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The next-to-leading order α_s corrections to Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (JIMWLK) equation

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The rapidity (energy) evolution of hadronic observables in scattering of a diluted, perturbative projectile, on a dense gluonic target is described in QCD by the JIMWLK equation [1]. This is a functional non-linear equation that is consistent with the QCD unitarity and reduces to the linear Balitsky, Fadin, Kuraev, Lipatov (BFKL) equation [2] when the scattering probability is low. Recently, there was major progress in this field. The NLO JIMWLK Hamiltonian correction was calculated by means of a comparison of the general structure of the NLO Hamiltonian with the NLO evolution equations of quark dipole [3] and Baryon [4]. The result found was suitable only for the case of JIMWLK Hamiltonian which is acting on gauge invariant structures [5] but was later on generalized for action on non-gauge invariant structures [6]. This review will present the lastest advancements in the field.

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1. Introduction to Gluon Saturation

When we accelerated a proton, what appeared to be a quark is resolved instead as a quark and (cloud of) gluons. More generally, by boosting a QCD state we increase the number of gluons in it. These newly created gluons are known as *Weizsacker Williams radiation*. The equation which describes this rapid growth of the gluon density for higher momentum transfer was derived by 1978 and known as the BFKL equation. Despite the fact that the BFKL equation describes the true pattern of exponentially growing gluon density, it suffers from two main problems which make it inadequate for working with energies beyond certain level. First, its solutions diffuse to the infrared, and second, it violates both unitarity and Froissart-Martin bound [1]. In order to address these problems we first have to understand their source. As long as the boost is not too high, each of the partons emits new gluons individually, while at some point if we keep increasing the boost these new gluons will overlap in space. At this point, known as ***saturation*** or ***color glass condensate (CGC)***, the radiation emission process will become a collective process instead, which is ignored in the BFKL equation.

The equation which replaces BFKL for the high-energy limit known as JIMWLK equation [1] and was derived by 97'. This equation takes into account saturation effects, while it reduces to BFKL equation [2] for the low energy limit. More importantly, this equation does not suffer from any of the problems mentioned for BFKL. JIMWLK is a perturbative equation. However, until recently only the leading order (LO) term in the series was known. This situation was recently changed and the next-to-leading (NLO) term was found [3][4].

There is a strong connection between the JIMWLK equation and a significant part of the experiments mentioned. JIMWLK describes *dense-dilute* scattering, a highly energetic particle (probe) which is scattered due to heavy nucleus. In principle, JIMWLK can equally be applied to small x DIS (ep) as to pp (proton-proton), pA (proton-nucleus) or forward AA (nucleus-nucleus) scattering and provides us with phenomenological predictions. Given appropriate initial conditions at large x, the solutions of JIMWLK equation allow one to compute a wide range of multi-particle final states in deeply inelastic scattering (DIS) and hadronic collisions which can be measured in LHC experiments. A prominent example is provided by inclusive DIS structure functions, F_2 and F_L, which are proportional to the forward scattering amplitude of a $q\bar{q}$ dipole on a nucleus.

2. The NLO correction to JIMWLK Equation

The perturbative expansion in α_s of the JIMWLK Hamiltonian has the following form:

$$H^{JIMWLK} = H^{LO}(\alpha_s) + H^{NLO}(\alpha_s^2) + H^{NNLO}(\alpha_s^3) + ... \tag{2.1}$$

Most of the efforts in this and the following sections will be concentrated on the term $H^{NLO}(\alpha_s^2)$, known as the next-to-leading order correction to JIMWLK. ***Why is the next-to-leading order***

[1] This bound restricts the cross sections growth, $\sigma \leq \alpha \ln^2 s$ for some constant α.

JIMWLK correction needed?
• Built-in information on the *running coupling* (up to α_s^2). The LO equation has a fix coupling constant which is usually set by hand to Q_s for analysis purposes. While this subscription is often useful it does not fully determine the length variable we should work with. For example in the case of dipole it is not clear if this coupling constant corresponds to the size of the original dipole $|x-y|$, or of the size of the produced dipoles $|x-z|$, $|z-y|$, or even a function of these variables. The solutions of the equation for each of this choices have a very different behavior. Here the running-coupling correction will follow naturally and no ambiguity will left.
• The NLO corrections are known to be large and therefore they might have a substantial impact on the cross section and scattering amplitudes.
• To get the *region of applicability* of the leading order equation.
• Important step towards *all order resummation*.

2.1 NLO JIMWLK for color singlet operators

The JIMWLK kernel describes the way expectation values of operators are related depending on the rapidity of Y:

$$\frac{d}{dY}\mathscr{O} = -H^{JIMWLK}\mathscr{O} \tag{2.2}$$

Where the JIMWLK Hamiltonian is obtained by computing the expectation value of the $\hat{S}$-matrix operator (expanded to first order in longitudinal phase space):

$$H^{JIMWLK} = \langle\psi|\hat{S}-1|\psi\rangle \tag{2.3}$$

To compute the NLO Hamiltonian, the wave function has to be computed to order g_s^3 and normalized to order g_s^4. Each emission off the valence gluons in the wave function brings a factor of color charge density J. At NLO at most two soft gluons can be emitted, and therefore the general form of the wave function at NLO is:

$$\begin{aligned}|\psi\rangle = &(1 - g_s^2\,\kappa_0 JJ - g_s^4(\delta_1 JJ + \delta_2 JJJ + \delta_3 JJJJ)\,|no\,soft\,gluons\rangle + \\ &+ (g_s\kappa_1 J + g_s^3\varepsilon_1 J + g_s^3\,\varepsilon_2 JJ)\,|one\,soft\,gluon\rangle + g_s^2(\varepsilon_3 J + \varepsilon_4 JJ)\,|two\,soft\,gluons\rangle \\ &+ g_s^2\,\varepsilon_5 J\,|quark-antiquark\rangle\,.\end{aligned} \tag{2.4}$$

Where the variables κ, ε and δ denote unknown functions. Given the general form of the wave function, based on the expected symmetry of $SU_L(N)\times SU_R(N)$ and unitarity, the Hamiltonian can be parametrized in terms of five kernels - $K_{JSJ}(x,y;z)$, $K_{JSSJ}(x,y;z,z')$, $K_{q\bar{q}}(x,y;z,z')$, $K_{JJSSJ}(w;x,y;z,z')$, and $K_{JJSJ}(w;x,y;z)$:

$$\begin{aligned}H^{NLO\ JIMWLK} &= \int_{x,y,z} K_{JSJ}(x,y;z)\left[J_L^a(x)J_L^a(y) + J_R^a(x)J_R^a(y) - 2J_L^a(x)S_A^{ab}(z)J_R^b(y)\right] \\ &+ \int_{xyzz'} K_{JSSJ}(x,y;z,z')\left[f^{abc}f^{def}J_L^a(x)S_A^{be}(z)S_A^{cf}(z')J_R^d(y) - N_cJ_L^a(x)S_A^{ab}(z)J_R^b(y)\right] \\ &+ \int_{x,y,z,z'} K_{q\bar{q}}(x,y;z,z')\left[2J_L^a(x)\,tr[S^\dagger(z)\,t^a\,S(z')t^b]\,J_R^b(y) - J_L^a(x)\,S_A^{ab}(z)\,J_R^b(y)\right]\end{aligned}$$

$$+\int_{w,x,y,z,z'} K_{JJSSJ}(w;x,y;z,z')f^{acb}\Big[J_L^d(x)\,J_L^e(y)\,S_A^{dc}(z)\,S_A^{eb}(z')\,J_R^a(w)$$
$$-J_L^a(w)\,S_A^{cd}(z)\,S_A^{be}(z')\,J_R^d(x)\,J_R^e(y)+\frac{1}{3}[J_L^c(x)\,J_L^b(y)\,J_L^a(w)-J_R^c(x)\,J_R^b(y)\,J_R^a(w)]\Big]$$
$$+\int_{w,x,y,z} K_{JJSJ}(w;x,y;z)\,f^{bde}\Big[J_L^d(x)\,J_L^e(y)\,S_A^{ba}(z)\,J_R^a(w)-J_L^a(w)\,S_A^{ab}(z)\,J_R^d(x)\,J_R^e(y)$$
$$+\frac{1}{3}[J_L^d(x)\,J_L^e(y)\,J_L^b(w)-J_R^d(x)\,J_R^e(y)\,J_R^b(w)]\Big]. \tag{2.5}$$

Where:

$$S^{ab}(x_\perp)=\left[P\exp\left(ig\int dx^+T_cA_c^-(x^+,x_\perp)\right)\right]^{ab} \tag{2.6}$$

And we denote also:

$$J_R^a(x_\perp)\equiv -tr\left[S(x)T^a\frac{\delta}{\delta S^\dagger(x)}\right] \qquad J_L^a(x_\perp)\equiv -tr\left[T^aS(x)\frac{\delta}{\delta S^\dagger(x)}\right] \tag{2.7}$$

As a part of my contribution to the field, together with Alex Kovner and Michael Lublinsky, we determined the explicit form of the kernels by means of comparison with the evolution equation produced from the above Hamiltonian for dipole with NLO BK [3]. This comparison turs out to be sufficient to fully determine three of the five kernels. However, it turned out that it can be done much quicker by using another important piece of information that was calculated by Grabosky last year [4], the evolution equation of the (connected part of) Baryon which is defined by:

$$B(x,y,z)=\varepsilon^{ijk}\varepsilon^{lmn}S^{il}(x)S^{jm}(y)S^{kn}(z) \tag{2.8}$$

The unabridged calculation of the determination of these kernels appears on [7].

2.2 NLO JIMWLK for color non-singlet operators

Since the determination of the kernels on the previous section relies only on the action of the Hamiltonian on color singlet operators, the kernels are determined only modulo terms that do not depend on (at least) one of the coordinates carried by one of the charge density operators J. One can add to the Hamiltonian an arbitrary operator proportional to $Q_{L(R)}=\int d^2xJ^a_{L(R)}(x)$ without altering its action on singlets, since $Q_{L(R)}$ annihilates any color singlet state. In order to find the missing terms we computed the evolution equations for $S^{ij}(x)$, $S^{ij}(x)S^{kl}(y)$, and $S^{ij}(x)S^{kl}(y)S^{mn}(z)$ and compared to the calculation in [7]. It was shown by inspection in [6], that three modifications of the kernels are needed in order that the NLO JIMWLK Hamiltonian be applicable to operate on non-gauge invariant operators. These additional terms do not contribute to evolution of gauge invariant operators.

2.3 The conformal properties of the NLO Hamiltonian

The leading order JIMWLK Hamiltonian is invariant under the following transformations (together known as conformal symmetry):

• **Scaling Symmetry**

$$x\to\alpha x$$

• Inversion

$$x \to \frac{1}{x}$$

In [8], together with Alex Kovner and Michael Lublinsky, we addressed the question whether these symmetries are still preserved at NLO JIMWLK Hamiltonian. Since QCD is not a conformal theory, we looked for NLO JIMWLK Hamiltonian corresponding to $\mathcal{N}=4$ theory, which is similar to QCD but with vanishing beta function.

It is straightforward to show that the NLO Hamiltonian is invariant under scaling but it turns out that not all the kernels are invariant under the naive inversion transformation $\mathscr{I}_0$. What we found is additional piece, $\mathscr{A}$:

$$\mathscr{I}_0 : H^{NLO\ JIMWLK} \to H^{NLO\ JIMWLK} + \mathscr{A} \tag{2.9}$$

One might however expect, that the Hamiltonian does possess an exact inversion (and conformal) symmetry, but that this symmetry is represented in a slightly different way than the naive transformation. This is generically the situation if one arrives at an effective theory by integrating out a subset of degrees of freedom. Say, one integrates over the subset $\{\alpha\}$ and obtains effective theory in terms of the remaining degrees of freedom $\{\beta\}$. If the cutoff separating α from β is not invariant under a symmetry of the full theory, the transformation of β involves α, that is $\delta\beta = f(\alpha,\beta)$. After the integration $f(\alpha,\beta)$ becomes some effective operator expressible in terms of β only. However this operator generically is not simply equal to $f(\alpha=0,\beta)$. This means, that the transformation of β in the effective theory looks somewhat different than in the original formulation before the integration of α. The situation in our case is very similar. The sharp rapidity cutoff used in deriving $H^{NLO\ JIMWLK}$ is not invariant under the conformal symmetry. Thus we expect that the naive form of conformal transformation should be modified, but that the symmetry itself is still the symmetry of $H^{NLO\ JIMWLK}$. If this is true, the anomalous piece $\mathscr{A}$ can be compensated if the Wilson lines S form a non-trivial representation of the conformal group such that

$$\begin{aligned} &\mathscr{I} : S(x) \to S(1/x) + \delta S(x)\,, \qquad \mathscr{I} : J_{L,R}(x) \to \frac{1}{x^2} J_{L,R}(1/x) + \delta J_{L,R}(x) \\ &\mathscr{I} : H^{LO} \to H^{LO} - \mathscr{A} \end{aligned} \tag{2.10}$$

where δS and δJ are perturbatively of the order α_s, such that the net anomaly is canceled and the total Hamiltonian remains invariant at NLO:

$$\mathscr{I} : H^{LO} + H^{NLO} \to H^{LO} + H^{NLO} \tag{2.11}$$

In order to explicitly construct such $\mathscr{I}$ we used the ansatz:

$$\mathscr{I} = (1 + \mathscr{C})\, \mathscr{I}_0 . \tag{2.12}$$

We managed to show in [8] that the explicit form of $\mathscr{C}$ is:

$$\mathscr{C} = -\frac{\alpha_s}{4\pi^2} \int_{x,y,z} \frac{(x-y)^2}{X^2Y^2} \ln\frac{z^2}{a^2} \left[J_L^a(x) J_L^a(y) + J_R^a(x) J_R^a(y) - 2 J_L^a S_A^{ab}(z) J_R^b(y) \right] \tag{2.13}$$

Having $\mathscr{I}$ we can construct conformal extensions of any operator including the Hamiltonian itself.

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