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Vortex Formation in a $U(1) \times U(1)'$ - $\mathcal{N} = 2$ - $D = 3$ Supersymmetric Gauge Model

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In this work, we study a $U(1) \times U(1)'-$ model that results from a dimensional reduction of the $\mathcal{N} = 1 - D = 4$ supersymmetric version of the Cremer-Scherk-Kalb-Ramond model with non-minimal coupling to matter. Field truncations are not carried out, two Abelian symmetries coexist and three vector fields are present; two of them are gauge bosons. Then, by considering the full $\mathcal{N} = 2 - D = 3$ supersymmetric model, we study the mechanism for magnetic vortex formation by means of the Bogomol'nyi relations, the magnetic flux and the topological charge in the presence of the two gauge potentials. A short discussion on the applications of our supersymmetric model and vortices are also presented .

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1. Introduction

The study of symmetries in Physics is of crucial importance as a tool for the understanding and the description of the Elementary Particles and their processes. The wide symmetry behind a Grand-Unified Theory (GUT) accomodates a large variety of phenomena in a single model. For instance, we have the Standard Model of the Elementary Particles (SM), where $SU(3)\otimes SU(2)\otimes U(1)$ describes three types of interactions. The existence of vortices is among the most interesting and important consequences of the property of superfluidity. In the Bose-Einstein Condensate (BEC), these vortices have been realized experimentally [1, 2, 3] and can be formed by domain-wall annihilation BEC [4]. On the other hand, (2+1) dimensions have been of strong theoretical and experimental interest for the experimental realization of monolayer graphene in 2004 [5, 6, 7], where it was observed that the to low-energy excitations behave like negatively-charged fermions satisfying a Dirac equation. These new possibilities open up a new interest on topological defects in lower-dimensional fermionic systems. The Dirac-type excitations in pure graphene are gapless; there appear specific impurities that affect the short-distance electron-electron interactions [8, 9, 10, 11]. This environment provides special nonperturbative quantum features. It is also possible to consider topological defects as a mechanism to introduce massive Dirac fermions [12] The study of vortex configurations in a supersymmetric context is an issue of interest due to the fact that supersymmetry (SUSY) is considered a fundamental symmetry and fermionic excitations are naturally coupled and taken into account in a supersymmetric scenario. In the context of the $U(N)\otimes U(N)$ gauge symmetry, it is important to highlight the construction of the Aharony-Bergman-Jafferis-Maldacena (ABJM) model [13] that includes a large class of Superconformal Chern-Simons (SCS) theories depending on the rank of the gauge group N and the Chern-Simons level k. In the ABJM theory, composite of M-branes and domain wall solutions, vortex-type solutions [14, 15, 16, 17, 18], and the classification of BPS conditions of intersecting M-branes [19] have been carefully studied. For the vortex-type solutions in the ABJM theory, $\mathcal{N}=1$ Chern-Simons vortex-type-regime have been obtained [17, 18] and the existence of the corresponding vortices with $\mathcal{N}=3$ supersymmetry has been discussed [20]. Vortex solutions in the non relativistic limit of the ABJM theory have been studied in Ref. [21]. The vortex also can appear in eleven dimensional supergravity scenarium [22] in most general form for AdS_4 configuration[23]. On the other hand, p-form potentials appear in many supersymmetric models. A 2-form field is referred to as the Kalb-Ramond field (KR) [24, 25]. This field appear in ten dimensions linked with the superstring theory[26]. Also, KR fields have already been studied in the physics of the topological insulators [27]. In this work, we wish to investigate the complete $\mathcal{N}=2-D=3$ gauge model with a $U(1)\times U(1)'$ symmetry. In a previous work [30], the truncated $\mathcal{N}=2-D=3$ model including the KR field has been considered and the vortex configurations have been worked out. The truncation consisted in identifying fields that appear from the dimensional reduction of an $\mathcal{N}=1-D=4$ model, as studied in [29, 28]. Here, we reconsider this model and discuss the full reduced model with two families of gauge potentials with a mixed Chern-Simons term and we focus on the analysis of vortex-type solutions in the presence of the second family of gauge fields. We could motivate the simultaneous presence of the two families of gauge potentials if we think to consider special situations where charged matter in interaction with an electromagnetic field is placed in a macroscopic external electric or magnetic field. In this situation, two electromagnetic fields are actually necessary. One of them is generated by the charged matter of the system under consideration; the other one, is a strong field that is not affected by the matter and it is imposed as an external condition; it is a sort of background field to which our system is subject. So, the non-identification of the different families of gauge fields may be plausible if we are to treat two different categories of electromagnetic fields appearing in the same system. The outline of this paper is as follows: in Section 2, we present some considerations about the Cremer-Scherk-Kalb-Ramond (CSKR) model in the supersymmetric $\mathcal{N}=2-D=3$ scenario. In Section 3, we devote our attention to showing the ingredients of the vortex magnetic configuration, we study the bosonic part of our SUSY model, the equations of motion and the critical coupling. In Section 4, we study the Bogomol'nyi equations and the minimal energy configuration of the vortex. Then, in Discussions and Remarks , the relation beetwen our $\mathcal{N}=2-D=3$ supersymmetric model with vortices in superfluid films is discussed and we also did some highlights about others applications.

2. The $\mathcal{N}=2-D=3$ SUSY model without truncation

In this section, we briefly review the $\mathcal{N}=2-D=3$ model that results from the dimensional reduction of the four-dimensinal CSKR model[30]. This model descends from the $N=1-D=4$ action that describes QED in the supersymmetric version coupled to the Kalb-Ramond field in a non-minimal way. This non-minimal coupling is unique. To see this, consider the pure Kalb-Ramond action coupled to an arbitrary current,

$$S_{K-R}=\int d^3x\left\{-\frac{1}{6}L_{\mu\nu\kappa}L^{\mu\nu\kappa}+J^{\mu\nu}B_{\mu\nu}\right\}, \tag{2.1}$$

where $L_{\mu\nu\kappa}$ is the field-strength 3-form. In momentum space, the field $B_{\mu\nu}(k)\equiv\widetilde{B}_{\mu\nu}$ can be expanded as follows:

$$\widetilde{B}_{\mu\nu}=\alpha k^{\mu}k^{\nu}+\beta_I k^{\mu}e_I^{\nu}+\gamma_I\bar{k}^{\mu}e_I^{\nu}+\delta_{IJ}e_I^{\mu}e_J^{\nu}, \tag{2.2}$$

where the basis vectors are taken as below:

$$k^{\mu}=\left(k^0,\overrightarrow{k}\right);\ \bar{k}^{\mu}=\left(k^0,-\overrightarrow{k}\right); \tag{2.3}$$

$$e_I^{\mu}=(0,\overrightarrow{e}_I);\ \overrightarrow{e}_I\cdot\overrightarrow{k}=0,\text{com } I=1,2. \tag{2.4}$$

With the help of the gauge symmetry for $B_{\mu\nu}$, $\widetilde{B}_{\mu\nu}$ can be shown to acquire the form

$$\widetilde{B}_{\mu\nu}=\delta_{IJ}e_I^{\mu}e_J^{\nu}. \tag{2.5}$$

So, the equations of motion in momentum space read as:

$$k^2\delta_{IJ}e_I^ie_J^j=\widetilde{J}^{ij}, \tag{2.6}$$

$$n\varepsilon^{ijk}k_k=\widetilde{J}^{ij}, \tag{2.7}$$

where $n=k^2\delta_{IJ}$. Equation (2.7) ensures that the current coupled to the Kalb-Ramond field is actually a topological current with the form:

$$J^{\mu\nu}=\varepsilon^{\mu\nu\kappa\lambda}\partial_\kappa j_\lambda. \tag{2.8}$$

This result denies the possiblity of writting down a symmetry group associated with the conservation of $J_{\mu\nu}$. In other words, the Yang-Mills version of the Kalb-Ramond model is not possible and this is actually shown as a no-go result in the work of [31]. Thus we have explained the origins of the non minimal coupling of the Kalb-Ramond theory in the $\mathcal{N}=1-D=4$ model.

Back to the $\mathcal{N}=2-D=3$ model [32], we write down the gauge-field sector of the bosonic action in components as:

$$S_{gauge} = \int d^3x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+2m\varepsilon^{\mu\nu\alpha}A_\mu\partial_\nu B_\alpha \tag{2.9}$$

$$-\frac{1}{2}G_{\mu\nu}G^{\mu\nu}\}, \tag{2.10}$$

where the index $\mu=0,1,2$, with $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ being the electromagnetic field-strength. B_μ is the vector given by the reduction of the 4-dimensional Kalb-Ramond field, $B^{3\mu}$, with a corresponding field-strength $G_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu$. Another vector field, the dual of $B^{\mu\nu}$ in $3D$, comes out which is defined by $B^{\mu\nu}=\varepsilon^{\mu\nu\rho}Z_\rho$. Having in mind that, in $(1+2)D$, the Kalb-Ramond field-strength may be written as a scalar,

$$L_{\mu\nu\kappa}=S\varepsilon_{\mu\nu\kappa}, \tag{2.11}$$

then

$$\partial_\mu Z^\mu=\frac{1}{2}\varepsilon^{\mu\nu\kappa}\partial_\mu B_{\nu\kappa}=S. \tag{2.12}$$

However, from the free field equations and the gauge transformation $Z'_\mu=Z_\mu+\varepsilon_{\mu\nu\kappa}\partial^\nu\xi^\kappa$, S is shown to be a constant, so that $B_{\mu\nu}$ does not correspond to a physical degree of freedom, unless it interacts with other fields.

The part of the $\mathcal{N}=2-D=3$ action involving the scalars is written as follows:

$$S_{scalar}=\int d^3x\{e^{-2gM}\nabla_\mu\varphi(\nabla^\mu\varphi)^*+P(\varphi)\,\partial_\mu M\partial^\mu M \tag{2.13}$$

$$+\frac{1}{2}\partial_\mu N\partial^\mu N+2N\left(m+gh|\varphi|^2e^{-2gM}\right)\left(\partial_\mu Z^\mu\right) \tag{2.14}$$

$$-g^2(\partial_\mu Z^\mu)^2|\varphi|^2e^{-2gM}+(\partial_\mu Z^\mu)^2\}, \tag{2.15}$$

where $P(\varphi)=1-g^2|\varphi|^2e^{-2gM}$. The covariant derivative, ∇_μ, is given by

$$\nabla_\mu\varphi=(\partial_\mu+ihA_\mu+igG_\mu)\varphi. \tag{2.16}$$

M and N are real scalars. The dual fields, F_μ and G_μ, are given by:

$$F_\mu=\frac{1}{2}\varepsilon_{\mu\nu\kappa}F^{\nu\kappa};\qquad G_\mu=\frac{1}{2}\varepsilon_{\mu\nu\kappa}G^{\nu\kappa}. \tag{2.17}$$

Adopting the parametrisations $\phi=e^{-gM}\varphi$ and $\partial_\mu Z^\mu=S$, we write down the remaining piece of the bosonic action, where the auxiliary field, Δ, is present and from which we can extract the potential of the model. We denote it by S_U and it is given by

$$S_U = \int d^3x \ \{-hN|\phi|^2 + 2h\Delta|\phi|^2 + \tag{2.18}$$

$$+2\Delta^2 - 4mM\Delta + \eta\Delta\}. \tag{2.19}$$

The equation of motion for the auxiliary field yields

$$\Delta = mM - \frac{h}{2}|\phi|^2 - \frac{\eta}{4}. \tag{2.20}$$

Once it is eliminated, the potential for the physical scalars takes the form below:

$$U = \frac{h^2}{2}\left(|\phi|^2 - \frac{2m}{h}M - v^2\right)^2 - h^2N^2|\phi|^2 \tag{2.21}$$

where $v^2 = \frac{\eta}{-2h}$. Once this potential has been built up, we are ready to discuss the symmetry-breaking pattern that yields the vortex formation.

3. Critical coupling and field equations

The equations of motion for the fields involved in our Lagrangian density are given below:

$$\partial_\nu\left[\left(1-g^2|\phi|^2\right)\left(\partial_\mu Z^\mu\right) + \left(m+gh|\phi|^2\right)N\right] = 0 \tag{3.1}$$

$$\left(\Box + 2h^2|\phi|^2\right)N - 2\partial_\mu Z^\mu\left(m+gh|\phi|^2\right) = 0 \tag{3.2}$$

$$\partial_\mu F^{\mu\nu} + 2mG^\nu = J^\nu \tag{3.3}$$

$$\partial_\mu G^{\mu\nu} + mF^\nu = \frac{g}{2h}\varepsilon^{\mu\kappa\nu}\partial_\mu J_\kappa, \tag{3.4}$$

where the current is $J_\mu = ih\left(\phi^*\nabla_\mu\phi - \phi\left(\nabla_\mu\phi\right)^*\right)$. We have three vector fields, two of them coupled by a Chern-Simons term, and the other one coupled to a scalar field. Despite this complicated mixing, Bogomoln'yi equations will be help us to understand the role of each field in vortex formation.

Decoupling the Eqs. 3.3 and 3.4 from one another, we obtain

$$(\Box + m^2)F^\nu = \varepsilon^{\mu\kappa\nu}\partial_\mu J_\kappa\left(\frac{gm}{h}+1\right) \tag{3.5}$$

$$(\Box + m^2)G^\nu = \left(\Box - \frac{hm}{g}\right)\left(\Box + \frac{-g}{2h}\right)J^\nu. \tag{3.6}$$

Using the critical coupling, $g = -\frac{h}{m}$, in the previous two equations yields:

$$\left(\Box + m^2\right)F^\nu = 0, \tag{3.7}$$

$$G^\nu = \frac{1}{2m}J^\nu. \tag{3.8}$$

The most general case in Eq. 3.8 involve the solution $G^\nu + F^\nu = \frac{1}{2m}J^\nu$. However, we will particularize the solution for the inhomogeneous case, then the G^μ field gives us

$$\varepsilon^{\nu\alpha\beta}\partial_\alpha B_\beta = \frac{1}{2m}J^\nu . \tag{3.9}$$

The value of the critical coupling, $g=-\frac{h}{m}$, reveals the purely topological character of the current, that shall be a relevant information in our analysis of the asymptotic behavior of the field configurations.

4. BP-states and asymptotic behavior

The explicit form of BPS-states can be worked out in a supersymmetric context. Based on that work, we could define new supersymmetric generators as follows,

$$Q_\pm = Q_\theta \mp i\gamma^0 Q_\tau , \tag{4.1}$$

where Q_θ e Q_τ are the Majorana-like generators for the $\mathcal{N}=2$ supersymmetry. The generators 4.1 render manifest one of the results of Houlsek and Spector [33],

$$\left\{Q_+,\overline{Q}_+\right\} = 4\gamma^0\left(P_0+Z\right);\ \left\{Q_-,\overline{Q}_-\right\} = 4\gamma^0\left(P_0-Z\right). \tag{4.2}$$

where Z it is a central charge of the extended supersymmetry.

Using these generators and setting to zero half of the fermionic variations, we can obtain BPS-states; however, here, we shall present another approach (more heuristic) that can be used also in the case of non supersymmetric models. To do so, we begin with the energy density of our model

$$\begin{aligned} E \ =& \int d^2x \Big\{ \frac{1}{2}\left(E_i^2+B^2\right) + P\left(e_i^2+b^2\right) + PS^2 + e^{-2gM}\left(D_0\varphi\right)^*\left(D_0\varphi\right) + \\ & + e^{-2gM}\left(D_i\varphi\right)^*\left(D_i\varphi\right) + P\left(\partial_0 M\right)^2 + P\left(\partial_i M\right)^2 + \frac{1}{2}\left(\partial_0 N\right)^2 + \frac{1}{2}\left(\partial_i N\right)^2 + U \Big\}, \end{aligned} \tag{4.3}$$

where, contrary to the work of ref. [30], the second family of gauge potentials is not truncated. And this is one of ours proposals: to understand the role of the $U(1)'$ factor and its corresponding gauge potential, B^μ, in the process of vortex formation.

Upon completion of squares,

$$\begin{aligned} E \ =& \int d^2x \Big\{ \frac{1}{2}\left[B \mp h\left(\frac{2m}{h}M - |\phi|^2 + v^2\right)\right]^2 + \frac{1}{2}\left(E_i \pm \partial_i N\right)^2 + P\left(G_0 \pm S\right)^2 \\ & + P\left(G_i \pm \partial_i M\right)^2 + e^{-2gM}|\left(D_0 \pm ihN\right)\varphi|^2 + e^{-2gM}|\left(D_1 \pm iD_2\right)\varphi|^2 \\ & \pm hB\left(\frac{2m}{h}M - |\phi|^2 + v^2\right) \mp E_i\partial_i N \mp 2PG_0 S \mp 2PG_i\partial_i M \mp 2e^{-2gM}NH_0 \\ & \mp e^{-2gM}\left(\frac{1}{h}\varepsilon_{ij}\partial_i H_j + hB|\varphi|^2\right) + U \Big\}, \end{aligned} \tag{4.4}$$

with

$$H_\mu = -\frac{ih}{2}\left(\varphi^* D_\mu \varphi - \varphi\left(D_\mu\varphi\right)^*\right). \tag{4.5}$$

Now, we drop all quadratic terms as we are interested in the minimum energy configuration. Then, we obtain the BPS-equations:

$$B \mp h\Big(\frac{2m}{h}M - |\phi|^2 + v^2\Big) = 0; \tag{4.6}$$

$$\partial_\mu Z^\mu = S = \pm G_0; \quad E_i \pm \partial_i N = 0; \tag{4.7}$$

$$G_i \pm \partial_i M = 0; \quad \Big(\nabla_1 \pm i\nabla_2\Big)\phi = 0. \tag{4.8}$$

Introducing Eq. 4.7 in 3.1 and 3.2 we recover Eqs 3.3 and 3.4, showing that BPS-states agree with the results from the equations of motion, as expected. It is worthy to mention that the field-strength for the Kalb-Ramond potential becomes the topological charge.

If asympotically we write $\phi = ve^{in\theta}$, then, from equation 4.8 , we get

$$\frac{1}{-i}\phi^{-1}\Big(\partial_1 \pm i\partial_2\Big)\phi = -e^{\pm i\theta}\frac{n}{r},$$
$$-e^{\pm i\theta}\frac{n}{r} = e\Big(A_1 \pm iA_2\Big) + g\Big(G_1 \pm iG_2\Big). \tag{4.9}$$

Therefore, in the minimum energy configuration both fields, A_μ and G_μ, participate of the vortex formation. However, for the critical coupling $\left(g = -\frac{h}{m}\right)$ and integrating Eq. 3.3, only the field that appears in the non-minimal coupling, G_μ, is relevant for the vortex configuration:

$$2m\int d^2xb = Q_{top} = 2m\Phi_{flux}. \tag{4.10}$$

By analyzing the critical coupling and the asymptotic behavior of Eq. 3.3, we see that the non-minimal coupling in the covariant derivative contributes directly to the topological current, in agreement with equation 3.9.

5. Discussions and Remarks

In this work, we have shown that the Kalb-Ramond current obeys a topological conservation law in four dimensions. So, it seems reasonable that the coupling of the KR field to any other model must be non-minimal. This also supports the non-existence of a non-Abelian generalization for these theories. Our result agrees with the "no-go" theorem [31] . In the study of vortex formation, the KR-field strength in $1+2$ dimensions is a simple constant and it couples to the present model as the topological charge of the vortex. This may also describe a non-trivial background. Also the non-minimal coupling of the vector field in the covariant derivative becomes directly identified with the topological current, which seems to stabilize the topological solutions for configuration of non-minimal energy. We analyzed how BPS-states in this model reduce the number of differential equations and give us some insight on the role of each field whenever half of the supersymmetry charges become zero. We see that the mixing of the minimal and non-minimal couplings contributes for the ansatz on the scalar field, in general. However, with the critical coupling, $g = -\frac{h}{m}$, only the non-minimal coupling is actually relevant for the vortex configuration. There are relations between a global vortex in the Abelian Higgs model and vortices in a superfluid has been exploited in [28]. This work is developed in $4D$ and basically two problems are found when we try to identify them. The first difference has to do with the energy density that falls off like $1/r^2$ in the case of the global vortex; on the other hand, vortices in a superfluid have non-zero energy density at infinity. The second main difference is related with the angular momentum, that is well-defined for vortices in a superfluid, but is zero for global vortices, when considering static configurations.These problems have been solved when Davis and Shellard [34] considered time dependent equations and a non-trivial background, In our case, (2+1) dimensions, we can use the anzatz $L^{ij} = S\varepsilon^{ij}$. in equations (2.11,2.12) and it gives us a constant field S that put a constraint in the equation to the Z_μ. An important fact to mention is that, in order to introduce a non-trivial background in our $N = 2-D = 3$ model, the SSB must also be realised by the Kalb-Ramond field. This has been done in $4D$ because the scalar action and the Kalb-Ramond action are simply related by a canonical transformation [28]. However in $3D$ the SSB cannot be realised by the Kalb-Ramond field and will be entirely described by a scalar field. Another relation of our $\mathcal{N} = 2-D = 3$ with Condensed Matter concerns the gauge action that contain the copling $m\varepsilon^{\mu\nu\alpha}A_\mu\partial_\nu B_\alpha$ In a lower-dimensional Condensed Matter system, the Chern-Simons-like term in equation (2.10) could also provide a non-trivial background. This mixing has been studied as an effective theory [35] in which a dynamical vortex is coupled with a superfluid film at zero temperature. In the $\varepsilon^{\mu\nu\alpha}A_\mu\partial_\nu B_\alpha$-term, the $A_\mu-$ field is chosen as the responsible for the vortex formation and the $B_\mu-$ field as the electromagnetic potential, which becomes part of the source that describes a uniform magnetic field. Also here, time-dependent equations must be considered. For this reason our results open a new window to study the graphene like materials and mechanisms to understand a mass gap, this subject is the subject of the next works. We also can be studied this model related with ABJM framework as discussed in the introduction. Finally, our perspectives are to study the possibility of having a minimal coupling of the KR model in higher dimensions and study whether or not this coupling is allowed in presence of a gravity background. It would also be interesting to explore further the relation between our $N = 2-D = 3$ and dynamical vortices in a superfluid film

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