

Two-loop applications of the program SecDec

Sophia Borowka*

Max Planck Institute for Physics, Föhringer Ring 6, 80805 Munich, Germany

E-mail: sborowka@mpp.mpg.de

Gudrun Heinrich

Max Planck Institute for Physics, Föhringer Ring 6, 80805 Munich, Germany

E-mail: gudrun@mpp.mpg.de

SECDEC is a program which can be used for the factorisation of poles and subsequent numerical evaluation of multi-loop integrals, in particular massive two-loop integrals. We show applications to two-loop master integrals entering the calculation of top quark pair production at NNLO, and to the dominant momentum dependent two-loop corrections to the neutral Higgs boson masses in the MSSM.

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*Speaker.

1. Introduction

Precision measurements are of primary importance in order to find deviations from the Standard Model and to identify their origin as “New Physics”. Certainly, this is only possible in conjunction with precise theory predictions, requiring the calculation of higher order corrections. While the calculation of corrections at next-to-leading order has reached an impressive level of automation meanwhile, corrections beyond one loop still require quite some efforts both on the conceptual and on the technical side before they can be produced in a largely automated way.

An important ingredient for the calculation of higher order corrections are the loop integrals entering the virtual corrections. Analytical expressions for integrals beyond one loop are only known for integrals depending on rather few mass scales. Therefore, as soon as several mass scales are involved, numerical methods to calculate these integrals are often indispensable. However, the latter can only be applied after extraction of possible ultraviolet and/or infrared poles contained in the diagrams.

The program SECDEC [1–3], performs the task of isolating dimensionally regulated singularities in an automated way, based on the algorithm of sector decomposition [4–6]. Other implementations of sector decomposition into public programs are also available [7–9]. However, the latter are more or less restricted to the Euclidean region, while SECDEC can deal with physical kinematics, including for instance mass thresholds.

In these proceedings, applications of the program SECDEC 2.1 to massive two-loop integrals will be presented.

2. Structure of the program

The program can be divided into two main branches, one for the computation of loop integrals, the other one for the treatment of more general parametric functions (corresponding to the directories *loop* and *general*). A flowchart of the program is depicted in Fig. 1. For a detailed description of the program we refer to [1–3], here we will only mention the main aspects, and describe features not highlighted previously.

The *loop* part has been extended in version 2.1 to be able to treat parametric integrals which are not in the canonical form as obtained directly from standard Feynman parametrisation. The integrals can have a different format, coming for example from variable transformations and/or analytical integrations over some of the Feynman parameters. Hence, as these functions differ from the standard representation which the program would derive from the propagators in an automated way, they have to be defined in an input file by the user. Contour deformation is available for these functions, as they are assumed to originate from a Feynman integral structure, where poles on the real axis are protected by the infinitesimal $i\delta$ prescription.

The setup in the directory *general* is designed to deal with even more general parametric functions, where the integrand can consist of a product of arbitrary length of polynomial functions to some power. However, these functions should have only endpoint singularities (i.e. dimensionally regulated singularities at the integration boundaries). Contour deformation is not available in this case because the correct sign of the imaginary part for the deformation into the complex plane cannot be inferred if the assumption of an underlying Feynman integral structure is dropped.

The procedure of iterated sector decomposition and subsequent numerical integration is the same for all the different types of input functions, and is described in [1, 2, 10].

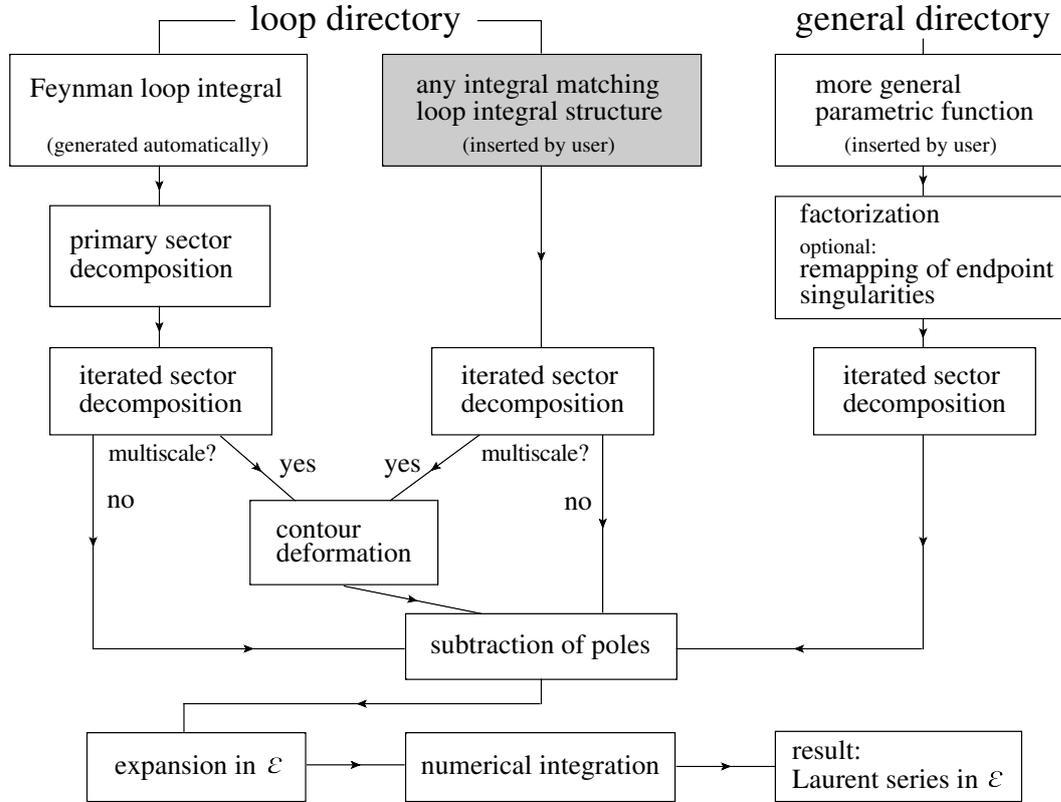


Figure 1: Flowchart showing the structure of the program SECDEC.

The current version 2.1.4 of the program can be downloaded from

<http://projects.hepforge.org/secdec>.

Unpacking the tar archive will create a directory called `SecDec-2.1.4`. Changing to this directory and running `./install` will compile the CUBA library [11] needed for the numerical integration. Prerequisites are Mathematica (version ≥ 6), Perl (installed by default on most Unix/Linux systems) and a C++ compiler, respectively a Fortran compiler if the Fortran option is used.

More details about the usage, in particular about the option `userdefined`, can be found in Refs. [1, 3], and also in the documentation coming with the program.

3. Applications

3.1 Non-planar massive two-loop diagrams entering NNLO $t\bar{t}$ production

The most complicated master topologies occurring in the two-loop corrections to $t\bar{t}$ production in the gg channel are the non-planar seven-propagator integrals shown in Fig. 2.

Analytic results for the integral containing a sub-diagram with a massive loop (called $ggt1$ here), are not available, while analytic results for the diagram corresponding to massless fermionic corrections in a sub-loop (called $ggt2$ here) have become available very recently [12]. However,

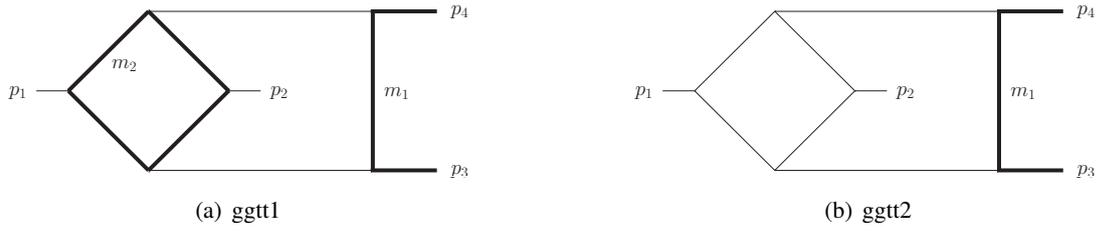


Figure 2: Massive non-planar two-loop box diagrams entering the heavy (a) and light (b) fermionic correction to the $gg \rightarrow t\bar{t}$ channel; the bold lines denote massive particles.

the numerical evaluation of $gg1$ with SECDEC is much easier than the one of $gg2$, due to its less complicated infrared singularity structure. While the leading poles of $gg2$ are of order $\mathcal{O}(\epsilon^{-4})$, and intermediate expressions during sector decomposition contain (spurious) poles where the degree of divergence is higher than logarithmic, the integral $gg1$ is finite and free from spurious singularities. Therefore we can evaluate $gg1$ with SECDEC 2.1 using the fully automated setup. In contrast, for $gg2$ it turned out to be advantageous to make some analytical manipulations beforehand. In particular, it was useful to perform one parameter integration analytically before feeding the integral into the decomposition and numerical integration algorithm. Further, we introduced special transformations to reduce the occurrence of spurious singularities, which are described in detail in [3, 13]. These manipulations lead to functions which were not in the “standard form” of Feynman parameterised loop integrals anymore. This entailed the development of a setup to treat “non-standard” parametric functions mentioned in Section 2, which has been made available for the user, as it can be beneficial in similar contexts.

Numerical results for the $gg2$ diagram are shown in Fig. 3, where we used the numerical values $p_3^2 = p_4^2 = m^2 = 1, s_{23} = -1.25, s_{13} = 2m^2 - s_{12} - s_{23}$, and we extract an overall factor of $-16 \Gamma(1 + \epsilon)^2$. We only show results for the finite part here, as it is the most complicated one and therefore more interesting than the pole coefficients.

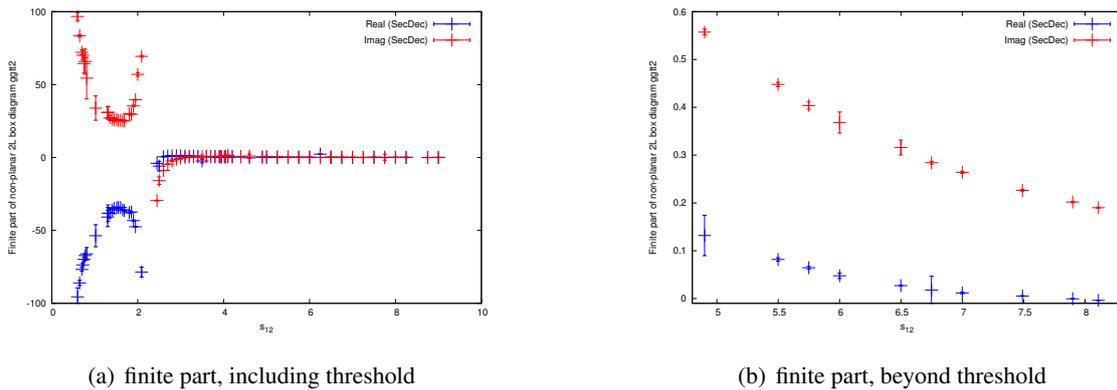


Figure 3: Results for the finite part of the scalar integral $gg2$, (a) for a larger kinematic range, (b) zoom into a region further away from threshold. The vertical bars denote the numerical integration errors.

Numerical results for the diagram $gg1$ are shown in Fig. 4 for both the scalar integral and an irreducible rank two tensor integral.

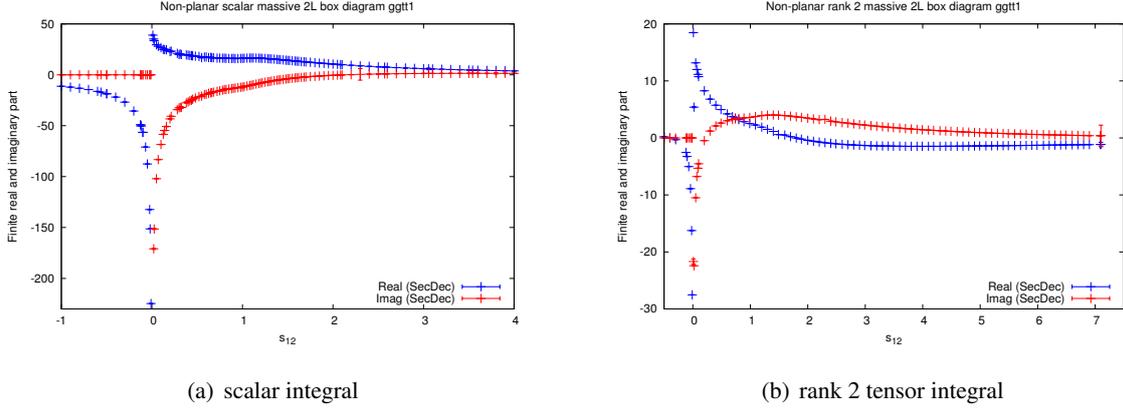


Figure 4: Results for the scalar integral $ggT1$ and the corresponding rank two tensor integral with $k_1 \cdot k_2$ in the numerator. We vary s_{12} and fix $s_{23} = -1.25, m_2 = m_1, p_3^2 = p_4^2 = m_1^2 = 1$.

For the results shown in Fig. 4 we used the numerical values $m_1^2 = m_2^2 = m^2 = 1, s_{23} = -1.25, s_{13} = 2m^2 - s_{12} - s_{23}$. We set $m_1^2 = m_2^2$ because this is the only case occurring in the process $gg \rightarrow t\bar{t}$ at two loops if the b -quarks are assumed to be massless. However, we also verified that the case $m_1^2 \neq m_2^2$ can be evaluated without a significant increase in computing time compared to the equal mass case.

The timings for one kinematic point for the scalar integral in Fig. 4(a) range from 11-60 secs for points far from threshold to 1.6×10^3 seconds for a point very close to threshold, with an average of about 500 secs for points in the vicinity of the threshold. A relative accuracy of 10^{-3} has been required for the numerical integration, while the absolute accuracy has been set to 10^{-5} . For the tensor integral, the timings are better than in the scalar case, as the numerator function present in this case smoothes out the singularity structure. The timings were obtained on a single machine using Intel i7 processors and 8 cores.

3.2 Corrections to the neutral CP even Higgs bosons in the MSSM

The Higgs sector of the MSSM consists of two doublets H_1 and H_2 , which can be written as

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1^0 + i\chi_1^0) \\ -\phi_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2^0 + i\chi_2^0) \end{pmatrix}. \quad (3.1)$$

The vacuum expectation values v_1 and v_2 define the angle $\tan\beta = v_2/v_1$. At tree level, the mass matrix of the neutral CP-even Higgs bosons in the ϕ_1, ϕ_2 basis can be written as

$$M_{\text{Higgs}}^{2,\text{tree}} = \begin{pmatrix} M_A^2 \sin^2 \beta + M_Z^2 \cos^2 \beta & -(M_A^2 + M_Z^2) \sin \beta \cos \beta \\ -(M_A^2 + M_Z^2) \sin \beta \cos \beta & M_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta \end{pmatrix},$$

where M_A is the mass of the CP-odd neutral Higgs boson A . The rotation to the basis formed by the Higgs bosons H^0, h^0 is given by

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}, \quad (3.2)$$

where

$$\tan(2\alpha) = \tan(2\beta) \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}, \quad -\frac{\pi}{2} < \alpha < 0. \quad (3.3)$$

The need for higher order corrections to the MSSM Higgs boson masses is obvious from the fact that at tree level, the mass of light CP-even Higgs boson is bound from above through the relation $m_h \leq \min(M_Z, M_A) |\cos(2\beta)|$.

The status of currently available two-loop self-energy corrections to the Higgs boson masses in the MSSM with real parameters (rMSSM) is the following. The corrections $\mathcal{O}(\alpha_s \alpha_t)$ [14–19], $\mathcal{O}(\alpha_t^2)$ [14, 20, 21], $\mathcal{O}(\alpha_s \alpha_b)$ [22], $\mathcal{O}(\alpha_t \alpha_b)$ [23], $\mathcal{O}(\alpha_b^2)$ [23] are known in the gaugeless limit (i.e. vanishing gauge couplings to the SM vector bosons), and using the approximation $p^2 = 0$ for the external momentum of the self-energies. The strong coupling constant is denoted by α_s as usual, while $\alpha_{\{t,b\}}$ denote the Yukawa couplings of the top (bottom) quarks, where $\alpha_{\{t,b\}} = y_{\{t,b\}}^2 / (4\pi)$. At three loops, the $\mathcal{O}(\alpha_s^2 \alpha_t)$ corrections are known [24–26] in the gaugeless limit and in the $p^2 = 0$ approximation. The largest uncertainties on the presently known results stem from the contributions of the momentum dependent self-energy corrections at two loop order. The latter have been calculated in [27, 28] based on an effective potential approach within the \overline{DR} scheme. However, in order to incorporate the corrections into the public program `FeynHiggs` [29, 30], it is more convenient to use on-shell renormalisation for all the masses. To this aim, we computed the self-energies $\Sigma_{\phi_i \phi_j}(p^2)$ at two loops at $\mathcal{O}(\alpha_s \alpha_t)$, keeping the full momentum dependence. We use the \overline{DR} scheme for the field renormalisation, and on-shell renormalisation for all other renormalisation constants, as explained e.g. in [17].

The reduction of the amplitude to scalar master integrals is performed with the help of the package `TwoCalc` [31] for the two-loop diagrams, while the one-loop counter term diagrams are reduced using `FormCalc` [32]. The two-loop topologies for which a full analytical result is not available, shown in Fig.5, are calculated numerically with `SECDEC`. In total, these amount

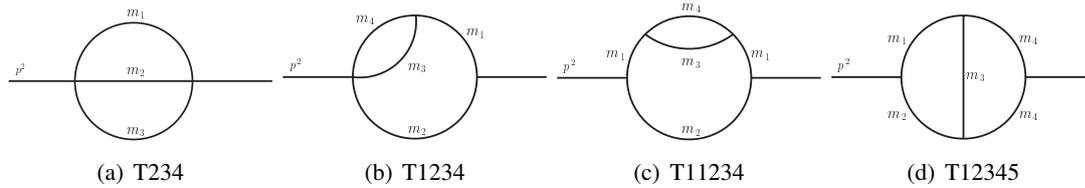


Figure 5: Two-loop two-point topologies treated with `SECDEC` with up to four different mass scales

to 37 different mass configurations, where the two-loop diagrams can have up to four different masses. We tested the numerical integration with `SECDEC` for integrals where the kinematic values differed by up to 14 orders of magnitude. The timings range between 0.01 secs and 100 secs with an achieved relative accuracy of 10^{-5} to 10^{-11} . Two representative results are shown in Fig. 6. The configuration in Fig. 6(a) has three different mass scales where we set $m_1 = m_2 = 173.2$ GeV, $m_3 = 826.8$ GeV and $m_4 = 1.5$ TeV. In Fig. 6(b), the numerical values $m_1 = 1173.2$ GeV, $m_2 = 826.8$ GeV, $m_3 = 1.5$ TeV and $m_4 = 173.2$ GeV are chosen.

More detailed results will be given elsewhere [33].

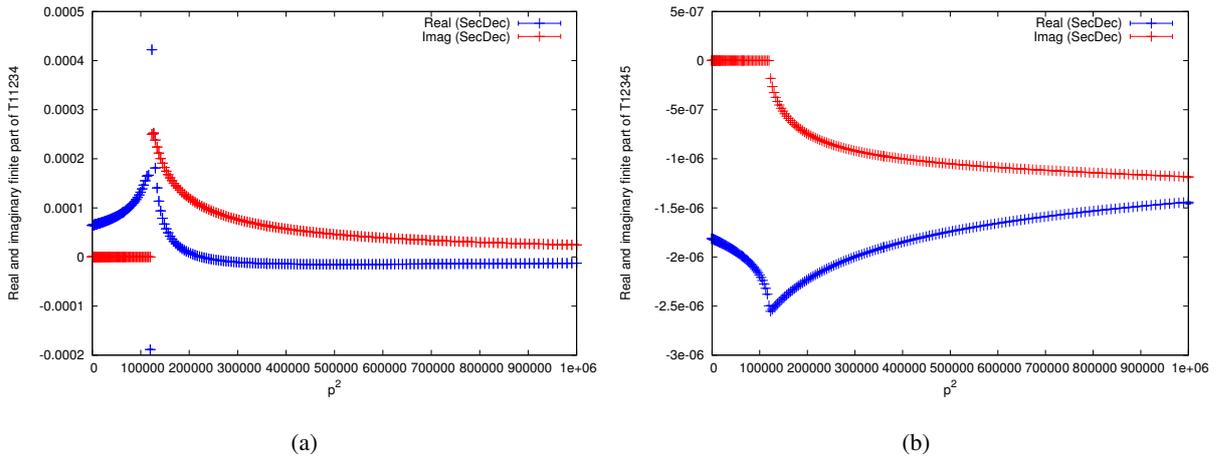


Figure 6: Numerical SECDEC results for the integrals of type (c) and (d) as defined in Fig. 5.

4. Conclusions

We have presented new features and applications of the program SECDEC, which can be used to calculate multi-loop integrals numerically in an automated way. Applications to non-planar two-loop master integrals occurring in $t\bar{t}$ production are shown, as well as the calculation of momentum dependent two-loop corrections to the masses of the neutral CP-even Higgs bosons in the MSSM. The program SECDEC is publicly available at <http://projects.hepforge.org/secdec>.

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