PROCEEDINGS OF SCIENCE

Form factor and width of a quantum string

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We show how the form factor for a quantum string can be obtained from field correlation functions calculated in lattice Monte Carlo simulations. As an example, we apply this technique for simulations of the Ising model. We demonstrate that the form factor shows the same logarithmic broadening as observed by other quantities. Various difficulties in finding the intrinsic width of a string are discussed.

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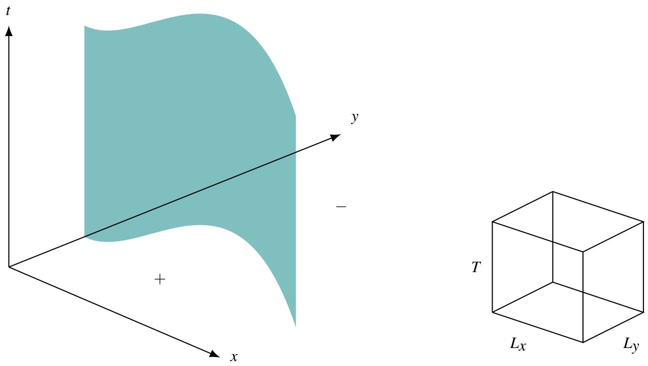

Figure 1: System setup. The twisted boundary conditions in the y direction create a domain wall, shaded.

1. Introduction

The form factor of a string is a well-defined quantum mechanical observable, which characterises the string's interaction with elementary particles. What properties would this form factor have? It provides a generalisation from the intrinsic width of a string [1], to a quantity that can be studied at various length scales. The intrinsic width is usually measured in a manner that is only sensitive to fluctuations in the string position, such as by measuring the expectation value of energy density in the vicinity of the string (see for example Ref. [2] for the Ising model and Ref. [3] for Yang-Mills theory). There has been some discussion on whether other observables could improve on this [4, 5].

In the present work we use the Ising model to test our ideas about string form factors [6]. Because it is exactly dual to the confining three-dimensional $\mathbb{Z}_2$ gauge theory we know that our conclusions about domain walls in the Ising model should also be at least qualitatively correct for confining strings.

The system has Hamiltonian

$$H = -\sum_{\langle \mathbf{x},\mathbf{x}' \rangle} k_{\mathbf{x},\mathbf{x}'} s(\mathbf{x}) s(\mathbf{x}'); \tag{1.1}$$

the spins $s(\mathbf{x})$ take the values ± 1. The sum is over all nearest-neighbour links. To create twisted boundary conditions we set $k_{\mathbf{x},\mathbf{x}'} = -1$ for one value of y and $+1$ everywhere else. The result is a domain wall, shown in Fig. 1.

2. String form factor and width

Let us write the string ground state as $|0\rangle$. It has zero momentum and linearly divergent energy $E_0 = \sigma L$ in the infinite-volume limit, where σ is the string tension. We can boost the string so that it has a momentum k, yielding the state $|k\rangle$, the energy of such a state being $E_k = \sqrt{k^2 + E_0^2}$. We can then define the form factor f as the matrix element $f_{\mathscr{O}}(k) = \langle k|\mathscr{O}(0)|0\rangle$ which, in the semiclassical limit [7], corresponds to the Fourier transform of $\mathscr{O}(x)$. For a step function-like domain wall, we obtain $f_s(k) = v(2i/k)$, where v is the vev.

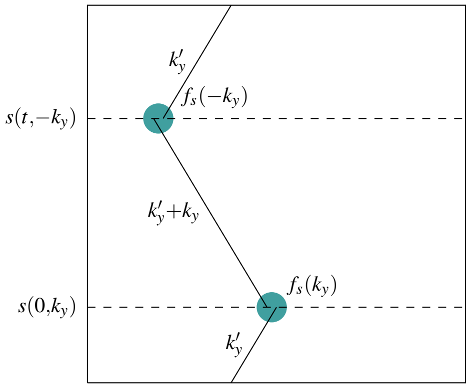

Figure 2: Schematic representation of the correlator $\langle s(0,k_y)s(t,-k_y)\rangle$. The interaction between the correlator and the domain wall is represented by the shaded circles at $t=0$ and $t=t$. The amplitude of this interaction is parametrised by a form factor $f_s(k_y)$, and the momentum supplied causes the domain wall to recoil.

In 1+1 dimensions, there is only one zero mode and at weak coupling the result is straightforward [8], but in 2+1 dimensions the additional fluctuations along the domain wall apply a multiplicative broadening factor, hence

$$|f(k)| \approx |f_0(k)|e^{-\frac{1}{2}k^2w^2}. \tag{2.1}$$

At long distances the intrinsic string behaviour must be recovered, so we require that $f_0(k) \to v(2i/k)$ as $k \to 0$, regardless of the microscopic details. The apparent width w is then given by

$$w^2 = \frac{1}{2\pi\sigma}\ln\frac{L}{c\xi}, \tag{2.2}$$

where ξ is the correlation length. Hence any intrinsic thickness must be inferred from $f_0(k)$, which is exponentially suppressed. The purpose of the current work is to confirm this picture of string broadening and establish whether, for the Ising model, any intrinsic structure can be discerned.

3. Correlators

We wish to measure the form factor f using translationally invariant correlators. Fortunately, finite-size effects in the t-direction and conservation of momentum can be exploited to find the form of the correlator at nonzero momentum in the presence of a domain wall. Restricting ourselves to the case where momentum is transferred entirely to the domain wall, we find

$$\langle s(0,k_y)s(t,-k_y)\rangle \approx |f(k_y)|^2 e^{-\frac{k_y^2}{2E_0}\frac{t(T-t)}{T}}, \tag{3.1}$$

in other words, the moving wall has energy $k_y^2/2E_0$.

States which excite Goldstone modes will have energy at least $4\pi/L$ higher, and will be suppressed at longer distances $t \gtrsim L$. We anticipate therefore a clean signal for the form factor. The qualitative behaviour of the correlator is shown in Fig. 2. In addition to this pure contribution from

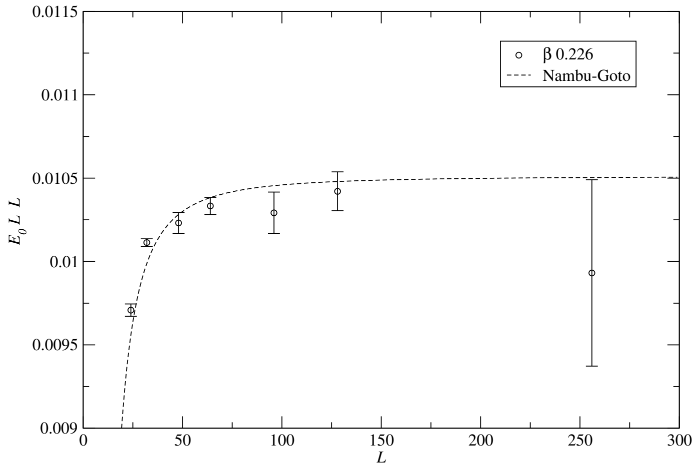

Figure 3: For $\beta = 0.226$ and various L, energy per unit length of the string measured using the correlator method. The exact Nambu-Goto result is also shown.

the movement of the domain wall, the correlator can excite particle states on either side of the wall, but these are more massive by a factor of at least the mass m of the scalar excitation in the model, and hence are also suppressed at long distances.

4. Results

First, we put Eq. (3.1) to use measuring the string tension as a function of L. For a closed Nambu-Goto string, the ground state energy is

$$E_0(L) = \sigma L \sqrt{1 - \frac{\pi}{3\sigma L^2}}. \tag{4.1}$$

For small k and large T, we can fit correlator data to Eq. (3.1) to obtain E_0, shown in Fig. 3. As with our previous calculations of the masses of kinks and monopoles, the fitting of the correlator Eq. (3.1) to lattice data is not necessarily the cleanest method of making this measurement; it is highly sensitive to metastability. As kinks, domain walls, strings and solitons are all extended objects, the measurement of observables associated with their motion can suffer from long autocorrelation times. Employing a cluster algorithm for the present simulations helps to mitigate this somewhat. Nevertheless, successfully measuring the string tension in the system with twisted boundary conditions is an important test of our calculation.

Next, as a further check that our measurement works correctly, we divide out by the exponential factor in Eq. (3.1) and obtain measurements of $|f(k)|$. A few examples of this calculation are shown in Fig. 4.

We carry out simulations for various L and β, then scale the resulting data by $\sqrt{\sigma} \propto \xi^{-1}$. This would be expected to collapse curves for different k and L onto a single line if the domain wall's intrinsic nature – whether infinitesimally thin like a step function, or broad like a kink – was the only phenomenon present. Its failure to do so leads us to conclude that broadening of the form Eq. (2.1) is taking place – at least at low k – and that w^2 depends on L.

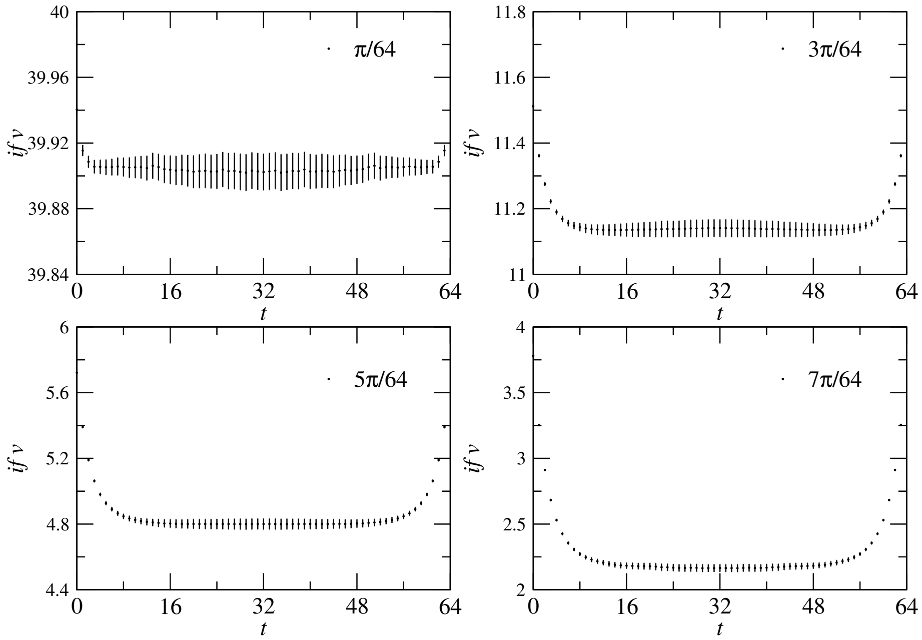

Figure 4: Measured $f(k)$ as a function of t for $\beta = 0.23$, $L = 64$ for several k. Note that the data presented here are *not* the raw correlator measurements for the states in question, but rather estimators of the form factor after the exponential weight has been removed. There is a 'plateau' at long distances; at short distances the correlator is dominated by excited states.

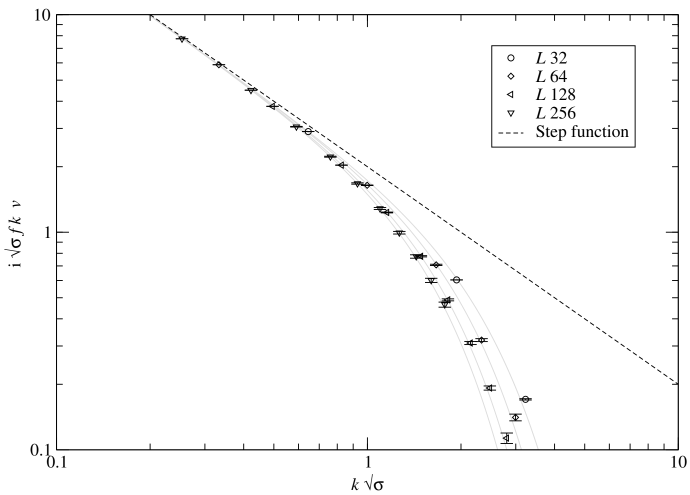

Figure 5: Form factor results for $\beta = 0.23$ and various L. Scaled by $\sqrt{\sigma} \propto \xi^{-1}$ the data do not fall onto a single curve, as one would have expected for point-like defects without any broadening by fluctuations.

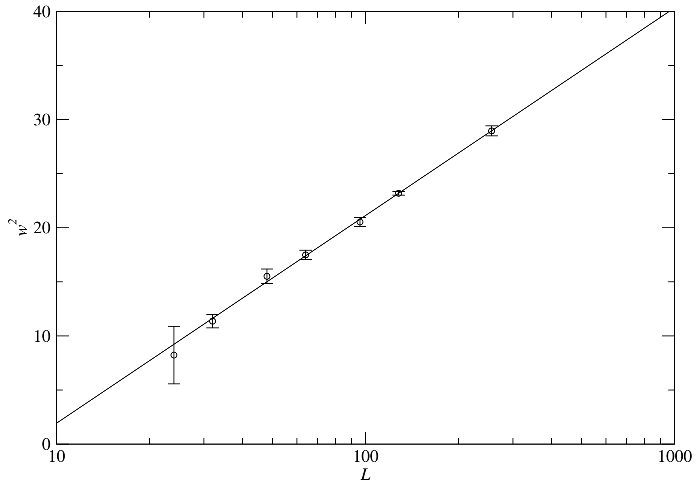

Figure 6: w^2 obtained by fitting our ansatz to the correlator data for different k. The data show excellent agreement with logarithmic broadening.

Also shown in Fig. 5 are fits to Eq. (2.1) assuming no 'intrinsic' width, in other words $f_0(k) = 2iv/k$ for all k. In such a fit the only free parameter is the broadened width $w(L)$. Plotting this quantity demonstrates unambiguously that this broadening is logarithmic, as shown in Fig. 6.

In the current simulations the data are not good enough to test the assumption that the domain walls are infinitesimally thin; one would probably need to go to larger volumes and higher k to see any discernible effect, and in any case far better statistics than are presented here would be required. The data presented here required about 10k hours of CPU time to produce, and a considerably larger investment of resources would be required to study the intrinsic width, if it exists.

5. Conclusions

We have demonstrated how we can use two-point correlators to measure the string tension and form factor – physically meaningful and, in principle, experimentally measurable quantities. The results for the string tension are in good agreement with theory, while the measured form factor is consistent with a string that is broadened logarithmically in the manner of Eq. (2.2). Hence we have demonstrated the broadening of the confining string using a novel, translationally invariant method. Unfortunately, we have insufficient data for form factors at high momenta to establish whether the string has an intrinsic width.

Further work is required to establish the applicability of the techniques used here to confining strings in SU(N) gauge theory.

Acknowledgments

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