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Complete fusion of weakly bound nuclei

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Existing Quantum Mechanics methods to study fusion reactions with weakly bound nuclei cannot evaluate separately the different contributions to the complete fusion cross section. We develop a semiclassical procedure that can calculate this cross section, apply it to ^{6}Li + ^{209}Bi collisions at energies just above the barrier and show that its prediction for the complete fusion cross section is in very good agreement with the data. We find that the contribution from the sequential fusion of the ^{6}Li fragments to the complete fusion cross section is substantial, reaching almost 40% of that from the direct process, which illustrates the importance of calculating correctly the different components of the complete fusion cross section.

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Interest in the study of collisions of weakly bound nuclei increased enormously over the last decade [Canto(2006)]. In such collisions, the breakup cross section tends to be very large and breakup couplings may have a strong influence on the cross sections for several other channels. An important example is the fusion process, which in this case becomes much more complex, as, in addition to the usual fusion reaction, in which the whole projectile merges with the target to form the compound nucleus, there are other fusion processes following the breakup of the weakly bound collision partner. There is the possibility that one or more, but not all, fragments are absorbed by the target whereas part of the projectile's mass escapes the interaction region. In this case we call the process *incomplete fusion* (ICF) whereas the fusion of all projectile's nucleons with the target is called *complete fusion* (CF). The CF cross section is the sum of the cross section for the direct fusion of the projectile with the target (DCF) and of the sequential fusion of all of the projectile's fragments (SCF).

Most experiments measure only the total fusion (TF) cross section, which is the sum of the cross sections for CF and ICF. However, for some particular projectile-target combinations, it is possible to perform separate measurements of the cross sections for CF and ICF. Important examples are the fusion reactions ^{6,7}Li+^{209}Bi [Dasgupta(2002), Dasgupta(2004)] and ^{9}Be + ^{208}Pb [Dasgupta(1999), Dasgupta(2004)], where the influence of the breakup channel on fusion was shown to be very strong. The ICF cross section, however, cannot be separated from the contribution from transfer processes leading to the same final nuclei. For this reason we concentrate, in the present work, on the CF process and how it is affected by the breakup channel.

Many theoretical approaches have been proposed to study fusion reactions with weakly bound nuclei (for a review see Ref. [Canto(2006)]), ranging from simple classical models [Hagino(2004), Diaz-Torres(2011)] to full quantum mechanical calculations [Hagino(2000), Diaz-Torres(2002), Diaz-Torres(2003), Keeley(2001)], using the Continuum Discretized Coupled Channel method (CDCC). In most CDCC calculations fusion is included by means of short-range imaginary potentials acting on each fragment. In this way, there is no correlation between absorptions of different fragments. Thus, one cannot know if the absorption by one of these potentials contributes to ICF or to SCF. Consequently, the calculation gives only the summed cross section for these processes, $\sigma_{\rm TF}$ [Keeley(1996)].

In the present work, we introduce a semiclassical method to evaluate both components (DCF and SCF) of the CF cross sections in collisions of weakly bound nuclei. Our method, which has been successfully applied to breakup reactions [Marta(2008)], consists of treating the projectile-target relative motion by classical mechanics while the intrinsic dynamics of the weakly bound projectile is handled by quantum mechanics.

We consider the reaction as a two step process. In the first part the breakup of the weakly bound projectile is described by the semiclassical procedure of Refs. [Marta(2002), Marta(2008)], which we summarise in what follows. We take the weakly bound projectile as consisting of two clusters, c_1 and c_2, moving around the projectile's center of mass. The collision dynamics is described by two vectors: $\mathbf{R}$, joining the centers of mass of the projectile and the target, and $\mathbf{r}$, joining the centers of c_1 and c_2. As the collision proceeds, the projectile-target interaction couples the intrinsic states of the system. In this way, the projectile, which is initially in its ground state, may suffer transitions to excited bound states, if any, and to continuum states. For a collision with given energy, E, and impact parameter, b, one determines a trajectory by classical mechanics and uses this trajectory to transform $\mathbf{R}$-dependence into time-dependence. The intrinsic dynamics is then treated as a time-dependent Quantum Mechanics problem.

Analogously to Refs. [Nunes(1998), Nunes(1999)], the interaction is given in terms of the fragment-target vectors,

$$\mathbf{r}_1 = \mathbf{R} + \frac{A_2}{A_\mathrm{P}}\mathbf{r}, \quad \mathbf{r}_2 = \mathbf{R} - \frac{A_1}{A_\mathrm{P}}\mathbf{r},$$

by the expression

$$V(\mathbf{R},\mathbf{r}) = V_1\left(\mathbf{R} + \frac{A_2}{A_\mathrm{P}}\mathbf{r}\right) + V_2\left(\mathbf{R} - \frac{A_1}{A_\mathrm{P}}\mathbf{r}\right), \tag{1}$$

where V_1 (V_2) is the interaction between c_1 (c_2) and the target.

Above, A_1 and A_2 are the mass numbers of c_1 and c_2, and $A_\mathrm{P} = A_1 + A_2$ is the mass number of the projectile. The potentials V_1 and V_2 contain nuclear and Coulomb parts. For the semiclassical calculation, as in Ref. [Marta(2008)], the interaction is split into an optical potential, V_0, which real part only affects the classical trajectory of the projectile-target system, while its imaginary part represent absorption from other channels, and a coupling interaction, $U(\mathbf{R},\mathbf{r})$, which leads to breakup.

$$V_0(\mathbf{R}) = V(\mathbf{R},\mathbf{r}=0) = V_1(\mathbf{R}) + V_2(\mathbf{R}), \tag{2}$$

and

$$U(\mathbf{R},\mathbf{r}) = V(\mathbf{R},\mathbf{r}) - \mathrm{Re}\{V_0(\mathbf{R})\}. \tag{3}$$

The derivation of the semiclassical coupled-channel equations was done in [Marta(2008)], where also the procedure for the discretisation of the continuum was described in detail. The study of the breakup follows the same procedure as in that work. In the present one we study the evolution of the system after the breakup took place, in particular the eventual fusion of one or both of the projectile fragments with the target nucleus.

To do this, as we have the breakup amplitudes $c_\alpha(b,t)$ along the projectile trajectory, we could in principle consider, at each point along that trajectory, that the two clusters appear with the velocities associated to the continuum state α and probabilities $|c_\alpha(b,t)|^2$. These are the initial conditions needed to determine whether those clusters fuse with the target nucleus.

Although the above procedure is feasible, it would require the evaluation of a very large number of fusion probabilities. In order to decrease the computational effort of the calculation, we have resorted to the following approximation. We calculate the breakup amplitudes until the classical trajectory reaches the point of closest approach, or the projectile-target distance becomes smaller than the radius of the effective barrier for the impact parameter b. We then consider that, when breakup takes place, the fragments are created at the point of closest approach of the projectile's classical trajectory. This assumption is reasonable, since the breakup probability distribution is strongly peaked in the region around the point of closest approach.

The direct complete fusion cross section is calculated as

$$\sigma_{\mathrm{DCF}} = \frac{\pi}{K^2}\sum_L (2L+1)\left(1 - P_L^{\mathrm{bup}}\right) T_L^{(\mathrm{P})}(K), \tag{4}$$

where the factor P_L^{bup} is the breakup probability, and $T_L^{(\mathrm{P})}(K)$ is the probability that the projectile fuses with the target when having momentum $\hbar K$ in the partial wave L. The fusion probabilities are approximated by the Hill-Wheeler formula.

We calculate the incomplete fusion cross section of fragment $c_i, \; i = 1,2$ by means of the expression

$$\sigma_{\mathrm{ICF}_i} = \frac{\pi}{K^2}\sum_L (2L+1)\int d^3\mathbf{k}\left|A^L(\mathbf{k})\right|^2 P_{\mathrm{F_i}}(\mathbf{k}), \tag{5}$$

and the sequential complete fusion cross section is

$$\sigma_{\mathrm{SCF}} = \frac{\pi}{K^2}\sum_L (2L+1)\int d^3\mathbf{k}\left|A^L(\mathbf{k})\right|^2 P_{\mathrm{SCF}}(\mathbf{k}). \tag{6}$$

Above,

$$A^L(\mathbf{k}) = \sum_{\nu_1\nu_2} A_{\nu_1\nu_2}(\mathbf{k}, t_f, b) \tag{7}$$

is the relative momentum distribution of the $\mathrm{c}_1 - \mathrm{c}_2$ system at the instant of closest approach or when it enters the strong interaction region, t_f, and we denote by $\hbar K$ and $L = Kb$ the relative momentum and the orbital angular momentum in units of $\hbar$ of the projectile-target relative motion, respectively. In Eq. (7), $A_{\nu_1\nu_2}(\mathbf{k}, t, b)$ is the scalar product

$$A_{\nu_1\nu_2}(\mathbf{k}, t, b) = \langle \Psi^{(-)}_{\nu_1\nu_2}(\mathbf{k}, t) | \Psi_{\mathrm{C}}(b, t)\rangle,$$

where the wavefunction $\Psi_{\mathrm{C}}(b,t)$ is the component of the system's wavefunction in the continuum, and $\Psi^{(-)}_{\nu_1\nu_2}(\mathbf{k},t)$ is the scattering wave function with incoming wave boundary conditions.

In Eq. (5), $P_{F_i}(\mathbf{k})$ is the probability that only the fragment $\mathrm{c}_i (i = 1,2)$ fuses with the target, given by

$$P_{F_1}(\mathbf{k}) \;=\; T^{(c_1)}_{l_1}(E_1)\,\left(1 - T^{(c_2)}_{l_2}(E_2)\right), \tag{8}$$

$$P_{F_2}(\mathbf{k}) \;=\; T^{(c_2)}_{l_2}(E_2)\,\left(1 - T^{(c_1)}_{l_1}(E_1)\right), \tag{9}$$

In Eq. (6), $P_{\mathrm{SCF}}(\mathbf{k})$ is the probability that both fragments fuse with the target,

$$P_{\mathrm{SCF}}(\mathbf{k}) = T^{(c_1)}_{l_1}(E_1)\;\, T^{(c_2)}_{l_2}(E_2)\,. \tag{10}$$

As before, the fusion probabilities are estimated by means of the Hill-Wheeler formula.

Using the above procedure we have calculated the complete and incomplete fusion cross sections for the ^{6}Li-^{209}Bi system, for which these data are available. For the interaction between each fragment and the target we used the Christensen-Winther potential [Broglia(2004), Christensen(1976)] and the continuum distretization was performed with states with energies up to 7 MeV and angular momenta up to $3\hbar$.

In Fig. 1 (a) we show the result of our calculations for the complete and total (complete + incomplete) fusion cross sections for the ^{6}Li-^{209}Bi system. The results shown were obtained under the assumption that, when the fusion calculation begins, the fragments are placed at a distance $1.2\,(A_1 1/3 + A_1 1/3)$ between them. The orientation of the vectors joining their centers, **r** is given by their relative velocity. From these assumptions the initial conditions for the calculation of their fusion cross section with the target are obtained. We should remark that the calculation is not very sensitive to these initial conditions: we have verified that if we start the fusion cross section calculations assuming that the two fragments are emitted from the center of mass of the projectile the results do not change much. These calculations are compared with the experimental measurements of Ref. [Dasgupta(2004)]. The agreement with the data is quite good, which gives support to the appropriateness of the simplifying assumptions introduced in the calculation, which, we should note, contains no adjustable parameters.

A recent experiment of Luong *el al.* [Luong(2011)] has been able to distinguish prompt breakup from delayed breakup. However, there is no experiment that can distinguish direct complete fusion from sequential complete fusion. From what we have mentioned above, if the SCF cross section were negligible, standard CDCC calculations would be applicable to these systems. Thus the importance of assessing the sequential fusion contribution to the CF cross section. Since in the semiclassical approach presented here direct complete fusion and sequential complete can be separately calculated, we can compare the cross section for each of them. In Fig. 1 (b) we show the calculated DCF and SCF cross sections, together with that of the complete fusion (DCF+SCF). We note that the contribution of the sequential process to the CF cross section is over one third of that of the direct one.

In conclusion, we have developed a semiclassical calculation procedure to study the influence of the breakup process in the fusion reaction of a weakly bound projectile with a heavy target, applied it to ^{6}Li + ^{209}Bi collisions at near-barrier energies, and compared its predictions for the CF cross section with the data of Dasgupta et al. [Dasgupta(2002)]. Our model was shown to reproduce the data very well. Further, our results indicate a sizable contribution of the sequential fusion process to the total complete fusion cross section. Therefore, an improper consideration of this process may lead to inaccurate predictions of both the complete and incomplete fusion cross sections.

Our calculations indicate that the semiclassical method has the potential to give a complete and accurate picture of the processes occurring in collisions induced by weakly bound nuclei, and other similar systems. In particular they could be applied to study collisions between molecules, atomic clusters, and other objects for which the small de Broglie wavelength of the relative motion justifies the use of a classical trajectory, while the internal states of the colliding partners require a quantal description. We stress that quantum-mechanical CDCC calculations for systems like ^{6}Li+^{209}Bi cannot evaluate separate cross sections for CF and ICF [Keeley(1996)]. Thus, this seems to be an important strength of the semiclassical approach. A purely classical treatment [Hagino(2004), Diaz-Torres(2011)] is able to correctly distinguish between all of these processes, but lacks, however, the inclusion of quantum effects, such as tunneling, and, most importantly,

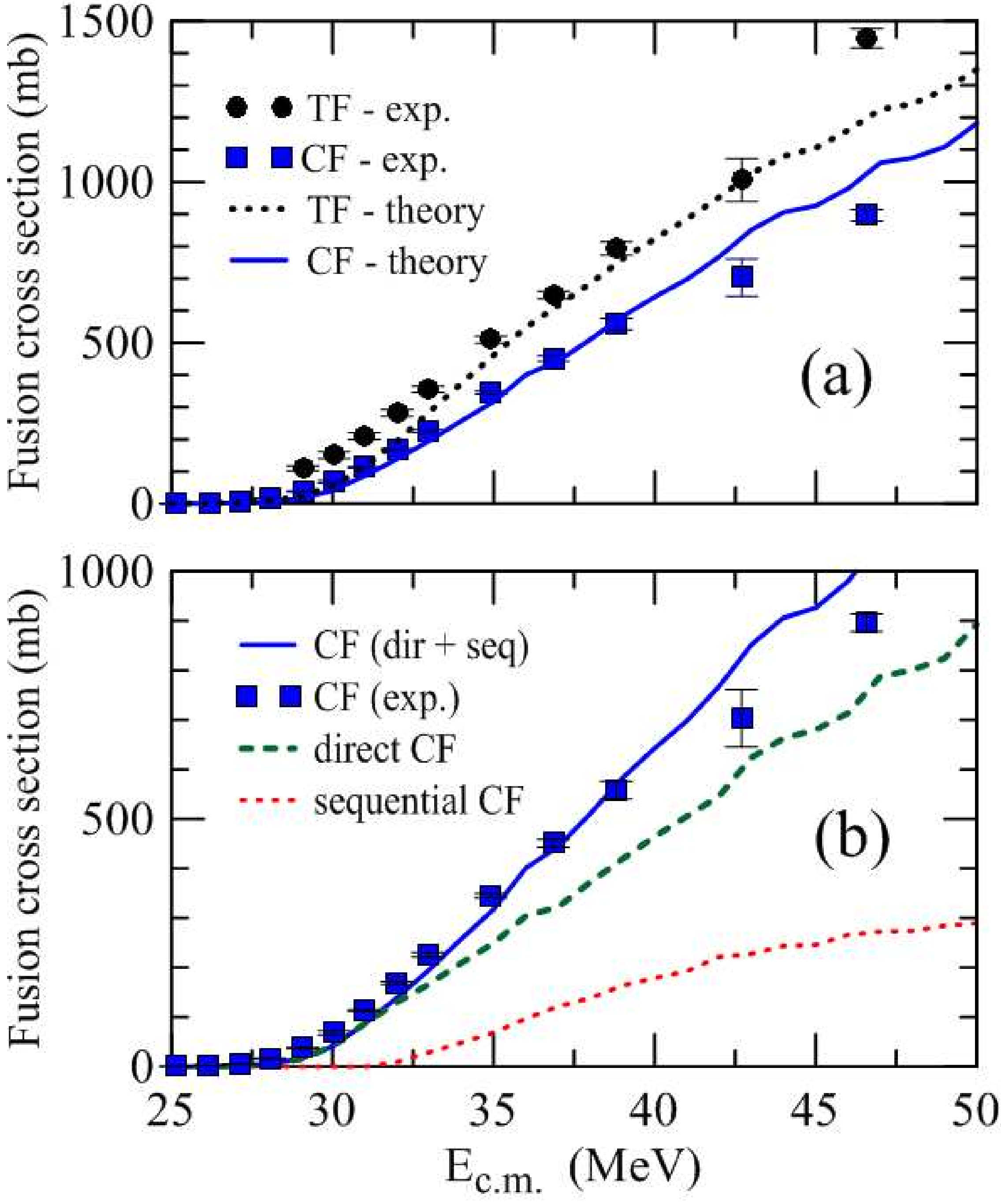

Figure 1: (Color online) Total fusion (TF) and complete fusion (CF) cross sections (a) and the components of the complete fusion cross section (b).

the quantum mechanical description of the excitation of the weakly bound projectile during the collision process.

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