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Asymptotic pion and kaon pair production in $\gamma\gamma$ collisions

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For the first time, to our knowledge, we have obtained analytical asymptotic formulas at Born level for the $\gamma\gamma$ production of lepton and pion pair and the production of pion and kaon pair with no approximation on particle masses. All these results are in agreement with computations using mass approximations. The computation method is the same as for previous computation of the production of two lepton pair with equal or unequal masses. We also present a Monte-Carlo based on the impact factor method, which gives a crude estimate of the production rate of this kind of events at the Photon Collider, LHC and the Future Linear Collider.

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1. Introduction

The total cross section of two identical lepton pair production at infinite energy in $\gamma\gamma$ collisions has been computed long time ago [1, 2, 4, 5]. The corresponding total and differential cross sections of different two pair produced has already been obtained with logarithmic approximation and including $\gamma\gamma$ polarisation [7]. Two types of diagrams contribute : peripheral and bremsstrahlung [9] . At the level of the total cross section, only the first ones give a non null constant value [5]. The main ingredients used in our present approach are the Factorisation Formula and the Helicity Amplitudes Computation [10]. The result is obtained in a closed form and details of interest are given.

1.1 The factorization formula

The differential cross-section corresponding to the peripheral contribution is given by [9] :

$$\frac{d\sigma}{dtdW^2dW'^2} = \frac{W^2W'^2}{8\pi^3s^2t^2}\left((1+ch^2\theta)\sigma_T\sigma'_T + sh^2\theta(\sigma_T\sigma'_L+\sigma_L\sigma'_T)+ch^2\theta\sigma_L\sigma'_L\right) \tag{1.1}$$

where s is the $\gamma\gamma$ invariant mass squared, W and W' the two pair masses, m and m' the two lepton masses and $-t$ the exchanged space-like photon squared mass. σ_T and σ_L are the transverse and longitudinal cross sections of virtual photoproduction [12] at one vertex :

$$\sigma_T = \frac{4\pi\alpha^2\beta W^2}{(W^2+t)^2}(\beta^2-2+2\frac{t}{W^2}-\frac{t^2}{W^4}+\frac{3-\beta^4+2t^2/W^4}{2\beta}\mathscr{L}) \tag{1.2}$$

$$\sigma_L = \frac{16\pi\alpha^2\beta t}{(W^2+t)^2}(1-\frac{1-\beta^2}{2\beta}\mathscr{L}), \quad \mathscr{L}=\ln(\frac{1+\beta}{1-\beta}), \quad \beta=\sqrt{1-\frac{4m^2}{W^2}} \tag{1.3}$$

and σ'_T, σ'_L the analogous ones at the other vertex. θ is the imaginary rotation angle between the two vertices planes [10] and $t_{min}(t_{max})$ the lower (upper) value of t. Since

$$sh^2\theta = \frac{4st(t-t_{min})(t_{max}-t)}{(W^2+t)^2(W'^2+t)^2} \tag{1.4}$$

we have for large center of mass energy : $sh^2\theta \simeq ch^2\theta$. In that case, the right hand side member of equation (1.1) gets a simple factorized expression :

$$\frac{d\sigma}{dtdW^2dW'^2} = \frac{W^2W'^2}{2\pi^3}\frac{(\sigma_T+\sigma_L)}{(W^2+t)^2}\frac{(\sigma'_T+\sigma'_L)}{(W'^2+t)^2} \tag{1.5}$$

2. Analytic formula for two pairs production at $\gamma\gamma$ infinite energy

2.1 Two lepton pairs production

After integrating over the invariant mass of each pair of different masses leptons, we obtain when the $\gamma\gamma$ invariant mass goes to infinity :

$$\sigma = \frac{8\alpha^4}{\pi}\int_0^\infty f(t,m)f(t,m')dt \tag{2.1}$$

where

$$f(t,m) = \int_{4m^2}^{\infty} \frac{W^2(\sigma_T+\sigma_L)}{4\pi\alpha^2(W^2+t)^2}\,dW^2$$
$$= \frac{1}{3}\int_{4m^2}^{\infty}\frac{dW^2}{W^2+t}\frac{d}{dW^2}\left(\beta+2L\right) = \frac{1}{3t}\int_0^1\left(1+\frac{4t-4m^2}{4m^2+t-t\beta^2}\right)d\beta \tag{2.2}$$
$$= \frac{1}{3t}\left(1+\frac{1}{2}v\left(5-\frac{1}{v^2}\right)\ln\left(\frac{1+v}{1-v}\right)\right),\; v=\sqrt{\frac{t}{t+4m^2}}$$

Making the following variable change :

$$t = \frac{4m^2(1-z)^2y^2}{(1-y^2)(1-z^2y^2)}, \quad z=\frac{m-m'}{m+m'}, \quad 0\le y\le 1 \tag{2.3}$$

we obtain an easier integrable expression where the product of logarithms has now disappeared and only logarithms and logarithms squared appear :

$$\sigma = \frac{8\alpha^4}{\pi}\int_0^1 c(y,z)g(y,z)g(y,-z)dy \tag{2.4}$$

with

$$c(y,z) = \frac{1}{18mm'(1-z^2)y^3}, \quad g(y,z)=a(y,z)+b(y,z)\ln\left(\frac{(1+y)(1-zy)}{(1-y)(1+zy)}\right) \tag{2.5}$$

$$a(y,z) = 1-zy^2, \quad b(y,z) = \frac{y^2\left((5-y^2)z^2-8z+5\right)-1}{2(1-z)y} \tag{2.6}$$

Now the integration is straightforward. All our calculations were tested with Mathematica [19]. We finally obtain the total cross section for the two lepton pair production in $\gamma\gamma$ at infinite energy :

$$\sigma^{\gamma\gamma\to 2l2L} = \frac{4\alpha^4}{9\pi mm'}\left\{\frac{19}{16}\left[2\left(\frac{1}{u}-u\right)\ln u-\left(\frac{1}{u}+u\right)\left(2+\ln^2 u\right)\right]+\left[\frac{25}{4}+\frac{19}{32}\left(\frac{1}{u}-u\right)^2\right]P(u)\right\} \tag{2.7}$$

where

$$P(u) = P\left(\frac{1}{u}\right) = \Lambda_3(u)-\Lambda_3(-u), \quad \Lambda_n(z) = \int_0^z \frac{\ln^{n-1}|t|}{1+t}\,dt \text{ (Kummer function [13])} \tag{2.8}$$

$$P(u) = \ln^2(u)\ln\left(\frac{1+u}{1-u}\right)-2\ln(u)\left(Li_2(u)-Li_2(-u)\right)+2\left(Li_3(u)-Li_3(-u)\right), u=\frac{m'}{m}\le 1 \tag{2.9}$$

When the two masses are very different, i.e. $m \gg m'$ in our case, the cross section given by Eq. (2.10)

$$\sigma \simeq \frac{28\alpha^4}{27\pi m^2}\left(\ln^2 u^2-\frac{103}{21}\ln u^2+\frac{485}{63}\right) \tag{2.10}$$

is in agreement with the computation of [7] . For equal masses, dividing by 2 the expression obtained in order to take into account the effect of identical particles, we get :

$$\sigma = \frac{\alpha^4}{\pi m^2}\left(\frac{175}{36}\zeta(3)-\frac{19}{18}\right) \tag{2.11}$$

This result (2.11) coincides with the well-known formula for identical pair production [1, 2, 20].

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2.2 Lepton and charged pion or kaon pairs production

In order to obtain an analytical asymptotic formula for the $\gamma\gamma$ production of lepton and pion pairs we compute the transverse (Eq. (2.12)) and longitudinal (Eq. (2.13)) cross sections of virtual photoproduction of two charged pions depicted as scalar point-like particles in QED [8].

$$\sigma_T^{\gamma\gamma^\star\to\pi^+\pi^-} = \frac{2\pi\alpha^2\beta W^2}{(W^2+t)^2}\left[2-\beta^2+\frac{t^2}{W^4}+\frac{-1+\beta^4+\frac{2t}{W^2}-\frac{2t}{W^2}\beta^2}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right)\right] \quad (2.12)$$

$$\sigma_L^{\gamma\gamma^\star\to\pi^+\pi^-} = \frac{4\pi\alpha^2\beta t}{(W^2+t)^2}\left[-3+\frac{3-\beta^2}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right)\right], \quad L=\ln(\frac{1+\beta}{1-\beta}), \quad \beta=\sqrt{1-\frac{4m_\pi^2}{W^2}} \quad (2.13)$$

The transverse and longitudinal cross sections of virtual photoproduction of two leptons are given by Eq. (1.2) and Eq. (1.3). At the impact factor approximation the Eq. (2.12) can be replaced by the Eq. (2.14), below, wherein the contact term which gives a zero contribution to the approximation of the impact factor has been omitted.

$$\sigma_T^{\gamma\gamma^\star\to\pi^+\pi^-} = \frac{2\pi\alpha^2\beta W^2}{(W^2+t)^2}\left[3-\beta^2+\frac{-3+2\beta^2+\beta^4}{2\beta}\ln\left(\frac{1+\beta}{1-\beta}\right)\right] \quad (2.14)$$

Using the change of variables depicted by Eq. (2.3) the integration of Eq. (1.5) can be made and the total cross section for the production of lepton and pion pair at infinite energy is obtained taking $u=\frac{m_l}{m_\pi}$ (m_l is the lepton mass) :

$$\sigma^{\gamma\gamma\to 2l2\pi} = \frac{\alpha^4}{72\pi m_\pi m_l}\left[-2\left(\frac{19}{u}+5u\right)\ln u+\left(\frac{19}{u}-5u\right)\left(2+\ln^2 u\right)+\left(\frac{5u^2}{2}+27-\frac{19}{2u^2}\right)P(u)\right] \quad (2.15)$$

The production of lepton and kaon pairs is also described by Eq. (2.15), above, exchanging pion mass with kaon mass. When the two masses are very different, which is the case of electron and pion pair production ($m_\pi \gg m_e$), we obtain Eq. (2.16) below

$$\sigma \simeq \frac{16\alpha^4}{27\pi m_\pi^2}\left[\ln^2\left(\frac{m_e}{m_\pi}\right)-\frac{8}{3}\ln\left(\frac{m_e}{m_\pi}\right)+\frac{163}{72}\right] \quad (2.16)$$

which is in agreement with [8].

2.3 Charged pion and kaon pair production

Function of $u=\frac{m_\pi}{m_K}$, the total cross section for the production of charged kaon and charged pion pairs in $\gamma\gamma$ collisions at infinite energy (Eq. (2.17)) is obtained replacing Eq. (1.2) and Eq. (1.3) respectively by Eq. (2.12) and Eq. (2.13) with the kaon mass used instead of the pion mass.

$$\sigma^{\gamma\gamma\to 2\pi 2K} = \frac{5\alpha^4}{144\pi m_\pi m_K}\left[2\left(u-\frac{1}{u}\right)\ln u+\left(\frac{1}{u}+u\right)\left(2+\ln^2 u\right)+\left(\frac{4}{5}-\frac{1}{2}\left(\frac{1}{u}-u\right)^2\right)P(u)\right] \quad (2.17)$$

2.4 Two charged pion pair production

For equal masses, dividing by 2 the expression (2.17) in order to take into account the effect of identical particles, we get Eq. (2.18) below,

$$\sigma^{\gamma\gamma\to 4\pi} = \frac{\alpha^4}{144\pi m_\pi^2}\left(7\zeta(3)+10\right) \tag{2.18}$$

which coincides with the formula for identical charged pions pair production [3]

2.5 Finite size pion corrections

In order to take in account the finite size pion in the QED Born amplitude for the $\gamma^\star(k_1)\gamma(k_2)\to \pi^+(L)\pi^-(\bar{L})$ reaction we use an idea proposed by Poppe [14]. The QED Born amplitude $(M_{\lambda_1,\lambda_2}(QED))$, function on the incoming photon helicities (λ_1,λ_2), is corrected by an overall $\Omega\left(t',u',t\right)$ form factor depending of Mandelstan variables $t'=(k_1-L)^2$, $u'=(k_1-\bar{L})^2$, and the photon virtuality $t=-k_1^2$. So, the amplitude $(M_{\lambda_1,\lambda_2}(\text{Finite Size}))$ in the finite size model is given by :

$$M_{\lambda_1,\lambda_2}(\text{Finite Size}) = \Omega\left(t',u',t\right) M_{\lambda_1,\lambda_2}(\text{QED}) \tag{2.19}$$

with

$$\Omega\left(t',u',t\right) = \frac{x_0^2}{t'+u'-2m_\pi^2}\left(\frac{u'-m_\pi^2}{x_0^2+t+m_\pi^2-t'}+\frac{t'-m_\pi^2}{x_0^2+t+m_\pi^2-u'}\right) \tag{2.20}$$

where x_0 is the scale set by the pion form factor. Expressed in units of energy we take $x_0=m_\rho=0.770$ GeV/c^2. In the finite size model, the maximum value for transverse (longitudinal) cross section is respectively the order of 397(32) nb for $(W_{\pi\pi}, t)$=(0.310(0.562) GeV/c^2,0(0.135) GeV^2) (see the fig. 3). The finite size pion corrections dump the QED born amplitude. This effect becomes more significant when the incoming photon virtuality t increases. So, the transverse (longitudinal) cross section in the finite size model is lower than Born cross section, typically of 13%(23%) [18%(26%)] at transfert $t=m_e^2[m_\mu^2]$ increasing strongly with the tranfert t, typically of 84%(83%) for $t=m_\rho^2$ (see also fig. 3). All relative numerical values given above were calculated at a $W_{\pi\pi}$ value which maximized the transverse or the longitudinal cross section.
In the case of $e^+e^-(\mu^+\mu^-)\pi^+\pi^-$ production, if we use at one vertex the $\gamma\gamma^\star\to\pi^+\pi^-$ finite size model cross section instead of that Born, the numerical value of cross section is now 302(12) pb, lower than 42%(67%) (see the fig. 1) in agreement with the fact that the main contribution to the cross section given by logarithmically large contribution comes from the region [8] :

$$m^2_{e(\mu)} \ll t\left(\simeq \vec{q}_t^{\,2}\right) \ll \min\left(W_{\pi\pi}^2, m_\rho^2\right) \tag{2.21}$$

In the two charged pion pair production, for the finite size model the cross section is now 0.29 pb, 42 % lower than the crude Born model (see fig. 2).

3. Inclusive cross section in $\gamma\gamma$ collisions at infinite energy

Using the impact factor method the energy distribution of particles moving along the momentum of one photon is given by Eqs. (9-33) of reference [7] and can be written in the following form :

$$\frac{d\sigma_a}{dx_+ dx'_+} = \frac{4\alpha^4}{\pi} \int_0^\infty f\left(t, X, x_+\right) f'\left(t, X', x'_+\right) \, dt \tag{3.1}$$

with $f\,(f')$ function depends on lepton or scalar nature of the particle pair produced at one vertex :

$$f\left(t, X, x_+\right) = \begin{cases} \frac{x_+ x_-}{t} F_s(X) & \text{(scalar)} \\ \frac{-2x_+ x_-}{t} F_s(X) + \frac{X}{t} \ln \frac{1+X}{1-X} & \text{(lepton)} \end{cases} \tag{3.2}$$

where $x_+(x_-) \in [0,1]$, with $x_+ + x_- = 1$, is the incoming first photon energy fraction carried by the positive (negative) particle of the pair produced at one vertex. The function f' and the variables $x'_+(x'_-)$ are similarly defined for particle pair produced by the second incoming photon at the other vertex.

For the two different pairs of charged scalar particles production (S and S'), du to the $x_\pm$ dependence in the scalar case of formula (3.2) also available for $x'_\pm$, we can use the method exposed in section (2.2) and obtain :

$$\frac{d\sigma_a^{\gamma\gamma \to S^+ S^- S'^+ S'^-}}{dx_+ dx'_+} = 18 x_+ x_- x'_+ x'_- \, \sigma^{\gamma\gamma \to S^+ S^- S'^+ S'^-} \tag{3.3}$$

Integrating Eq. (3.3) over x_+ and x'_+ and multiply by a factor 2 to take into account the exchange particle pairs between vertices (see reference [7]) we find the total cross section (2.17).

For the scalar particle and lepton pair production (S and l'), using the fraction energy dependence in the lepton case of formula (3.2) and the result (3.3), we obtain :

$$\frac{d\sigma_a^{\gamma\gamma \to S^+ S^- l'^+ l'^-}}{dx_+ dx'_+} = 3 x_+ x_- \left[\sigma^{\gamma\gamma \to S^+ S^- l'^+ l'^-} + 2\left(1 - 6x'_+ x'_-\right) \sigma_{\text{scal.}(l')}^{\gamma\gamma \to S^+ S^- l'^+ l'^-} \right] \tag{3.4}$$

where $\sigma^{\gamma\gamma \to S^+ S^- l'^+ l'^-}$ is given by the formula (2.15) and $\sigma_{\text{scal.}(l')}^{\gamma\gamma \to S^+ S^- l'^+ l'^-}$ is the total cross section of scalar particle and lepton pair production in the case where the leptons (l') are treated as scalar particles and its expression is given by the formula (2.17) where m' is the lepton mass and m the scalar particle mass.

Integrating Eq. (3.4) over the lepton variable x'_+ we get :

$$\frac{d\sigma_a^{\gamma\gamma \to S^+ S^- l'^+ l'^-}}{dx_+} = 3 x_+ x_- \sigma^{\gamma\gamma \to S^+ S^- l'^+ l'^-} \tag{3.5}$$

which where $m \gg m'$ gives :

$$\frac{d\sigma_a^{\gamma\gamma \to S^+ S^- l'^+ l'^-}}{dx_+} \simeq \frac{4\alpha^4}{9\pi m^2} x_+ x_- \left[\ln^2 u^2 - \frac{16}{3} \ln u^2 + \frac{163}{18} \right] \tag{3.6}$$

and is in agreement with Eq. (14) of reference [8] and in the case of $\pi^+\pi^-e^+e^-$ production agrees with the exact expression (3.5) within a relative accuracy of 5 10^{-6}.
For the two different pairs of leptons production (l and l'), using the same method of calculation expressions (3.3) and (3.4) we get :

$$\frac{d\sigma_a^{\gamma\gamma\to l^+l^-l'^+l'^-}}{dx_+dx'_+} = \frac{1}{2}\sigma^{\gamma\gamma\to l^+l^-l'^+l'^-} + (1-6x_+x_-)\,\sigma_{\text{scal.(l)}}^{\gamma\gamma\to l^+l^-l'^+l'^-} + \left(1-6x'_+x'_-\right)\sigma_{\text{scal.(l')}}^{\gamma\gamma\to l^+l^-l'^+l'^-} + 2\,(1-6x_+x_-)\left(1-6x'_+x'_-\right)\sigma_{\text{scal.(l,l')}}^{\gamma\gamma\to l^+l^-l'^+l'^-}$$

where $\sigma^{\gamma\gamma\to l^+l^-l'^+l'^-}$ is the QED total cross section of two different lepton pair production given in reference [11], $\sigma_{\text{scal.(l)}}^{\gamma\gamma\to l^+l^-l'^+l'^-}$ ($\sigma_{\text{scal.(l')}}^{\gamma\gamma\to l^+l^-l'^+l'^-}$) the total cross section of two different lepton pair production in the case where the leptons l (l') are treated as scalar particles computed using the formula (2.15) and $\sigma_{\text{scal.(l,l')}}^{\gamma\gamma\to l^+l^-l'^+l'^-}$ is the total cross section of two different lepton pair production in the case where the leptons l and l' are treated as scalar particles and its expression is given by the formula (2.17) where m (m') is the mass of lepton l (l').
For better comparison with the reference [7], the expression (3.7) can be written in the form :

$$\frac{d\sigma_a^{\gamma\gamma\to l^+l^-l'^+l'^-}}{dx_+dx'_+} = a - x_+x_-b - x'_+x'_-c + x_+x_-x'_+x'_-d \tag{3.7}$$

with

$$\begin{aligned}
a &= \frac{1}{2}\sigma^{\gamma\gamma\to l^+l^-l'^+l'^-} + \sigma_{\text{scal.(l)}}^{\gamma\gamma\to l^+l^-l'^+l'^-} + \sigma_{\text{scal.(l')}}^{\gamma\gamma\to l^+l^-l'^+l'^-} + 2\sigma_{\text{scal.(l,l')}}^{\gamma\gamma\to l^+l^-l'^+l'^-}\\
b &= 6\sigma_{\text{scal.(l)}}^{\gamma\gamma\to l^+l^-l'^+l'^-} + 12\sigma_{\text{scal.(l,l')}}^{\gamma\gamma\to l^+l^-l'^+l'^-}\\
c &= 6\sigma_{\text{scal.(l')}}^{\gamma\gamma\to l^+l^-l'^+l'^-} + 12\sigma_{\text{scal.(l,l')}}^{\gamma\gamma\to l^+l^-l'^+l'^-}\\
d &= 72\sigma_{\text{scal.(l,l')}}^{\gamma\gamma\to l^+l^-l'^+l'^-}
\end{aligned} \tag{3.8}$$

which when $m \gg m'$ give :

$$\begin{aligned}
a &\simeq \frac{8\alpha^4}{\pi m^2}\left(\frac{1}{8}\ln^2 u^2 - \frac{1}{2}\ln u^2 + 1\right)\\
b &\simeq \frac{8\alpha^4}{\pi m^2}\left(\frac{1}{6}\ln^2 u^2 - \frac{13}{18}\ln u^2 + \frac{40}{27}\right)\\
c &\simeq \frac{8\alpha^4}{\pi m^2}\left(\frac{1}{4}\ln^2 u^2 - \frac{1}{2}\ln u^2 + 2\right)\\
d &\simeq \frac{8\alpha^4}{\pi m^2}\left(\frac{1}{3}\ln^2 u^2 - \frac{7}{9}\ln u^2 + \frac{77}{27}\right)
\end{aligned} \tag{3.9}$$

and are in agreement with the expression (37) of reference [7]. In the case of $\mu^+\mu^-e^+e^-$ production use the formula (3.7) instead of the exact (3.7) leads a relative accuracy lower than 2 10^{-5} (see upper and lower part in fig. 4).

4. Pseudo pairs configuration Monte Carlo

In order to estimate the experimental production rate of two pairs including cuts, we have built a Monte-Carlo fully integrated in ROOT [15]. The pseudo pairs configuration phase space event is made using the Cellular Monte Carlo Event Generator FOAM [16]. In the case of electron and muon pairs production in $\gamma\gamma$ collision, upper right in fig. 6 shows perfect agreement between first, Helicity Amplitudes computed without approximation (red triangles) and the numerical integration of the factorization formulae (1.1) (blue and pink (with invariant mass cut) lines), second Impact Factor Method which gives us the dominant term at low angle and high energy (black points) and formula (2.7) (green curve) at infinite energy.

4.1 ILC result's

As an example of our Monte Carlo results, we show in fig. 5, for a future linear collider, the pseudo-rapidity distribution (η). This is an important variable that allows to check if the pairs are contained in the detector. Fig. 5 shows that pions ($|\eta_{\mathrm{MAX.}}| \simeq 4$ (top, middle, lower figure, blue line)), electrons ($|\eta_{\mathrm{MAX.}}| \simeq 6.5$ (top figure,red line)), muons ($|\eta_{\mathrm{MAX.}}| \simeq 4$ (middle figure, red line)) and products of $u\bar{u}$ fragmentation (lower figure, green line) can be seen in very forward detector (FTD and VTX). As a conclusion, due to the mass of particles composing the pseudo pairs, many particles are produced in the beam pipe, but a significant fraction can be seen at low angle. The cross section was computed using the expression (4.1) below

$$\sigma = \int_{z_{min}}^{z_{max}} dz\, 2z \int_{\frac{z^2}{z_{max}}}^{z_{max}} \frac{dy}{y}\, f_{\gamma/e}(y) f_{\gamma/e}(\frac{z^2}{y}) \sigma^{\gamma\gamma \to\ \pi^+\pi^-l^+l^-}\left(z\sqrt{S_{e^+e^-}}\right), \quad y = \frac{E_\gamma}{E_{beam}} \tag{4.1}$$

where $f_{\gamma/e}(y)$ is the equivalent photon approximation flux [6, 10]. Fig. 6 lower right shows the production of pion and muon pairs as a function of electron-positron center of mass energy. This kind of plot is relevant when the energy $\sqrt{S_{e^+e^-}}$ is high as this is the case for a the future linear collider. We think that in an environment where the production of pions is very high, due to a lot of production processes, lepton pairs can be used for tagging pion pairs coming from this particular mode of production. But as we see in fig. 6 lower right, there is a strong dependence of the visible cross section to the angular and energy cuts applied. Event if we do not want to detect specially this kind of $\gamma\gamma$ production, more studies have to be made, because this is a potential background for detectors at very low angle, and for the time being, it is never taken into account.

4.2 LHC result's

To study mechanisms of pion pair production at LHC in $\gamma\gamma \to\ \pi^+\pi^-l^+l^-$, the cross section is computed in the following way

$$\sigma = \int_{z_{min}}^{z_{max}} dz 2z \int_{\frac{z^2}{z_{max}}}^{z_{max}} \frac{dy}{y} f_{\gamma/p}(y) f_{\gamma/p}(\frac{z^2}{y}) \sigma^{\gamma\gamma \to\ \pi^+\pi^-l^+l^-}\left(W_{\gamma\gamma}\right) \tag{4.2}$$

where

$$f_{\gamma/p}(y) = f_{\gamma(el)/p}(y) + f_{\gamma(inel)/p,Q^2}(y) \tag{4.3}$$

$y = \frac{E_\gamma}{E_{beam}}$ and Q^2 is the resolution scale at which the proton is probed. We use for the photon content of the proton, the elastic and inelastic contributions given respectively by [17] and [18]. Fig. 6 upper left, shows few events, depending strongly on experimental cuts. But in the elastic case the signature is clear. To proceed and conclude on a realistic estimation of the number of events, we need the simulation of all background event in order to reject them. The use of Roman Pots to tag the proton can help a lot, but the statistics can decrease drastically.

In the case where the pion pair is accompanied by a beauty pair, for example see lower left part of fig. 6, we use for the gluon content of the proton, the CTEQ6 parton density function and the expression of the central cross section is :

$$\sigma^{\gamma g \to \pi^+\pi^- Q\bar{Q}} = \frac{1}{8} 4 e_Q^2 \frac{\alpha_s}{\alpha} \frac{1}{2} \sigma^{\gamma\gamma \to l\bar{l}Q\bar{Q}} \quad (4.4)$$

We need also in this case a realistic LHC estimation of background (pion decay in flight, simulation, pile-up ,$\cdots$) to see if we can extract this type of signal. Anyway this kind of events has to be estimated, because the pseudo pair configuration can lead to some "strange" events. Considering a pion pair and a $Q\bar{Q}$ pair, produced at low angle, back to back in the γg center of mass, once boosted to the laboratory system, some pion pairs might escape the detector, while $Q\bar{Q}$ pairs are still visible from one side of the detector.

5. Conclusions

For the first time, at our knowledge, we have obtained analytical asymptotic formulae (2.7, 2.15, 2.17) at Born level for the $\gamma\gamma$ production of two lepton pairs, pion and lepton pairs and pion and kaon pairs respectively, without any approximation on particle masses. In the case where the masses are equal or very different these formulas (2.10, 2.11, 2.16, 2.18) coincide with the literature. We think that this work is the basis for further studies, including the threshold behavior and the global approach of QED production of four leptons or mixed QED QCD processes at future colliders, as well as the gluon-gluon case which is under study. We have also presented a Monte-Carlo which can generate this type of pseudo-pair events. The studies are in progress in order to evaluate some realistic numbers of expected event at LHC, PLC and the future colliders.

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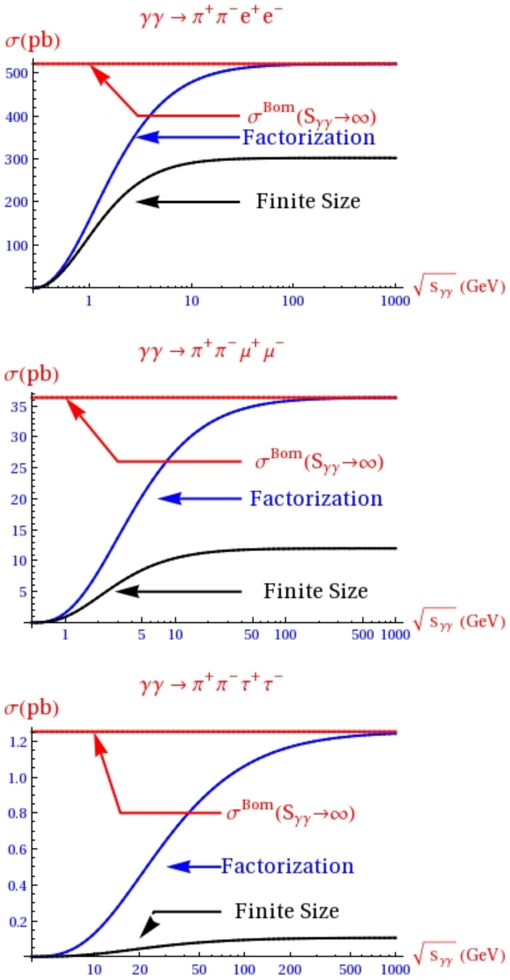

Figure 1: $\gamma\gamma$ production of pion and lepton pair - formula (2.15) (red curve)- numerical integration of formula (1.1) in Born approximation (blue curve)- numerical integration of formula (1.1) using finite size model (black curve).

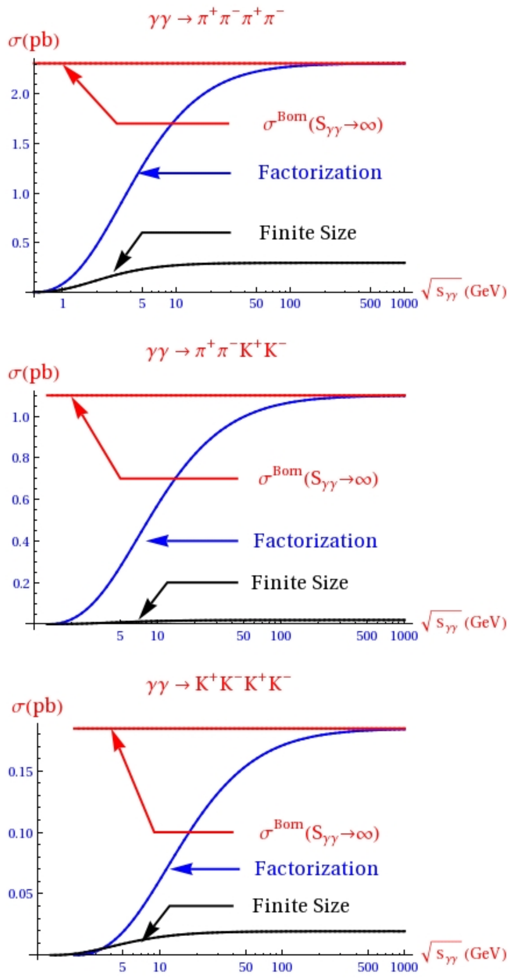

Figure 2: $\gamma\gamma$ production of pion and kaon pairs - formulas (2.18) and (2.17) (red curve)- numerical integration of formula (1.1) in Born approximation (blue curve)- numerical integration of formula (1.1) using finite size model (black curve).

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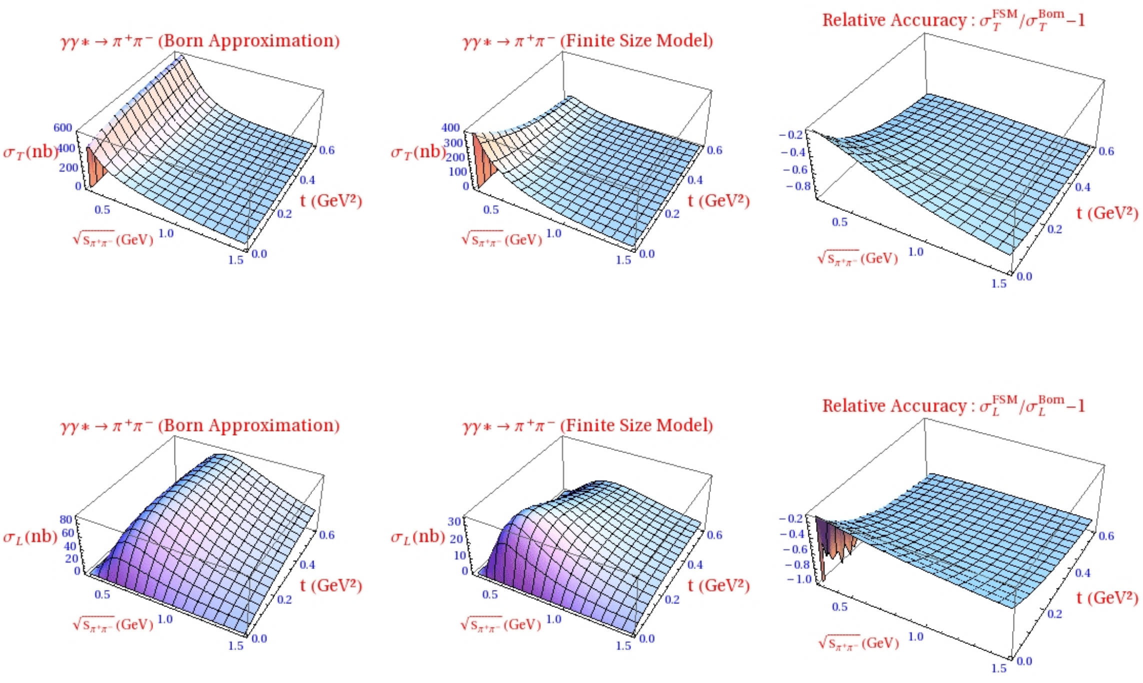

Figure 3: Transversal and longitudinal cross section in Born approximation and In the Finite Size Model and relative accuracy (details are explained in the text).

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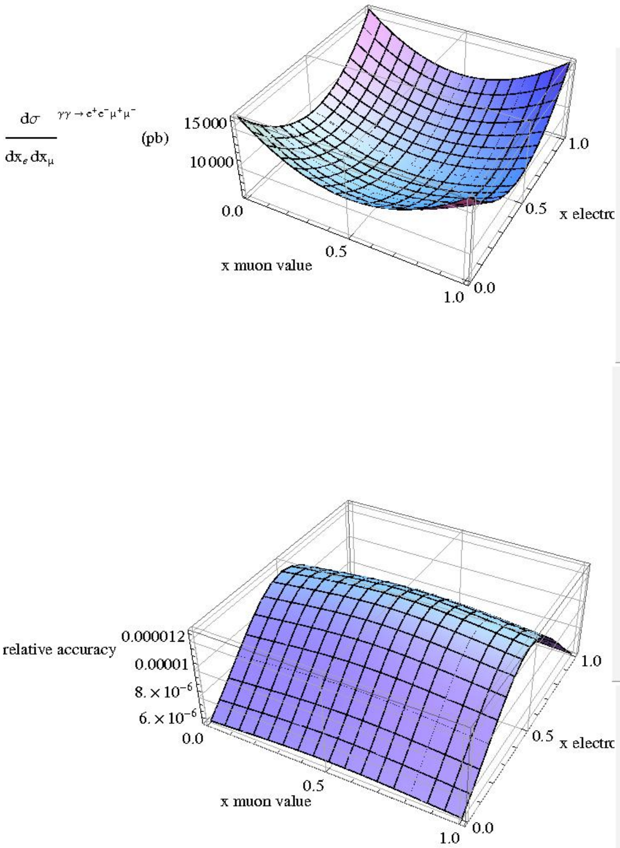

Figure 4: upper: Inclusive cross section in $\gamma\gamma$ collisions at infinite energy :$\mu^+\mu^-e^+e^-$ - formula (3.7)- lower : relative accuracy of $\mu^+\mu^-e^+e^-$ production if we use the formulae (3.7) instead of the exact (3.7)

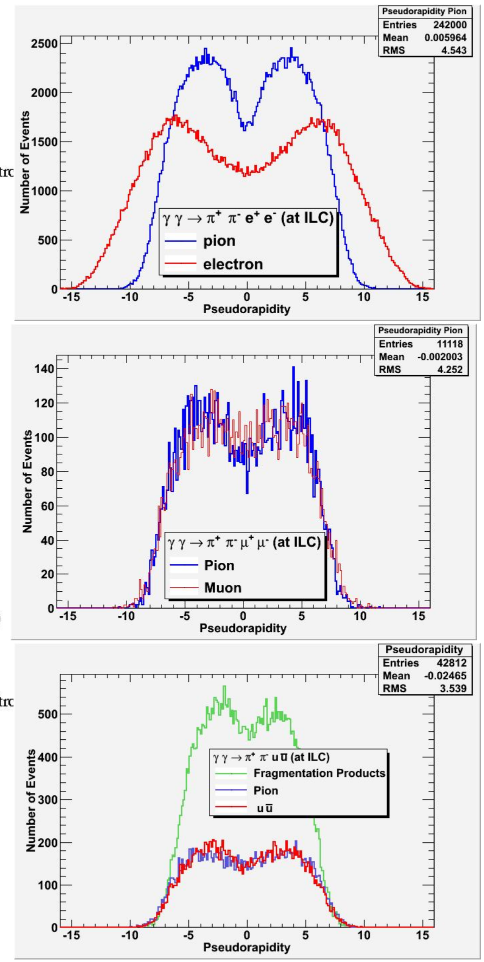

Figure 5: Pion and lepton (or u-quark) pair production at ILC (details are explained in the text)

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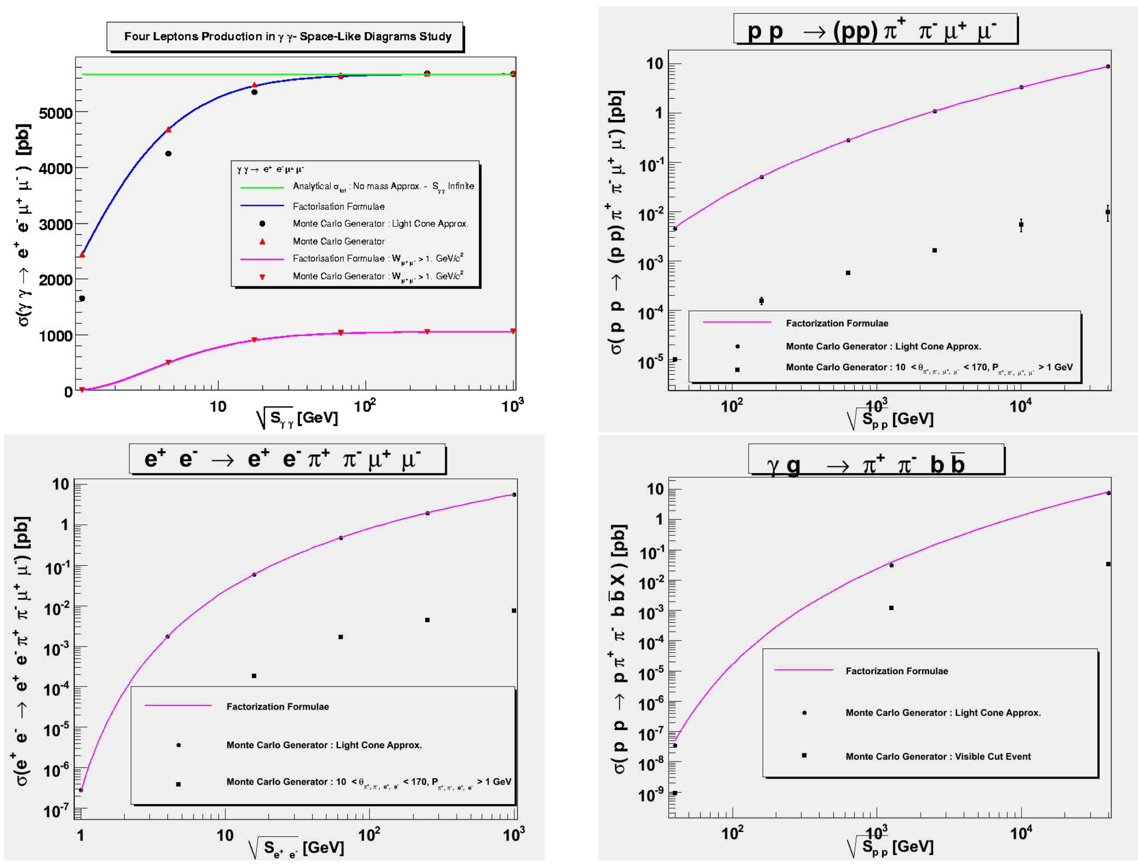

Figure 6: Electron and muon pair production at PLC (upper right) - pion and muon pair production at ILC (lower right) - pion and muon pair production at LHC (upper left) - pion and beauty quark pair production at LHC (lower left) - details are explained in the text.