

# Application of the spatial efficiency $\varepsilon(\vec{r})$ of a HPGe detector to determine the specific activity of radioactive material in cylindrical extended sources

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In the present work a methodology to determine of the specific activity (intrinsic spatial efficiency methodology) for a high-resolution gamma spectroscopy system is presented. In order to determine the specific activity, mathematical expressions for the intrinsic spatial efficiency and the absolute efficiency for a cylindrical volume sample were obtained from basic concepts of solid angle and gamma attenuation.

This methodology allows us to determine the specific activity of radioactive species present in homogeneous volume samples, such as soil, water and construction materials, without the direct use of reference material for quantification of radiation levels. Reference materials were used to validate the method.

The advantage of this method is that allows specific activity determination without the need of making matrix effect corrections (shape and size of the sample, sample density, self-attenuation), which are the principal error sources in this type of measurements.

*X Latin American Symposium on Nuclear Physics and Applications (X LASNPA),  
1-6 December 2013  
Montevideo, Uruguay*

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†This work was supported in part by Center for Experimental Physics (CEFEX), University of Chile.

## 1. Introduction

Gamma spectrometry is a widely used technique for the determination and quantification of radionuclides, natural and artificial, in environmental samples [1, 2, 3]. In this type of analysis the samples are voluminous due to the low concentration of radioactive material.

In order to determine the specific activity the knowledge of the absolute efficiency of the spectroscopy system is required. This is accomplished using reference materials in the same experimental conditions (matrix, size and shape) as the samples [1, 2, 3].

The specific activity of the sample is determined by the equation

$$A_s = \frac{N}{\varepsilon \Delta t m I_\gamma} f_1 f_2 f_3 \dots \quad (1.1)$$

Where  $N$  represents the Full Energy Peak (FEP) area,  $\varepsilon$  is the absolute efficiency for the volume source setting,  $\Delta t$  is the counting time,  $m$  the mass of the sample and  $I_\gamma$  is the  $\gamma$  emission probability. The terms  $f_i$  are correction factors due to electronic (true coincidence, pile-up and dead time), dry mass, waiting or cooling time, decays at the moment of measurement, self-attenuation, shape and size.

## 2. Theory

### 2.1 Detection Full Energy Peak (FEP) Efficiency

In general there are two efficiency types [4]: The absolute efficiency defined as,

$$\varepsilon_{abs} = \frac{\{\text{N}^\circ \text{ of photons detected in photopeak}\}}{\{\text{N}^\circ \text{ of photons emitted by source}\}}, \quad (2.1)$$

which depends on detector characteristics and the material between source to detector. And the intrinsic efficiency,

$$\varepsilon_{int} = \frac{\{\text{N}^\circ \text{ of photons detected in photopeak}\}}{\{\text{N}^\circ \text{ of photons incident in the detector}\}}, \quad (2.2)$$

which depend only on detector physics and chemical characteristics. Both efficiencies are related by,

$$\varepsilon_{abs} = \varepsilon_{int} f, \quad (2.3)$$

where  $f$  represents the fraction of photons reaching the detector and depends on the source-detector position and the attenuation of photons.

$$f = \int_A \frac{\cos(\alpha)}{4\pi d^2} \prod_i e^{-\mu_i d_{att,i}} dA, \quad (2.4)$$

where  $d$  is the distance from source to detector active zone,  $\mu_i$  are the attenuation coefficients of the materials between the source and the detector active zone, and finally  $d_{att,i}$  are the distances traveled by the photons in those materials.

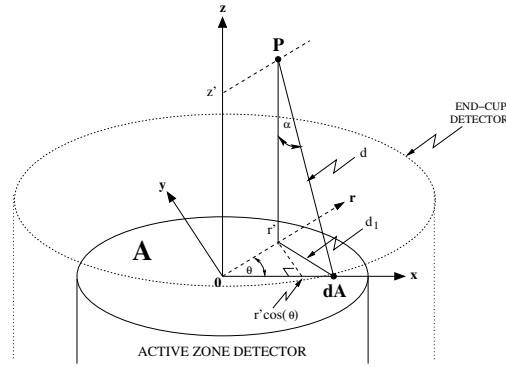


Figure 1: Possible trajectory of a photon emitted by a point source, P, impacting on an elemental area,  $dA$ , of the front face active zone of the detector, of area A.

## 2.2 Intrinsic FEP spatial efficiency

In Figure 1 we can see a possible trajectory for the photons, emitted by a point source, that impact on the front face active zone of the detector. If the medium between the source and end-cup is air, only the solid angle is important because the  $\gamma$  attenuation in air is negligible.

It has been shown that the intrinsic FEP efficiency depends on the source-detector position [5, 6, 7]. Considering this condition and previous definitions, (3) to (5), the intrinsic spatial efficiency is given by,

$$\varepsilon_{int}(r', z') = \frac{N_{det}(r', z')}{n_{tot} f_{\Omega}(r', z')}. \quad (2.5)$$

Where  $f_{\Omega}$  corresponds to eq. (2.4) with  $\mu = 0$ , i.e, the solid angle fraction. It is given by,

$$f_{\Omega} = \int_0^{R_d} \int_0^{2\pi} \frac{z' r_d d\theta dr_d}{4\pi (r_d^2 - 2r_d r' \cos(\theta) + r'^2 + z'^2)^{3/2}}. \quad (2.6)$$

Here, we are not considering attenuations in the internal region of the detector, i.e, between end-cup and active zone of the detector.

## 2.3 Absolute FEP efficiency for a volume source

Now, In Figure 2, we can see the same situation but with the source embedded in a medium of attenuation coefficient  $\mu$ .

The absolute efficiency for a source, radioactively homogeneous, of volume  $V'$  and attenuation coefficient  $\mu$  is given by,

$$\varepsilon_{\mu, V'} = \frac{\int_{V'} \varepsilon_{int}(r', z') f_{\Omega, \mu}(r', z') dV'}{V'}. \quad (2.7)$$

From eq. (2.3) we can see that the term inside the integral corresponds to the absolute FEP spatial efficiency for a point source embedded in a medium of attenuation coefficient  $\mu$  instead of air,

$$\varepsilon_{abs, \mu}(r', z') = \varepsilon_{int}(r', z') f_{\Omega, \mu}(r', z'). \quad (2.8)$$

Therefore eq. (2.7) corresponds to average, on the volume of the volume source, of the absolute FEP spatial efficiency in a medium of attenuation coefficient equal to the source.

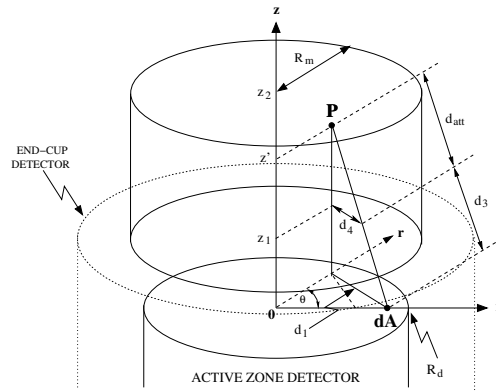


Figure 2: A point source, P, embedded in a cylindrical voluminous sample of attenuation coefficient  $\mu$ , irradiating a detector. The distance traveled under the influence of attenuation is  $d_{att}$ .

In eq. (2.8),  $f_{\Omega,\mu}$  is determined by eq. (2.4). If the volume source is a cylinder, from Figure 2,

$$f_{\Omega,\mu}(r',z') = \int_0^{R_d} \int_0^{2\pi} \frac{z' e^{-\mu(z'-z_1)} \sqrt{1+(1/z')^2 (r_d^2 - 2r_d r' \cos(\theta) + r'^2)}}{4\pi (r_d^2 - 2r_d r' \cos(\theta) + r'^2 + z'^2)^{3/2}} r_d dr_d d\theta. \quad (2.9)$$

This expression is valid if the cylindrical source radius,  $R_m$ , is greater or equal than the detector radius,  $R_d$ .

## 2.4 Specific activity of a volume source: Intrinsic spatial efficiency methodology

In analogy with eq. (1.1) the specific activity is given by,

$$A_s = \frac{N}{\epsilon_{\mu,\nu} \Delta t m I_\gamma} f_1 f_2 f_3 \dots \quad (2.10)$$

The methodology presented here permits evaluation of specific activity without corrections for self-attenuation, shape and size.

## 3. Experimental

### 3.1 Spectroscopy system

The spectroscopy system is composed by a coaxial HPGe detector Ortec GEM-10195, standard electronic units, MCA Nucleus and the software PCA-II.

### 3.2 Determination of intrinsic spatial efficiency

The determination of the spatial efficiency was performed by mapping the FEP area obtained with a radioactive source, of known activity, on the  $r - z$  plane, such as is shown in Figure 3. Then, using the solid angle and the total number of  $\gamma$  rays emitted by the source the intrinsic spatial efficiency was determined, eq. (2.5).

The radioactive  $^{137}\text{Cs}$  source was planar with a circular active zone of 3.5 mm of diameter. The source position was taken in its geometric centre.

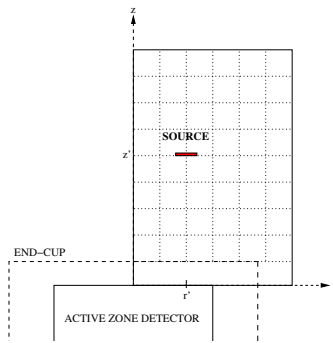


Figure 3: This figure shown a grid with the source positions to determine the spatial efficiency.

Measurements in other angular positions are not necessary due to the axial symmetry of the system. This is true, normally, for radial positions less than the detector radius.

Next, the FEP area mapping on  $r - z$  plane and the intrinsic FEP spatial efficiency of the detector are shown in Figure 4 and Figure 5, respectively.

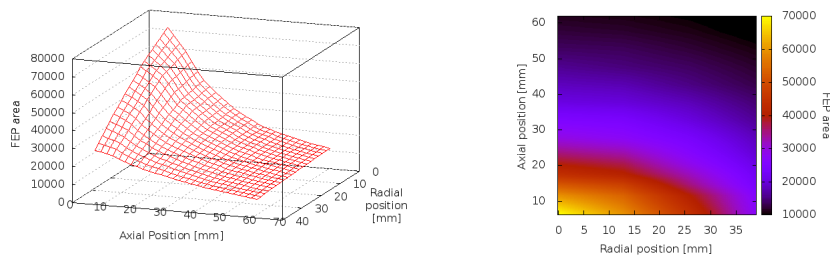


Figure 4: The FEP area mapped on the  $r - z$  plane obtained with a source of  $^{137}\text{Cs}$ . A 3-D graph and 2-D map are shown.

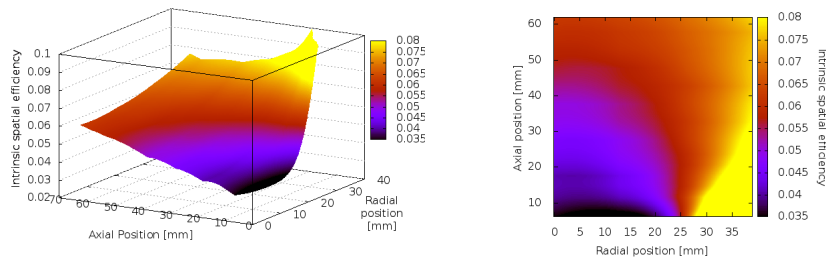


Figure 5: The intrinsic FEP spatial efficiency mapped on the  $r - z$  plane. A 3-D graph and 2-D map are shown.

### 3.3 Determination of the specific activity

This method was tested using matrix reference materials, obtained from International Atomic

Energy Agency (IAEA), whose characteristics are shown in the Table 1.

Table 1: Specific activity (dry mass) reported for the matrix reference materials.

Characteristics of reference materials		
Matrix	Title	Specific activity $^{137}\text{Cs}$ [Bq/kg]
Moss-Soil	IAEA-447 [8]	$425 \pm 10$
Soil	sample 1 [9]	$52.6 \pm 1.1$
Sediment lake	IAEA-SL2 [10]	$2.4 \pm 0.3$
Liquid	sample 3 [9]	$16.72 \pm 0.1$
Grass	IAEA-372 [11]	$11320 \pm 360$

The FEP area, for  $^{137}\text{Cs}$  (661,7 keV), was obtained for each reference material. The experimental set-up is shown in Figure 6.

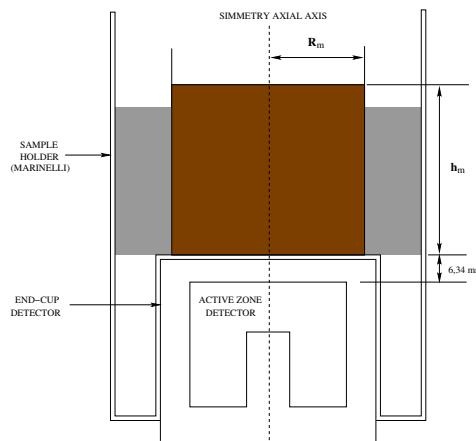


Figure 6: Experimental setting to determine the specific activity. We can observe a cylindrical source, of radius  $R_m$  and high  $h_m$ , coaxial with the detector, a marinelli beaker has been used as sampler holder. The closest distance between the volume source and the active zone of the detector are shown.

With the intrinsic spatial efficiency and the geometric factor, eq. (2.9), the absolute spatial efficiency was determined using eq. (2.8). It is necessary to know the attenuation coefficient, this was determined by mean the transmission method.

Then, by numerical integration of eq. (2.7), the absolute efficiency for the setting was computed and finally, by eq. (2.10), the specific activity was determined.

#### 4. Results

In Table 2 the results obtained for the specific activity of the matrix reference materials are presented.

These results show a good agreement between the reported activities of the reference material and our measurement.

Table 2: In this table the informed dry mass specific activity,  $A_{s,\text{ref}}$ , the dry mass specific activity obtained in this work,  $A_{s,\text{our}}$ , and the percentage relative difference are shown.

	Results				
	Reference Material				
	Sediment lake IAEA-SL2	Moss-Soil IAEA-447	Soil sample 1	Grass IAEA-372	Water sample 3
$A_{s,\text{ref}}$ [Bq/kg]	$2.4\pm 0,3$	$425\pm 10$	$52.6\pm 1.1$	$11320\pm 185$	$16.7\pm 0.1$
$A_{s,\text{our}}$ [Bq/kg]	$2.5\pm 0.6$	$398\pm 9$	$50.2\pm 4.1$	$10146\pm 111$	$18.6\pm 3.8$
$\Delta\%$	5	6.6	4.6	10.1	11.2

## 5. Conclusion

The methodology presented here has been successful applied to for the determination of the specific activity of  $^{137}\text{Cs}$  for an homogeneous cylindrical volume source and for different matrix materials. This can be applied in the case of environmental samples.

A general expression, in terms of the shape, size and matrix source, for the determination of the absolute efficiency for a radioactively homogeneous volume source was obtained (eq. (2.7)).

## 6. Acknowledgement

The partial support of the Center for Experimental Physics, CEFEX, Faculty of Sciences, University of Chile and the Master Degree scholarship of Department of Physics, Faculty of Sciences, University of Chile are acknowledged.

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