PROCEEDINGS OF SCIENCE

$\mathscr{O}(\alpha_s\alpha)$ corrections to Drell–Yan processes in the resonance region

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Drell–Yan-like W-boson and Z-boson production in the resonance region allows for some high-precision measurements that are crucial to carry experimental tests of the Standard Model to the extremes, such as the determinations of the W-boson mass and the effective weak mixing angle. We describe how the Standard Model prediction can be successfully performed in terms of a consistent expansion about the resonance pole, which classifies the corrections in terms of factorizable and non-factorizable contributions. The former can be attributed to the W/Z production and decay subprocesses individually, while the latter link production and decay by soft-photon exchange. At next-to-leading order we compare the full electroweak corrections with the pole-expanded approximations, confirming the validity of the approximation. At $\mathscr{O}(\alpha_s\alpha)$, we describe the concept of the expansion and explicitly give results on the non-factorizable contributions, which turn out to be phenomenologically negligible. Our results, thus, prove the factorizability of the $\mathscr{O}(\alpha_s\alpha)$ corrections into terms associated with initial-state and/or final-state corrections.

11th International Symposium on Radiative Corrections (Applications of Quantum Field Theory to Phenomenology) (RADCOR 2013),
22-27 September 2013
Lumley Castle Hotel, Durham, UK

*Speaker.

PoS(RADCOR 2013)020

1. Introduction

Drell–Yan-like W- or Z-boson production is among the most important standard candle processes at the LHC. Apart from delivering important information on parton distributions and allowing for the search for new gauge bosons in the high-mass range, these processes allow for high-precision measurements in the resonance regions. The weak mixing angle might be measured from data with LEP precision, and the W-boson mass M_W with an accuracy exceeding 10 MeV.

In the past two decades, great effort was made in the theory community to deliver precise predictions matching the required accuracy (for a list of references, see Ref. [1]). QCD corrections are known up to next-to-next-to-leading order, electroweak (EW) corrections up to next-to-leading order (NLO). Both on the QCD and on the EW side, there are further refinements such as leading higher-order effects, resummations, matched parton showers. In view of fixed-order calculations, the largest missing piece seems to be the mixed QCD–EW corrections of $\mathscr{O}(\alpha_s\alpha)$. Knowing the contribution of this order will also answer the question how to properly combine QCD and EW corrections in predictions. In Ref. [2] this issue is quantitatively discussed with special emphasis on observables that are relevant for the M_W determination, revealing percent corrections of $\mathscr{O}(\alpha_s\alpha)$ that should be calculated. First steps towards this direction have been taken by calculating two-loop contributions [3], the full $\mathscr{O}(\alpha_s\alpha)$ correction to the W-decay width [4], and the full $\mathscr{O}(\alpha)$ EW corrections to W/Z+jet production including the W/Z decays [5].

In this short article, we briefly report on our effort [1] to calculate the $\mathscr{O}(\alpha_s\alpha)$ corrections to Drell–Yan processes in the resonance region via the so-called *pole approximation*, which is based on a systematic expansion about the resonance pole. Specifically, we sketch the salient features of the approach, discuss its success at NLO, and present results on the so-called non-factorizable contributions at $\mathscr{O}(\alpha_s\alpha)$, which comprise the most delicate contribution to the PA.

2. Pole approximation for NLO corrections

The general idea [6] of a pole approximation (PA) for any Feynman diagram with a single resonance is the systematic isolation of all parts that are enhanced by a resonance factor $1/(p^2-M_V^2+\mathrm{i}M_V\Gamma_V)$, where p, M_V, and Γ_V are the momentum, mass, and width of the resonating particle V, respectively. For the considered case of Drell–Yan production, V is a W or a Z boson. For W production different variants of PAs have been suggested and discussed at NLO already in Refs. [7, 8]. For the virtual corrections we follow the PA approach of Ref. [8]. Note, however, that we apply the PA to the real corrections as well, in contrast to Ref. [8] where they were based on full matrix elements.

Schematically each transition amplitude has the form

$$\mathscr{M} = \frac{W(p^2)}{p^2-M_V^2+\Sigma(p^2)} + N(p^2), \tag{2.1}$$

with functions W and N describing resonant and non-resonant parts, respectively, and Σ denoting the self-energy of V. The resonant contributions of $\mathscr{M}$ are isolated in a gauge-invariant way as follows,

$$\mathscr{M} = \frac{W(\mu_V^2)}{p^2-\mu_V^2}\,\frac{1}{1+\Sigma'(\mu_V^2)} + \left[\frac{W(p^2)}{p^2-M_V^2+\Sigma(p^2)} - \frac{W(\mu_V^2)}{p^2-\mu_V^2}\,\frac{1}{1+\Sigma'(\mu_V^2)}\right] + N(p^2), \tag{2.2}$$

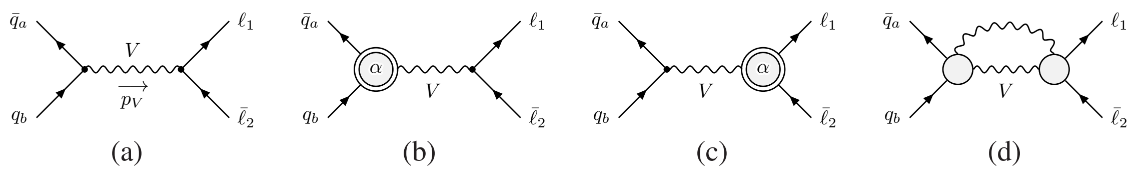

Figure 1: Generic diagrams for the lowest-order amplitude (a), for the EW virtual NLO factorizable corrections to production (b) and decay (c), as well as for virtual non-factorizable corrections (d), where the empty blobs stand for all relevant tree structures and the ones with "α" inside for one-loop corrections of $\mathscr{O}(\alpha)$.

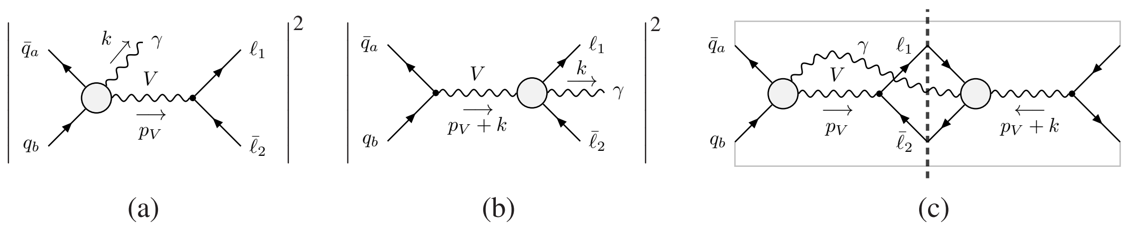

Figure 2: Generic diagrams for the real photonic NLO factorizable corrections to production (a) and decay (b), as well as for real non-factorizable corrections, where the blobs stand for all relevant tree structures.

where $\mu_V^2 = M_V^2 - \mathrm{i}M_V\Gamma_V$ is the gauge-invariant location of the propagator pole in the complex p^2 plane. Equation (2.2) can serve as a basis for the gauge-invariant introduction of the finite decay width in the resonance propagator, thereby defining the so-called *pole scheme*. In this scheme the term in square brackets is perturbatively expanded in the coupling α including terms up to $\mathscr{O}(\alpha)$, while the full p^2 dependence is kept. An application of this scheme to Z-boson production is, e.g., described in Ref. [9] in detail.

The PA for the amplitude results from the r.h.s. of (2.2) upon neglecting the last, non-resonant term and asymptotically expanding the term in square brackets in p^2 about the point $p^2 = \mu_V^2$, where only the leading, resonant term of the expansion is kept. The first term on the r.h.s. of (2.2) defines the so-called *factorizable* corrections in which on-shell production and decay amplitudes for V are linked by the off-shell propagator; these contributions are illustrated by diagrams (b) and (c) of Figure 1. The term on the r.h.s. of (2.2) in square brackets contains the so-called *non-factorizable* corrections which receives resonant contributions from all diagrams where the limit $p^2 \to \mu_V^2$ in $W(p^2)$ or $\Sigma'(p^2)$ would lead to (infrared) singularities. At NLO, this happens if a soft photon of energy $E_\gamma \lesssim \Gamma_V$ is exchanged between the production process, the decay part, and the intermediate V bosons; a generic loop diagram is shown in Figure 1(d). Figure 1 visualizes the generic diagrams containing one-loop corrections, Figure 2 shows the corresponding real-photon emission counterparts. Although in principle the real-emission corrections can be based on full amplitudes without further approximations, the consistent application of the PA to both virtual and real corrections is necessary to make a separate discussion of factorizable and non-factorizable contributions possible.

Conceptually the evaluation of the non-factorizable corrections is the most delicate among all PA contributions, but they possess rather interesting features. When the invariant mass of the resonance is integrated over, their contribution vanishes [10], i.e. they only tend to distort the resonance without changing the normalization of the cross section. Since they only involve soft photons, they take the form of a global correction factor to the lowest-order matrix element squared, with a non-trivial dependence on the virtuality $(p^2-\mu_V^2)$ which gave the corrections their name. The virtual correction factor can be explicitly calculated with quite compact results, even for double resonances for which their calculation is described in detail in the literature [11]. The real corrections are better evaluated numerically to keep some flexibility in the treatment of photons in the event selection, using extended eikonal currents [11] that take into account the resonance distortion by soft photons of energy $E_\gamma \lesssim \Gamma_V$. Virtual and real non-factorizable corrections involve soft IR divergences each, but their sum is IR finite. Collinear singularities do not occur at all, since all relevant diagrams are of interference type.

In spite of the simple general idea of the PA, its consistent implementation in higher-order calculations involves subtle details. For instance, care has to be taken that subamplitudes appearing before or after the V resonance are based on subamplitudes with on-shell V bosons, otherwise gauge invariance cannot be guaranteed. Setting $p^2 = M_V^2$, instead of the problematic complex value $p^2=\mu_V^2$, in the $\mathscr{O}(\alpha)$ corrections is certainly allowed in $\mathscr{O}(\alpha)$ approximation. However, the procedure is not unique, because the phase space is parametrized by more than one variable. The *on-shell projection* $p^2 \to M_V^2$ has to be defined carefully. Different variants may lead to results that differ within the intrinsic uncertainty of the PA, which is of $\mathscr{O}(\alpha/\pi \times \Gamma_V/M_V)$ in the resonance region when applied to $\mathscr{O}(\alpha)$ corrections. However, care has to be taken that virtual and real corrections still match properly in the (soft and collienear) infrared limits in order to guarantee the cancellation of the corresponding singularities.

Based on our results derived in Ref. [1], Figure 3 exemplarily shows the NLO QCD and EW corrections to the transverse-mass and transverse-lepton-momentum distributions for W^+ production at the LHC and, in particular, illustrates the structure and quality of the PA applied to the EW corrections. The distributions show the well-known Jacobian peaks at $M_{\mathrm{T},\nu l} \sim M_\mathrm{W}$ and $p_{\mathrm{T},l} \sim M_\mathrm{W}/2$, respectively, which play a central role in the measurement of the W-boson mass M_W at hadron colliders. The EW corrections significantly distort the distributions and shift the peak position. Note also the extremely large QCD corrections above the peak in the $p_{\mathrm{T},l}$ distribution, which are induced by the recoil of the W boson against the hard jet of the real QCD correction. The lower panels of Figure 3 compare the full NLO EW corrections (without photon-induced processes from $q\gamma$ collisions) to the result of the PA, which is also broken up into factorizable corrections to the initial/final state and non-factorizable contributions. Near the Jacobian peaks, the PA turns out to be good within some 0.1%. Interestingly, the impact of the non-factorizable corrections is suppressed to the 0.1% level and, thus, phenomenologically negligible. A similar conclusion even holds for the factorizable initial-state corrections when one takes into account that the percentage correction to the $p_{\mathrm{T},l}$ distribution actually should be normalized to the full cross section including QCD corrections, which are overwhelming above the Jacobian peak. Thus, the relevant part of the NLO EW corrections near the peaks entirely results from the factorizable final-state corrections, where the bulk originates from collinear final-state radiation from the decay leptons.

More details and results on the PA at NLO are discussed in Ref. [1], in particular for Z production, for which the PA works similarly well.

3. Pole approximation at $\mathscr{O}(\alpha_s\alpha)$

The concept of the PA, described at NLO in the previous section, can be carried over to higher

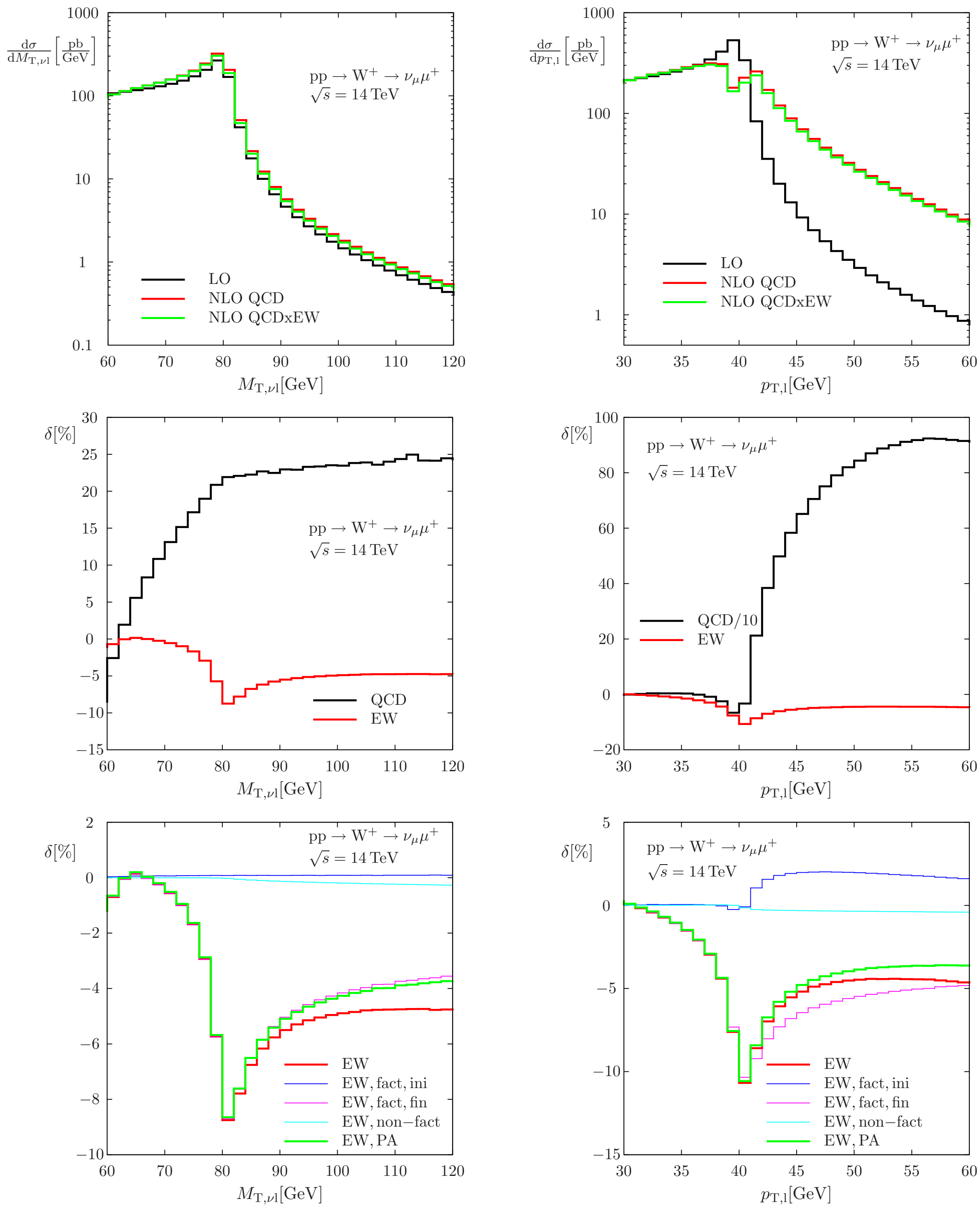

Figure 3: Distributions in the transverse mass (left) and transverse lepton momentum (right) for W^+ production at the LHC, with the upper plots showing the absolute distributions, the middle plots the full relative NLO QCD and EW corrections, and the lower plots the relative NLO EW corrections in PA broken up into its factorizable and non-factorizable parts (taken from Ref. [1]).

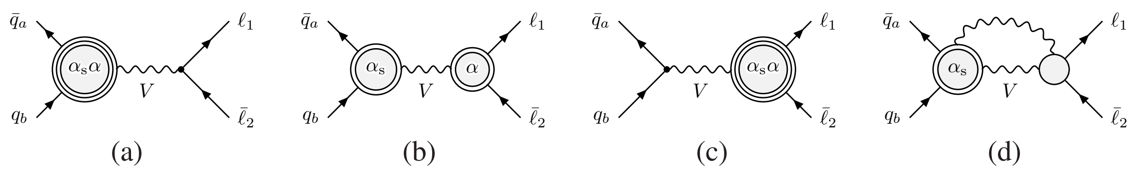

Figure 4: Generic diagrams for the various contributions to the virtual factorizable (a–c) and non-factorizable (d) corrections of $\mathcal{O}(\alpha_s\alpha)$ in PA, with α_s, α, and $\alpha_s\alpha$ in the blobs indicating the order of the included loop corrections.

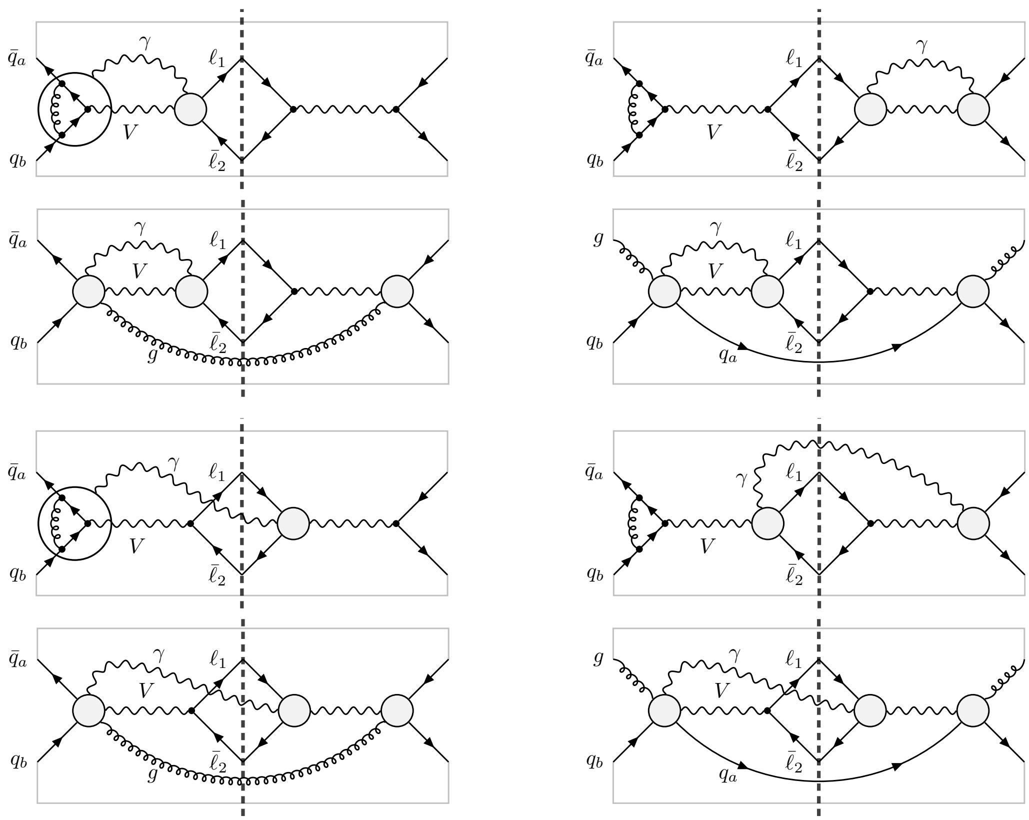

Figure 5: Generic diagrams for the various contributions to the non-factorizable corrections of $\mathcal{O}(\alpha_s\alpha)$, with blobs representing all relevant tree structures.

orders in a straightforward way, even though additional technical complications occur. Figure 4 generically illustrates the various types of purely virtual factorizable and non-factorizable corrections to the Drell–Yan process at $\mathcal{O}(\alpha_s\alpha)$. The corresponding real and mixed virtual–real contributions involve a plethora of different interference diagrams. Exemplarily we depict the generic graphs for the non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections in Figure 5, since we present numerical results for those below.

As at NLO, the non-factorizable corrections originate from the virtual exchange or real emission of soft photons with energies $E_\gamma \lesssim \Gamma_V$, however, without any restriction on the kinematics of virtual gluons or real jet radiation. In Ref. [1] we discuss in detail the factorization properties of the virtual and real photonic parts of the non-factorizable $\mathcal{O}(\alpha_s\alpha)$ corrections, which result from the soft nature of the effect. Using gauge-invariance arguments borrowed from the classic YFS

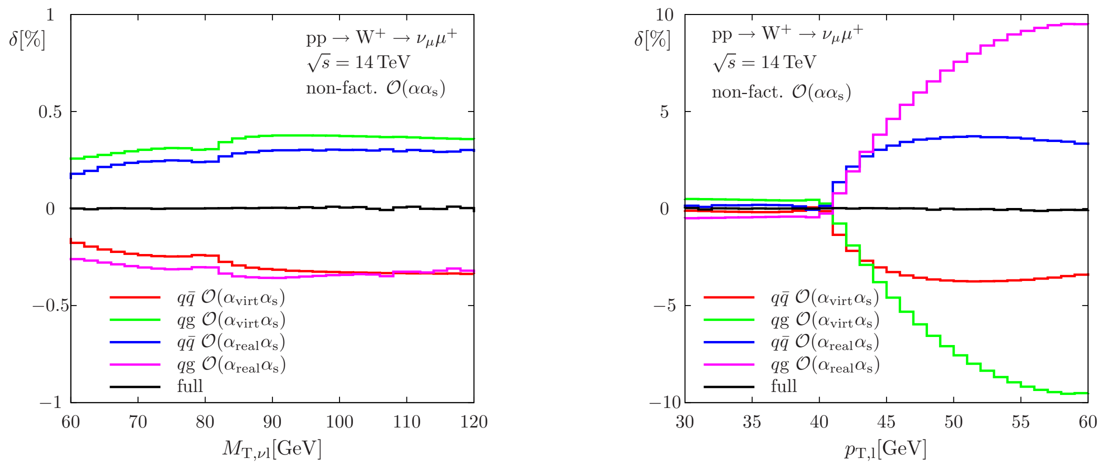

Figure 6: Relative non-factorizable corrections of $\mathscr{O}(\alpha_s\alpha)$ to the distributions in the transverse mass (left) and transverse lepton momentum (right) for W^+ production at the LHC, broken up into contributions of partonic $q\bar{q}/qg$ initial states and virtual/real photonic contributions (taken from Ref. [1]).

paper [12], we show that this factorization of the photonic factors even hold to any order in the strong coupling α_s. We have verified this statement diagrammatically and, for the purely virtual corrections, also with effective-field-theory techniques [13]. Both the virtual and real photonic corrections can be written as correction factors to squared matrix elements containing gluon loops or external gluons, i.e. the necessary building blocks are obtained from tree-level and one-loop calculations.

Figure 6 shows our numerical results on the $\mathscr{O}(\alpha_s\alpha)$ non-factorizable corrections to the $M_{\mathrm{T},\nu l}$ and $p_{\mathrm{T},l}$ distributions of W^+ production at the LHC. The contributions induced by virtual photons, indicated by α_{virt}, is infrared regularized upon adding the real soft-photon counterpart which accounts for the emission of photons of energy $E_\gamma < \Delta E \ll \Gamma_V$. Note that the photon-energy cut ΔE, which we numerically set to 10^{-4} times the partonic centre-of-mass energy, is much smaller than the width Γ_V of the resonating gauge boson. Although the individual contributions from virtual and real photons roughly reach the 0.5% level and even grow to several percent in the $p_{\mathrm{T},l}$ distribution, the sum of all non-factorizable corrections, which does not depend on ΔE, is way below the 0.1% level and, thus, phenomenologically negligible. Similar results on Z production, shown in Ref. [1], lead to the same conclusion for the neutral-current case.

Of course, one could have speculated on this suppression, since the impact of non-factorizable photonic corrections is already at the level of some 0.1% at NLO. However, the $\mathscr{O}(\alpha_s\alpha)$ corrections mix EW and QCD effects, so that small photonic corrections might have been enhanced by the strong jet recoil effect observed in the $p_{\mathrm{T},l}$ distribution. This enhancement is seen in the virtual and real corrections separately, but not in their sum. Furthermore, the existence of gluon-induced (qg) channels implies a new feature in the non-factorizable corrections. In the $q\bar{q}$ channels, and thus in the full NLO part of the non-factorizable corrections, the soft-photon exchange proceeds between initial- and final-state particles, whereas in the qg channels at $\mathscr{O}(\alpha_s\alpha)$ the photon is also exchanged between final-state particles. The known suppression mechanisms in non-factorizable corrections work somewhat differently in those cases [11].

4. Conclusions

Here we have briefly summarized the main results of Ref. [1], where we have shown how the yet unknown $\mathscr{O}(\alpha_s\alpha)$ corrections to Drell–Yan processes can be approximated by the leading term in an expansion about the resonance pole. The quality of such an approximation achieved at NLO strongly supports the expectation that this approach is sufficient for observables that are dominated by the resonance, which include, e.g., the ones relevant for precision determinations of the W-boson mass. The pole approaximation classifies corrections into factorizable and non-factorizable contributions. Our results show that the latter, which link production and decay subprocesses via soft-photon exchange, are phenomenologically negligible. The phenomenologically relevant corrections, thus, are of factorizable nature and can be attributed to corrections to the initial or final states. Again, the pattern of contributions to the pole approximation at NLO gives a clear picture on the expected hierarchy in higher orders. At $\mathscr{O}(\alpha_s\alpha)$, the bulk of corrections will be contained in the combination of QCD corrections to the production and EW corrections to the decay processes, with a particular enhancement induced by final-state radiation off the charged leptons. The completion of our calculation of $\mathscr{O}(\alpha_s\alpha)$ corrections by the factorizable parts is in progress.

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