excited charm baryon states including $\Lambda_c(2595)$, $\Lambda_c(2625)$. The J^P quantum numbers for most excited heavy baryons have not been determined experimentally, but are assigned by the Particle Data Group on the basis of quark model predictions. So it is very interesting to assign quantum numbers to excited heavy baryons according to hypercentral quark model. So in this paper, we present radial and orbital excited states of Λ_c^+ and Σ_c^+ in the hypercentral approach which is found to be successful in the study of three body problems.

2. Methodology

The Hamiltonian of the 3-body baryonic systems given in terms of the Jacobi co-ordinates ρ, λ can be expressed in terms of the hypercentral coordinate [2, 3, 4], x as

$$H = \frac{P_\rho^2}{2m_\rho} + \frac{P_\lambda^2}{2m_\lambda} + V(\rho,\lambda) = \frac{P_x^2}{2m} + V(x) \tag{2.1}$$

The hyperradial *Schödinger* equation corresponds to the Hamiltonian given by above Eqn, can be written as

$$\left[\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} - \frac{\gamma(\gamma+4)}{x^2}\right]\psi_{\omega\gamma}(x) = -2m[E - V(x)]\,\psi_{\omega\gamma}(x) \tag{2.2}$$

where γ is the hyper angular quantum number and is given by $\gamma = 2n + l_\rho + l_\lambda$, n = 0, 1, . . .; l_ρ and l_λ are the angular momenta associated with the $\vec{\rho}$ and $\vec{\lambda}$ variables and ω denotes the number of nodes of the spatial three-quark wave functions. Now for finding solution to the hyperradial *Schrödinger* equation, we consider the transformation, $\phi_{\omega\gamma}(x) = x^{\frac{5}{2}}\ \psi_{\omega\gamma}(x)$. Eqn. (2.2) then reduces to the form

$$\left[-\frac{1}{2m}\frac{d^2}{dx^2} + \frac{\frac{15}{4} + \gamma(\gamma+4)}{2mx^2} + V(x)\right]\phi_{\omega\gamma}(x) = E\phi_{\omega\gamma}(x) \tag{2.3}$$

The hyperradial wave function $\phi_{\omega\gamma}(x)$ is a solution of the reduced *Schrödinger* equation for the interacting potential defined by $V(x) = -\frac{2}{3}\frac{\alpha_s}{x} + \beta x^{\nu}$. If we compare Eqn. (2.3) with the usual three dimensional radial *Schrödinger* equation, the following correspondance between angular momentum with hyper angular momentum as mentioned in [5] can be made as $l(l+1) \rightarrow \frac{15}{4} + \gamma(\gamma+4)$.

For computing the mass difference between different degenerate baryonic states, we consider the spin dependent part of the usual one gluon exchange potential (OGEP) given by [6]. Accordingly, the spin-dependent part, $V_{SD}(r)$ contains three types of the interaction terms, such as the spin-spin, the spin-orbit and the tensor part given by [6]

$$V_{SD}(x) = V_{SS}(x)\left[S(S+1) - s_\rho(s_\rho+1) - \frac{3}{4}\right] + V_{\Gamma S}(x)\left(\vec{\Gamma}\cdot\vec{S}\right) + V_T(x)\left[S(S+1) - \frac{3(\vec{S}\cdot\vec{x})(\vec{S}\cdot\vec{x})}{x^2}\right] \tag{2.4}$$

The hyper spin-orbit term containing $V_{\Gamma S}(x)$ and the hyper tensor term containing $V_T(x)$ describe the fine structure of the baryon states, while the spin-spin term containing $V_{SS}(x)$ proportional to $2(\vec{s_\rho}\cdot\vec{s_\lambda}) = \left[S(S+1) - s_\rho(s_\rho+1) - \frac{3}{4}\right]$ gives the spin hyperfine splitting. The spin-orbit term in hypercentral model containing $V_{\Gamma S}(x)$ proportional to $2(\vec{\Gamma}\cdot\vec{S}) = \left[J(J+4) - \Gamma(\Gamma+4) - \frac{15}{4} - S(S+1)\right]$. The coefficient of these spin-dependent terms of Eqn (2.4) can be written in terms of the vector $\left(V_V = -\frac{xe^{-x}}{\Gamma(\Gamma+4)}\right)$ and scalar ($V_S = \beta x^{\nu}$) parts of the static potential as

$$V_{\Gamma S}(x) = \frac{1}{2\, m_\rho m_\lambda\, x}\left(3\frac{dV_V}{dx} - \frac{dV_S}{dx}\right) \tag{2.5}$$

$$V_T(x) = \frac{1}{6 m_\rho m_\lambda}\left(3\frac{d^2V_V}{dx^2} - \frac{1}{x}\frac{dV_V}{dx}\right) \tag{2.6}$$

$$V_{SS}(x) = \frac{1}{3\, m_\rho m_\lambda}\nabla^2 V_V \tag{2.7}$$

The baryon spin average mass is then obtained by the sum of the quark masses plus the binding energy as $M_{CW} = \sum_i m_i + BE$. The ground state and the radial and the orbital excited state masses of heavy flavour baryons are determined by the sum of spin average masses (M_{CW}) with spin-hyperfine interaction as $M_B = M_{CW} + \langle V_{SD}(x)\rangle$. We fix potential parameter β and hyperfine parameter A for each choices of ν using ground state experimental mass of $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ charm baryons (Figure 1). The regularization parameter x_0 is 1 $GeV^{-1}$and quark mass parameters are $m_u = 338$ MeV, $m_d = 350$ MeV, and $m_c = 1275$ MeV. For radial excited states, potential parameter β is found to vary as $\beta = \beta_0\sqrt{\omega+\gamma+\frac{3}{2}}$ and for orbital excited states, potential parameter β is found to vary as $\beta = \beta_0\sqrt{\omega+\gamma+\frac{1}{2}}$ where β_0 correspond to the ground state ($\omega = 0$, $\gamma = 0$) potential strength.

3. Result and Discussion

We have studied the radial and orbital excited states of single heavy charm baryon by solving six dimensional Schrödinger equation numerically with hypercentral potential of the hypercoulomb plus power potential. The computed orbital excited states of Λ_c^+ are in good agreement with experimental known state $\Lambda_c(2595)$, $\Lambda_c(2625)$ as well as other theoretical prediction at potential index

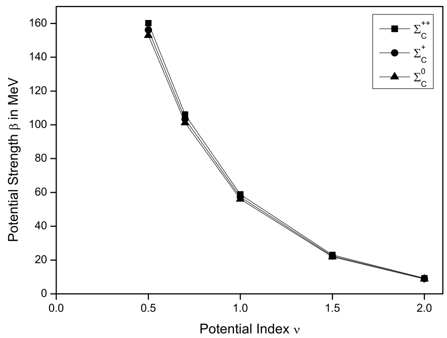

Figure 1: Variation of potential strength β to fix the ground state masses of single heavy charm baryons with respect to potential index ν.

Table 1: Mass of radially excited states including spin interaction for single charm Σ_c^+ baryon with different choices of ν.

Potential index	Mass of Radially Excited states			
	Present	Others	Present	Others
	$1^2S_{\frac{1}{2}}$	[10]	$1^4S_{\frac{3}{2}}$	[10]
0.50	2452.9	2452.9	2517.15	2517.5
1.00	2452.9		2517.15	
2.00	2452.9		2517.15	
	$2^2S_{\frac{1}{2}}$		$2^4S_{\frac{3}{2}}$	
0.50	2828.69	2901	2966.73	2936
1.00	2918.97		3052.56	
2.00	3056.18		3194.32	
	$3^2S_{\frac{1}{2}}$		$3^4S_{\frac{3}{2}}$	
0.50	3111.91	3271	3278.23	3293
1.00	3329.72		3495.09	
2.00	3681.39		3865.38	
	$4^2S_{\frac{1}{2}}$		$4^4S_{\frac{3}{2}}$	
0.50	3385.35	3581	3569.31	3598
1.00	3747.23		3934.42	
2.00	4362.34		4581.82	

Table 2: Mass of first negative parity states including spin, tensor and spin-orbit interaction for Single charm Σ_c^+ baryon with different choices of ν.

Baryon	Potential index ν	present	others
$\Sigma_c^+(1^2P_{\frac{1}{2}})$	0.5	2697.45	2748 [7]
	1	2764.22	2792^{+14}_{-5} [9]
	2	2961.07	2795 [10]
$\Sigma_c^+(1^2P_{\frac{3}{2}})$	0.5	2693.10	2763 [7]
	1	2741.23	2770 [8]
	2	2817.70	2761 [10]
$\Sigma_c^+(1^4P_{\frac{1}{2}})$	0.5	2699.45	2768 [7]
	1	2775.72	2770 [8]
	2	3032.75	2805 [10]
$\Sigma_c^+(1^4P_{\frac{3}{2}})$	0.5	2695.21	2776 [7]
	1	2752.73	2805 [8]
	2	2889.37	2799 [10]
$\Sigma_c^+(1^4P_{\frac{5}{2}})$	0.5	2689.56	2790 [7]
	1	2722.08	2815 [8]
	2	2698.23	2790 [10]

$\nu = 1$. Experimental known $\Sigma_c(2792)$ state is identified as first negative parity orbital excited state of Σ_c^+ with $J^P = \frac{1}{2}^-$. Same prediction is found in relativistic quark-diquark picture by D. Ebert et al, [10]. We found that the computed excited states are in good agreement with experimental known states at potential index $\nu = 1$. Thus, the $hCPP_{\nu\approx 1}$ model adequately represent the three-body interactions among the quarks constituting the baryons.

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Table 3: Mass of radially excited states including spin interaction for single charm Λ_c^+ baryon with different choices of ν.

Baryon	Potential Index	Present	Others
		$1^2S_{\frac{1}{2}}$	[10]
$\Lambda_c^+(\mathbf{1S})$	0.50	2286.46	2286.46
	1.00	2286.46	
	2.00	2286.46	
		$2^2S_{\frac{1}{2}}$	
$\Lambda_c^+(\mathbf{2S})$	0.50	2661.69	2766.6
	1.00	2751.97	
	2.00	2889.18	
		$3^2S_{\frac{1}{2}}$	
$\Lambda_c^+(\mathbf{3S})$	0.50	2944.91	3130.00
	1.00	3162.72	
	2.00	3514.39	
		$4^2S_{\frac{1}{2}}$	
$\Lambda_c^+(\mathbf{4S})$	0.50	3218.35	3437.00
	1.00	3580.23	
	2.00	4195.34	

Table 4: Mass of first negative parity states including spin, tensor and spin-orbit interaction for Single charm Λ_c^+ baryon with different choices of ν.

Baryon	Potential Index	Present	Others
$\Lambda_c^+(1^2P_{\frac{1}{2}})$	0.5	2530.45	2598 [10]
	1	2597.22	2592 [9]
	2	2794.07	2625 [7]
			2630 [8]
$\Lambda_c^+(1^2P_{\frac{3}{2}})$	0.5	2526.10	2627 [10]
	1	2574.23	2628 [9]
	2	2650.70	2636 [7]
			2640 [8]

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